Can Borrowing Costs Explain the Consumption Hump?

Nick L. Guo*

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Abstract

In this paper, a wedge between borrowing and saving interest rates is incorporated into an otherwise standard life cycle consumption-saving model. This model can generate both the right size and the right location of the consumption hump over the life cycle. The model does not rely on any of the known mechanisms for producing a consumption hump. The work in this paper highlights a simple but strong mechanism to account for the consumption hump. Borrowing and a wedge between borrowing and saving interest rates, therefore, should be included in many life cycle models pertaining to consumption and saving.

Key words: Borrowing Costs, Consumption Hump.

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*University of Wisconsin-Whitewater. Tel.: +1 262 472 7023. Email: nickguo@gmail.com
1 Introduction

In this paper, I ask a simple question: In a model economy, can we explain the consumption hump over the life cycle purely on the basis of a difference between borrowing and saving interest rates? The answer to that question is: Yes.

It has been well documented that the consumption profile of a typical U.S. household increases in the early years of life, peaks around age 50, and then decreases gradually thereafter (see Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2007) among others). The location of the peak typically falls between ages 45-55. The size of the peak, defined as consumption at the peak divided by consumption when the agent is young, is estimated to lie between 1.1 and 1.25. The hump-shaped consumption pattern is robust even after controlling many factors: Gourinchas and Parker (2002) show that the hump is not affected when adjusting for family size; and Fernández-Villaverde and Krueger (2011) find a hump in consumption even after controlling for spending on durable goods.

The hump-shaped consumption profile over the life cycle cannot be generated in the simplest form of the life cycle model with complete markets and rational agents. In a complete market, the agent smooths her consumption and the consumption profile should be monotone, depending on the relationship between the interest rate and the time discount rate. If the time discount rate is smaller (bigger) than the interest rate, consumption will increase (decrease) over time. Consumption will stay constant if the interest rate equals the time discount rate.

Since the interest rate is so crucial in shaping consumption over the life cycle, it's worth examining the empirical evidence on the actual interest rates that households face. In fact households face dual interest rates: the risk-free saving interest rate is significantly lower than the average rates when households take unsecured loans.\textsuperscript{1} And unsecured loans are largely taken by young households (see Athreya, Tam, and Young (2012) and Davis, Kubler, Willen (2006) report the historical wedge between the two interest rates around 6% – 9%.

\textsuperscript{1}Davis, Kubler, and Willen (2006) report the historical wedge between the two interest rates around 6% – 9%.
Motivated by these facts, this paper incorporates borrowing costs into an otherwise standard life cycle model. The paper asks whether borrowing costs, in isolation, can produce an empirically plausible consumption hump over the life cycle. It shows that, under reasonable specifications, as long as the time discount rate falls between the savings and borrowing rates, the model can generate the right size and the right location of the consumption hump over the life cycle.

The mechanism behind the model is intuitive. In the early ages, the agent borrows and the effective interest rate is the borrowing rate, which is bigger than the time discount rate. Thus, consumption increases in this phase. The agent postpones consumption in order to pay back loans as quickly as possible. During the saving phase, the interest rate on saving is applied, which is lower than the time discount factor. Thus, the agent will allocate more resources for consumption in the early portion of this phase. Hence there is a hump in consumption allocations over the life cycle.

This paper highlights a simple but strong mechanism to account for the consumption hump. The mechanism used in this paper does not require a hump-shaped wage profile, or any other features such as income or longevity uncertainty, or any form of bounded rationality. The result in this paper suggests that differential saving and borrowing interest rates should be considered in many life cycle models which study consumption and saving decisions.

The mechanism highlighted in this paper is a substitute for using borrowing constraints to explain the consumption hump (see Hubbard, Skinner, and Zeldes (1994) and Gourinchas and Parker (2002) among others), while remaining consistent with the stylized facts: 1) many U.S. households do borrow, and 2) many U.S. households already have debt when they start working. This paper contributes a channel that can be potentially combined with other possible explanations in order to fully account for empirical consumption patterns.²

²For instance, Bullard and Feigenbaum (2007) notice that substitutable leisure and consumption in preferences, together with a hump-shaped wage profile can deliver hump shaped consumption. Hansen and
2 Setup

In this model, time is continuous. The agent enters the world at time $t = 0$ and dies at $t = T$. The agent receives non-asset income $y(t)$, where $t \in [0, T]$ and she begins with debt ($B(0) > 0$) and with no savings ($S(0) = 0$). Savings grow at interest rate $r_s$, and debt grows at interest rate $r_b$, with $r_s < r_b$. At each time $t$, given savings account balance, $S(t)$, and borrowing account balance, $B(t)$, the agent chooses her consumption rate, $c(t)$, saving rate, $s(t)$, and borrowing rate, $b(t)$, in order to maximization her utility. To better illustrate the consumption dynamics over the life cycle, I assume that the instantaneous utility function is $\ln(c)$.

At time 0, an individual maximizes her life time utility by solving the following problem $(P0)$:

\[
(P0) : \max_{c(t),s(t),b(t)} \int_0^T e^{-\rho t} \ln(c(t))dt,
\]

where $\rho$ is the instantaneous discount rate, subject to:

\[
\frac{dS(t)}{dt} = r_s S(t) + s(t), \quad (2)
\]

\[
\frac{dB(t)}{dt} = r_b B(t) + b(t), \quad (3)
\]

\[
b(t) + y(t) = c(t) + s(t), \quad (4)
\]

with nonnegativity conditions:

\[
S(t), B(t) \geq 0 \quad \text{for} \quad \forall t \in [0, T], \quad (5)
\]

İmrohoroğlu (2008) and Feigenbaum (2008) use mortality risk as an explanation. Leaving rationality can also generate the right consumption profile, see for example Caliendo and Aadland (2007).
and with initial and terminal conditions

\[ S(0) = 0, B(0) > 0 \quad \text{given,} \quad S(T) = 0, B(T) = 0. \]  

(6)

Replacing consumption rate \( c \) by \( b + y - s \), problem \((P0)\) is equivalent to problem \((P1)\):

\[ (P1): \quad \max_{s(t), b(t)} \int_{0}^{T} e^{-\rho t} \ln (b(t) + y(t) - s(t)) \ dt, \]  

(7)

subject to conditions (2), (3), (5), and (6).

The necessary conditions of \((P1)\) include satisfaction of (2), (3), (5), (6), and:

\[
\begin{align*}
-\frac{e^{\rho t}}{b(t) + y(t) - s(t)} &+ \lambda_s(t) = 0, \\
\frac{e^{\rho t}}{b(t) + y(t) - s(t)} &+ \lambda_b(t) = 0, \\
\frac{d\lambda_s(t)}{dt} &=-r_s\lambda_s(t) - \mu_S(t), \\
\frac{d\lambda_b(t)}{dt} &=-r_b\lambda_b(t) - \mu_B(t), \\
\mu_S(t)S(t) &\geq 0, \quad \text{and} \quad \mu_S(t) \geq 0, \\
\mu_B(t)B(t) &\geq 0, \quad \text{and} \quad \mu_B(t) \geq 0,
\end{align*}
\]

(8), (9), (10), (11), (12), (13)

where \( \lambda_s, \lambda_b \) are costate variables for \( s, b \), while \( \mu_S, \mu_B \) are the multiplier functions for the nonnegativity conditions for \( S(t) \) and \( B(t) \).

The following lemmas characterize some of the properties of the solution.

**Lemma 1** Optimal functions \( S(t) \) and \( B(t) \) satisfy:

\[ S(t)B(t) = 0. \]  

(14)

Proof: At \( t = 0 \) or \( t = T \), (14) is satisfied. Suppose \( S(t) > 0 \) and \( B(t) > 0 \) for some
$t \in (0, T)$, then there is

$$\mu_S(t) = \mu_B(t) = 0,$$

(15)

according to necessary conditions (12) and (13). Insert (15) into the multiplier equations:

$$\frac{d\lambda_s(t)}{dt} = -r_s\lambda_s(t),$$

(16)

$$\frac{d\lambda_b(t)}{dt} = -r_b\lambda_b(t).$$

(17)

Optimality conditions (8) and (9) imply that:

$$\lambda_s(t) = -\lambda_b(t).$$

(18)

Together with (16) and (17), we have:

$$r_s\lambda_s(t) = r_b\lambda_s(t).$$

(19)

Since $r_b > r_s$, (19) implies that $\lambda_s(t) = 0$, which in turn implies that either $t = \infty$ or $b(t) + y(t) - s(t) = \infty$. It’s a contradiction. Q.E.D.

According to Lemma 1, there does not exist such a time that the agent has both savings and borrowing accounts positive. In the borrowing phase(s), the agent borrows, and she does not have a positive balance in savings. Similarly, in the saving phase(s), the agent only saves and has a borrowing balance equal to zero. Switch points might exist when either $B(t)$ or $S(t)$ changes from positive to zero. According to Seierstad and Sydsaeter (1977), the costate variables $\mu_S(t)$ and $\mu_B(t)$ may be discontinuous at the switch points. However, the following lemma proves continuity of the optimal consumption path.

**Lemma 2** The optimal consumption rate is continuous with respect to time.

Proof: Since $\lambda_s(t)$ is continuously differentiable, the optimality condition (8) implies the
continuity of consumption. Q.E.D.

Even though there might exist many switch points when the agent stops borrowing or stops saving, the following discussion will be focused on the case where there is only one switch in the optimal control. By “one switch case” we mean there exists a time \( \hat{T} \in (0, T) \), such that \( B(t) > 0 \) for \( t \in (0, \hat{T}) \) and that \( S(t) > 0 \) when \( t \in (\hat{T}, T) \). In this case, the agent pays back the debt in the first phase, and she then starts saving. The savings finally are dissolved at death. The rest of the paper will be focused on the “one switch” case for two reasons. First, the “one switch” case is consistent with the data. In the data, households hold debt when they are young before they start saving. Second, in the numerical examples shown below, the optimal control has the “one switch” property. In the following lemma, the necessary conditions to have “one switch” are laid out.

**Lemma 3** If there is only one switch in the savings and borrowing balances, then there exists a switch time \( \hat{T} \in (0, T) \), optimal control variables \( s(t) \) and \( b(t) \), state variables \( S(t) \) and \( B(t) \), costate variables \( \lambda(t) \), \( \mu_S(t) \), and \( \mu_B(t) \) such that: I) for \( t \in [0, \hat{T}) \),

\[
\frac{dB(t)}{dt} = r_b B(t) + b(t), \tag{20}
\]
\[
\frac{dS(t)}{dt} = 0, \quad S(t) = 0, \quad s(t) = 0, \tag{21}
\]
\[
e^{-\rho t} \frac{b(t) + y(t)}{B(t)} = \lambda(t), \tag{22}
\]
\[
\frac{d\lambda(t)}{dt} = -r_s \lambda(t) - \mu_S(t) = -r_b \lambda(t), \tag{23}
\]
\[
B(t) > 0, \quad \mu_B(t) = 0, \quad \mu_S \geq 0, \tag{24}
\]
and II) for \( t \in (\hat{T}, T) \),

\[
\frac{dS(t)}{dt} = r_s S(t) + s(t), \tag{25}
\]

\[
\frac{dB(t)}{dt} = 0, \quad B(t) = 0, \quad b(t) = 0, \tag{26}
\]

\[
e^{-\rho t} \frac{e^{-\rho t}}{y(t) - s(t)} = \lambda(t), \tag{27}
\]

\[
\frac{d\lambda(t)}{dt} = -r_s \lambda(t) = -r_b \lambda(t) + \mu_B(t), \tag{28}
\]

\[
S(t) > 0, \quad \mu_S(t) = 0, \quad \mu_B \geq 0, \tag{29}
\]

and that at \( T \): \( B(T) = S(T) = 0 \), and at the switch point \( \hat{T} \): \( \lambda(\hat{T}^-) = \lambda(\hat{T}^+) \).

The conditions (20)-(29) are derived from the necessary conditions for the original problem (P1) and the definition of “one switch”. Since we have strictly concave utility, and a convex constraint, the conditions (20)-(29) should also be sufficient.

Given the properties of the optimal control, I simplify the agent’s optimal control problem with one control variable: the bank balance or net worth of wealth \( k(t) \). If the bank account from the last period is negative \( (k(t-) < 0^3) \), i.e., the individual was in debt, then the debt grows at the borrowing rate \( r_b \). If the bank account from the last period is positive \( (k(t-) > 0) \), then the savings grow at rate \( r_s \). The individual starts life with a negative balance in the savings account \( (k(0) < 0) \). An individual maximizes her lifetime utility by solving the following problem

\[
(P2): \max \int_0^T e^{-\rho t} \ln(c(t)) dt, \tag{30}
\]

subject to

\[
\frac{dk(t)}{dt} = r_b k(t) + y(t) - c(t), \text{ if } k(t-) < 0, \tag{31}
\]

\[
\frac{dk(t)}{dt} = r_s k(t) + y(t) - c(t), \text{ if } k(t-) > 0, \tag{32}
\]

\(^3k(t-)\) is formally defined as \( \lim_{\tau \to t} k(\tau) \)
where \( k(t-) \) is the balance in the bank account when it’s infinitely close and before time \( t \) for all \( t \in (0, \bar{T}) \), and
\[
k(0) < 0 \quad \text{given, } k(\bar{T}) = 0. \tag{33}
\]

Upon solving this system given the right parameterizations, we will be able to study the shape of the consumption profile over the life cycle.

3 Main result

**Proposition 1** If
\[
r_s < \rho < r_b
\]
and \((P0)-(P2)\) has the “one switch” property, then consumption is hump shaped over the life cycle.

**Proof.** The necessary and sufficient conditions for “one switch” optimal control are that there exists a switch time \( \hat{T} \), costate variable \( \lambda(t) \), optimal control \( c(t) \), and state \( k(t) \) and they satisfy for \( t \in [0, \hat{T}) \):
\[
e^{-\rho t} \frac{c(t)}{c(t)} = \lambda(t), \tag{34}
\]
\[
\frac{dk(t)}{dt} = r_b k(t) + y(t) - c(t), \tag{35}
\]
\[
\frac{d\lambda(t)}{dt} = -r_b \lambda(t). \tag{36}
\]
For \( t \in (\hat{T}, T] \):

\[
\frac{e^{-\rho t}}{c(t)} = \lambda(t),
\]

\[
\frac{dk(t)}{dt} = r_s k(t) + y(t) - c(t),
\]

\[
\frac{d\lambda(t)}{dt} = -r_s \lambda(t),
\]

and that:

\[
k(\hat{T}) = k(T) = 0,
\]

\[
k(t) < 0 \quad \text{for} \quad t \in (0, \hat{T}),
\]

\[
k(t) > 0 \quad \text{for} \quad t \in (\hat{T}, T),
\]

and the matching condition:

\[
\lambda(\hat{T}^-) = \lambda(\hat{T}^+).
\]

The optimality condition (34) and multiplier equation (36) imply:

\[
c(t) = c(0)e^{(r_b - \rho)t} \quad \text{for} \quad t \in [0, \hat{T}),
\]

which can be inserted into the state equation (35):

\[
\frac{dk(t)}{dt} = r_b k(t) + y(t) - c(0)e^{(r_b - \rho)t}.
\]

Rewrite this new state equation:

\[
\frac{dk(t)}{dt} e^{-r_b t} - r_b k(t) e^{-r_b t} = y(t) e^{-r_b t} - c(0) e^{-\rho t},
\]
and then integrate from $t = 0$ to $t = \hat{T}$:

$$k(t)e^{-rs(t-\hat{T})} \bigg|_0^{\hat{T}} = \int_0^{\hat{T}} y(t)e^{-rs} dt - c(0) \int_0^{\hat{T}} e^{-\rho t} dt. \quad (47)$$

Using the initial condition $k(0)$ and (40), (47) gives:

$$-k(0) = \int_0^{\hat{T}} y(t)e^{-rs} dt - c(0) \int_0^{\hat{T}} e^{-\rho t} dt. \quad (48)$$

Similarly, the optimality condition (37) and multiplier equation (39) imply:

$$c(t) = c(\hat{T})e^{(r_s - \rho)(t-\hat{T})} \quad \text{for} \quad t \in [\hat{T}, T), \quad (49)$$

which can be inserted to the state equation (38):

$$\frac{dk(t)}{dt} = r_s k(t) + y(t) - c(\hat{T})e^{(r_s - \rho)(t-\hat{T})}. \quad (50)$$

Rewrite this new state equation:

$$\frac{dk(t)}{dt}e^{-r_s(t-\hat{T})} - r_s k(t)e^{-r_s(t-\hat{T})} = y(t)e^{-r_s(t-\hat{T})} - c(\hat{T})e^{-\rho(t-\hat{T})}, \quad (51)$$

and then integrate from $t = \hat{T}$ to $t = T$:

$$k(t)e^{-r_s(t-\hat{T})} \bigg|_{\hat{T}}^{T} = \int_{\hat{T}}^{T} y(t)e^{-r_s(t-\hat{T})} dt - c(\hat{T}) \int_{\hat{T}}^{T} e^{-\rho(t-\hat{T})} dt. \quad (52)$$

Using the condition in (40), (52) becomes:

$$0 = \int_{\hat{T}}^{T} y(t)e^{-r_s(t-\hat{T})} dt - c(\hat{T}) \int_{\hat{T}}^{T} e^{-\rho(t-\hat{T})} dt. \quad (53)$$

Recalling that the matching condition (43) implies continuity of consumption at the
switch \( \hat{T} \), henceforth, there is:

\[
c(\hat{T}) = c(0)e^{(r_b - \rho)\hat{T}}. \tag{54}
\]

Inserting (54) into (53) and reorganizing the equation gives:

\[
0 = e^{-r_b\hat{T}} \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s(t-\hat{T})}dt - c(0) \int_{\hat{T}}^{\bar{T}} e^{-\rho t} dt. \tag{55}
\]

Combine (48) and (55):

\[
c(0) \int_{0}^{\hat{T}} e^{-\rho t} dt = k(0) + \int_{0}^{\hat{T}} y(t)e^{-r_b t} dt + e^{-r_b \hat{T}} \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s(t-\hat{T})} dt, \tag{56}
\]

or equivalently:

\[
c(0) = \frac{1}{\int_{0}^{\hat{T}} e^{-\rho t} dt} \left[ k(0) + \int_{0}^{\hat{T}} y(t)e^{-r_b t} dt + e^{-r_b \hat{T}} \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s(t-\hat{T})} dt \right]. \tag{57}
\]

The economic meaning of consumption at time zero is described as follows. The agent discounts all future non asset income to the present time. The discount rate she uses during the time she is in debt is the borrowing rate \( r_b \), while the discount rate she uses when she has positive saving is the savings interest rate \( r_s \). To be more specific, from time zero to the switch time, the agent’s total non asset income has present value \( \int_{0}^{\hat{T}} y(t)e^{-r_b t} dt \), where the discount rate is \( r_b \); from the switch time to death, the agent’s non asset income is valued as \( \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s(t-\hat{T})} dt \) as if the agent stands at the switch point, using discount rate \( r_s \). This value is then discounted to time zero using discount rate \( r_b \). The total present value of life time labor income is thus:

\[
w(0) = k(0) + \int_{0}^{\hat{T}} y(t)e^{-r_b t} dt + e^{-r_b \hat{T}} \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s(t-\hat{T})} dt, \tag{58}
\]

which multiplies the consumption share \( \frac{1}{\int_{0}^{\hat{T}} e^{-\rho t} dt} \) to give the consumption rate at time zero.
as shown in equation (57).

Moreover, equating the consumption \(c(0)\) derived from the first phase in (48) and that derived in (57) determines the switch \(\hat{T} \in (0, \bar{T})\):

\[
\frac{k(0) + \int_{0}^{\hat{T}} y(t)e^{-\rho t} dt}{\int_{0}^{T} e^{-\rho t} dt} = \frac{1}{\int_{0}^{\hat{T}} e^{-\rho t} dt} \left[ k(0) + \int_{0}^{\hat{T}} y(t)e^{-\rho t} dt + e^{-r_s \hat{T}} \int_{\hat{T}}^{\bar{T}} y(t)e^{-r_s (t-\hat{T})} dt \right]. \tag{59}
\]

To summarize the optimal control with “one switch” to problem \((P2)\): the switch \(\hat{T} \in (0, \bar{T})\) is determined in equation (59); the optimal consumption is:

\[
c^* (t) = \begin{cases} 
  w(0) / \int_{0}^{\hat{T}} e^{-\rho t} dt & \text{if } t = 0, \\
  c^* (0) e^{(r_s - \rho)t} & \text{if } t \in (0, \hat{T}), \\
  c^* (0) e^{(r_s - r_s) \hat{T} + (r_s - \rho)t} & \text{if } t \in [\hat{T}, \bar{T}],
\end{cases} \tag{60}
\]

and the optimal asset is:

\[
k^* (t) = \begin{cases} 
  \left\{ k(0) + \int_{0}^{t} [y(\tau) - c^* (\tau)]e^{-r_s \tau} d\tau \right\} e^{r_s t} & \text{if } t \in (0, \hat{T}), \\
  \left\{ \int_{\hat{T}}^{t} [y(\tau) - c^* (\tau)]e^{-r_s \tau} d\tau \right\} e^{r_s t} & \text{if } t \in [\hat{T}, \bar{T}].
\end{cases} \tag{61}
\]

Given the assumption \(r_s < \rho < r_b\) and based on the consumption profile in (60), it’s obvious that:

\[
\frac{\dot{c^*} (t)}{c^* (t)} = \begin{cases} 
  r_b - \rho > 0 & \text{if } t \in [0, \hat{T}), \\
  r_s - \rho < 0 & \text{if } t \in (\hat{T}, \bar{T}).
\end{cases} \tag{62}
\]

Thus the consumption profile is non-monotone. The optimal consumption profile increases exponentially in the borrowing phase and it then decreases exponentially in the saving phase. Q.E.D.

The reason why consumption increases in the first phase is discussed below. Since the agent is in debt in the early age, the borrowing rate is the effective interest rate in this
period. Because the borrowing rate is higher than the time discount rate, the agent wants to postpone her consumption to the latter of this period and pay back loans quickly.

The agent wants to decrease consumption in the saving phase. This is because the effective interest rate in that phase is the savings interest rate, which is lower than the time discount rate. The agent thus appears more impatient and wants to consume more in the early years within this period.

The location of the peak of the consumption profile $\hat{T}$ is determined in equation (59) and the size of consumption peak is:

$$\frac{c^*(\hat{T})}{c^*(0)} = e^{(r_b - \rho)\hat{T}}.$$  \hspace{1cm} (63)

Equations (59) and (63) give discipline to calibrate parameters in order to generate the right size and the right location of consumption peak. It can be seen from (63) that given the right location of the consumption peak, the size of the peak is then completely determined by the difference between the borrowing rate and the time discount factor.

4 Numerical example

In this section, I show that a hump shaped consumption profile is the optimal control under reasonable parameterizations. Moreover, optimal consumption is hump shaped without assuming a hump shape wage profile, and without the help of any of the other mechanisms that are known to produce a hump (uncertainty, borrowing constraint, irrationality, etc.).

I assume the agent enters the model at age 25, retires at 65 and dies at 80. In the benchmark model, I assume the borrowing interest rate is 5% ($r_b = 5\%$) and the savings interest rate is 1% ($r_s = 1\%$). While the instantaneous time discount rate is 4.5% ($\rho = 4.5\%$), so the condition:

$$r_s < \rho < r_b$$
is satisfied.

I assume income $y(t)$ is normalized as 1 during the working age, and it goes to zero after retirement. So:

$$
y(t) = \begin{cases} 
1 & \text{if } t \in [0, 40), \\
0 & \text{if } t \in [40, 55]
\end{cases}
$$

I also assume that the initial debt is 1. With this parameterization, I firstly test the necessary and sufficient conditions for existence of “one switch” optimal control, and then I solve the system for the optimal $c(t)$ and $k(t)$. Figure 1 shows the optimal consumption profile together with the income profile, and Figure 2 illustrates the optimal asset profile over the life cycle.

In this example, the peak of the consumption happens at $\hat{T} = 21.3$ or at age 46.3. The size of the peak, defined as $\frac{c(\hat{T})}{c(0)}$ is about 1.11.

Both the location and size of the consumption peak obtained in this exercise are consistent with empirical findings. For example, the location of the peak typically falls between ages 45 to age 55, and the size of the peak is estimated to lie between 1.1 and 1.25.

Moreover, in the robustness check, both the location and size of the peak are consistent and stable when I consider other reasonable values of borrowing interest rates and time discount factors.

5 Conclusion

In the United States, empirical evidences suggest that: 1) households hold debt, and their borrowing rates are higher than savings interest rate; 2) the typical consumption profile over the life cycle is hump shaped. This paper asks the question of whether we can explain the consumption hump purely on the basis of a difference between borrowing and saving interest rates. And this paper finds the answer is “Yes.”

In an model economy, the agent is allowed to borrow and save at the same time, while
the borrowing interest rate is higher than the saving interest rate. This paper illustrates that it’s possible to generate a hump shaped consumption profile over the life cycle when the time discount factor lies between the savings and borrowing interest rates.

In the early years of life, the agent is in debt, thus the effective interest rate is the borrowing interest rate. Since the borrowing interest rate is higher than the time discount rate, consumption in this phase increases. The agent then builds up savings in order to prepare for retirement. In this period, the effective interest rate is the savings interest rate, which is lower than the time discount rate. Thus consumption in this period decreases from the beginning. Hence, consumption is peaked in-between. With these reasonable parameterizations, this model generates both the right location and the right size of the consumption peak.

This paper highlights the simplest possible explanation to the consumption hump, which has been overlooked in previous research on the subject. The mechanism used in this paper does not require a hump-shaped wage profile, or any other features such as uncertainty or bounded rationality.

This paper illustrates a wedge between borrowing and saving interest rates in itself can generate a hump shaped consumption profile. My explanation presents a new channel that can be potentially combined with other mechanisms in a more sophisticated model with consumption-saving decision; and in those sophisticated models, each channel can be disentangled and tested for its quantitative importance.

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References


FIGURE 1: Optimal consumption profile and income profile over the life cycle

\[ \hat{T} = 46.3 \]

FIGURE 2: Optimal asset profile over the life cycle

\[ \hat{T} = 46.3 \]