Monetary Policy and Endogenous Asset Pricing Risk Premium

Ieng Man Ng

Thesis Advisor: Dr Timothy Kam

Economics Honours Thesis

Research School of Economics

The Australian National University
Abstract. I study how monetary policy transmission mechanism is linked to asset pricing dynamics, and the underlying dynamic outcomes of the economy, in a New Keynesian model with Epstein and Zin preference features. In particular, I investigate how the time varying risk component in asset pricing kernel influences the household's underlying optimal risky intertemporal consumption choice relating to asset returns. I find that the model can generate countercyclical properties in asset pricing dynamics for a household who prefers early resolution of uncertainty. These countercyclical features of the model provides a plausible account of the equity risk premium puzzle, and of the co-movement between the equity risk premium with the business cycle. Numerical results show that the mean of quarterly equity risk premium increases as the household's level of risk aversion increases. This suggests that a more risk averse household dislikes a more risky asset with an asymmetric distribution of returns and higher downside risks at the tail. A historical large spread in equity risk premium is not only because of higher potential earning rewards associated with equity, but is also due to the underlying additional risk compensation as required by investors investing in riskier assets. Furthermore, volatility of asset returns is lower in the New Keynesian production economy with recursive preferences features. The model can explain a mean effect (e.g., equity risk premium) but it has some weaknesses in accounting for asset returns at higher order moments to match the empirical data. From the analysis of this thesis, time variation of risk and higher order moments are crucial elements in studying asset prices but they do not necessarily fit well in a monetary policy DSGE model.

Keywords: Non-linear New Keynesian Model; Epstein and Zin Preferences; Asset Pricing; Monetary Policy Transmission Mechanism; Third-order Approximation.

Acknowledgment. I would like to thank my thesis advisor, Dr Timothy Kam. I am indebted to him for his ideas and guidance on this thesis. I am also thankful for his patience in teaching, and thoughtful comments along the journey. It has been an honour for me to be one of his students. Also, I would like to thank my friend, Brenda Tan, for her grammatical editing of my thesis.
CHAPTER 1

Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are widely used in macroeconomics and monetary policy advice. To have some understanding of how monetary policy may work, it is important to model and quantify the transmission mechanism of monetary policy. The objective of this thesis is to study how the household’s optimal risky intertemporal consumption choice is linked with asset returns, and how an alternative preference (Epstein and Zin features) matters for this asset pricing dynamic with monetary policy transmission mechanism in a New Keynesian model.

As a motivation, consider figure 1: The observed equity return is higher than government bond return, which is known as equity premium puzzle, Mehra and Prescott (1985).

According to figure 1, it shows a countercyclical co-movement of equity risk premium with the business cycle. In particular, the equity risk premium is high (low) during recession (boom). This observation is also supported by an empirical study

1Historical data was taken from St Louis Fred to construct this graph. \( \log \left( \frac{\text{real output}}{\text{real potential output}} \right) \) is used as a proxy for the business cycle. Real GDP and real potential GDP are both measured in 2009 dollars. Equity risk premium is the difference of returns between on a relatively more risky asset (corporate bond) and riskless asset (government bond).
on time varying equity risk premium, which was done by Gagligardini, Ossola and Scaillet (2016). The study showed that the risk premium is large and volatile in crisis periods. In Chapter 4 of this thesis, I will discuss the interaction of model's equilibrium with asset pricing dynamics. This is to provide economic intuitions on these observations in empirical data.

Standard DSGE models assume that households with constant relative risk aversion (CRRA) utility does not give much economic intuitions on understanding the equity risk premium puzzle. The main problem with CRRA preferences in studying asset pricing problem is that it cannot separate the household's intertemporal elasticity of substitution and level of risk aversion in their economic trade-offs. Epstein and Zin (1989) introduced a class of preferences that can overcome this problem. In order to have better a understanding of the link between asset pricing dynamics and the monetary policy transmission mechanism, I have therefore introduced Epstein and Zin preferences into a monetary policy DSGE model.

Epstein and Zin preferences have been included in the model as it contains important features for studying asset pricing problems. In the past literature, Epstein (1988) has studied the importance of time preferences for asset prices in a Lucas asset pricing model. Later on, Epstein and Zin (1989) have studied equilibrium asset prices in a consumption-based capital asset pricing model. Since these two models are in a partial equilibrium framework, they are silent on the connection from asset pricing dynamics to aggregate performance of the economy. In this thesis, the monetary policy DSGE framework, with recursive preferences, will allow us to study problems in asset pricing dynamics with monetary policy transmission mechanism. In short, this mechanism will link asset returns to the household's stochastic Euler equations, which will then pin down her underlying optimal economic trade-offs accordingly. Ultimately, the household's optimal risky intertemporal consumption choice will lead to impacts on aggregate performance of the economy.

Epstein and Zin preferences introduce two new features in the New Keynesian (NK) model, which typically features preference functions of the CRRA class. First, Epstein and Zin preferences can break the link between the household's level of risk aversion and the degree of intertemporal elasticity of substitution. This enriches the expected utility preferences by allowing a household to have early or late resolution of uncertainty. The resolution of uncertainty occurs when the household's future consumption choice is not directly observable. That is, she has to form an expectation on future consumption conditional on the resolution of uncertainty (e.g., risky states). The implication of a household who prefers early resolution of uncertainty is that she prefers to have a smoother consumption path across periods and risky states. This intuition can be thought of as a time-diversification in the household's
intertemporal consumption choice. The household chooses her optimal risky intertemporal consumption choice in relation to asset returns. Such optimal economic trade-off is governed by her stochastic Euler equation.

Second, Epstein and Zin preferences induce a time varying risk component which is absent under the standard DSGE model with CRRA preferences. In particular, the time variation of risk component acts as an extra term in the household’s stochastic Euler equations. This will result in an additional factor underlying the agent’s optimal economic trade-offs with asset returns. The time varying risk component measures the household’s continuation value of future consumption relative to her certainty equivalent value. For example, for a household who prefers early resolution of uncertainty, she attaches more weight on the low continuation value of future consumption relative to her certainty equivalent value. This is because she dislikes future consumption risks more and thus requires a higher conditional expected value on future consumption (e.g., higher certainty equivalent value). Further, the time varying risk component is increasing in the level of risk aversion which governs a higher asset pricing kernel. Consequently, it has greater weight on the household’s optimal risk intertemporal consumption trade-offs with asset returns.

The model can generate countercyclical properties in asset pricing for household with Epstein and Zin preferences who prefers early resolution of uncertainty. The monetary policy DSGE model with recursive preferences will provide us with better economic intuitions on understanding the fundamentals behind equity premium puzzle. We can also understand how the asset pricing dynamics with time variation of risk in this model can generate the co-movement of equity risk premium with the business cycle that we see in figure 1. In Chapter 4, I will explain in more detail how the asset pricing in this economy with Epstein and Zin preferences, especially the preference with early resolution of uncertainty, interact with the model’s equilibrium.

Numerical results in Chapter 5 show that the spread of (mean) equity risk premium is increasing in household’s level of risk aversion. This is due to the impacts of time varying risk component in household’s stochastic Euler equations. Intuitively, a more risk averse household dislikes returns with an asymmetric distribution (e.g., positively skewed to the right) that associated with higher downside risk at the tail (e.g., leptokurtic distribution). This implies a larger equity risk premium is required by a household who has Epstein and Zin preferences with early resolution of uncertainty. From the numerical experiments, I find that asset pricing dynamics is relatively less volatile for a household with Epstein and Zin preferences, relative to CRRA preferences. This suggests some weaknesses of the model in accounting for asset prices at higher order moments to match the empirical data. I will discuss more on this under section 5.2 in Chapter 5.
Key modeling approaches of this thesis include (1) separable utility function with Epstein and Zin preferences, (2) cost of capital adjustment, (3) monopolistic goods market, (4) cost of price adjustment (Rotemberg, 1982) and (5) central bank’s policy rate is used as a proxy for the period gross return on nominal bond. The reasons for these are as follows. Firstly, in a general equilibrium setting, income effect acts as an indirect effect on consumption in response to change in labour income. This indirect effect arises from labour demand which might dilute the direct effect on consumption in response to change in monetary policy rate. Hence, the separable utility function with Epstein and Zin preference can eliminate income effect. It allows us to mainly focus on problems with asset pricing, such that how household’s optimal risky intertemporal consumption choice is related to asset returns. We can then study how the transmission mechanism transmission links the policy rate to have impacts of aggregate performance of the economy. Secondly, costly capital adjustment is widely used in studying asset prices in a production economy to address the equity risk premium problem. Rotemberg (1982) pricing implies sticky prices. Cost of capital adjustment, sticky prices and monopolistic goods market structure are leading to market distortions in equilibrium allocation. This suggests that there is a role for monetary policy intervention to stabilize fluctuations over the business cycle. Lastly, the Federal Funds Rate (policy rate) move quite closely with the 90 days T-bill rate, according to figure 2. Given this feature, the central bank’s policy rate is used as a proxy for the one-period gross return on nominal bond in this model.

![Federal Funds Rate V.S. 90-day Treasury Bill Rate](image)

**Figure 2.** Data source: St Louis Fred.

Given that the central bank policy rate is used as a proxy for the nominal bond return rate, the change in monetary policy rate will enter directly to the household’s
stochastic Euler equation in the model. It therefore has direct influence on the household’s optimal economic trade-offs relating to asset returns. These modeling approaches allow us to study how the time varying risk component captured in Epstein and Zin preferences matter for monetary policy transmission mechanism with household’s optimal risky trade-offs, as well as the link between asset pricing dynamics and aggregate performance of the economy.

In Chapter 2, I will discuss the related literature background, and contribution of the monetary policy DSGE model with recursive preferences in this thesis. In Chapter 3, I will explain the model setup and equilibrium conditions, as well as solution method. In Chapter 4, I will explain intuitions on how the asset pricing dynamics with Epstein and Zin features is linked to household’s optimal economic trade-offs, and its interaction with equilibrium outcomes of the economy. In Chapter 5, I will first discuss the experiments design in this thesis and then, I will explain the impulse responses of the economy subject to exogenous shock. This allows us to see the dynamic properties of the model and gain a deeper understanding on the mechanism with Epstein and Zin features. Lastly, Chapter 6 concludes this thesis.
CHAPTER 2

Related Literature Review and Contribution

Efforts and contributions of many researchers in monetary macroeconomics has led to the framework of New Keynesian (NK) model (Calarida, Gali and Gertler, 1999; Gali and Gertler, 2007). This New Keynesian model has a core structure with regard to the Real Business Cycle model, except there are market distortions in equilibrium allocation. This is due to sticky prices and monopolistic goods market in NK model. Hence, there is a role for monetary policy intervention to minimize fluctuations over the business cycle. This dynamic stochastic general equilibrium (DSGE) framework is widely used for monetary policy advice and business cycle analysis.

Mehra and Prescott (1985) found the equity premium, which is defined as equity returns minus bond returns, has been high on average in empirical data. This is known as equity premium puzzle. The large spread in equity premium implies that investors have high level of risk aversion in the standard DSGE model. This is because it assumes that the household has a constant relative risk aversion (CRRA) preference. A household with this type of preference cannot separate the link between the level of risk aversion and intertemporal elasticity of substitution. This causes a problem in studying asset pricing dynamics, which is the main motivation of introducing recursive preferences into a monetary policy DSGE model in this thesis. The main reason why I have chosen Epstein and Zin preferences is because this class of preferences capture the important features in studying asset prices.

Preferences of intertemporal risk attitudes were initially studied by Kreps-Porteus (1978). This utility functional form studies the agent’s time preference for early or late resolution of risk. Epstein and Zin (1989) made an extension from Kreps-Porteus utility. Epstein and Zin developed a class of recursive preferences that can break the link between household’s level of risk aversion and intertemporal elasticity of substitution. This allows the household to have preference for early or late resolution of uncertainty. It also captures a time varying risk component in agent’s intertemporal consumption trade-offs. Epstein and Zin provided an useful preferences framework in studying asset pricing and portfolio choice.

Epstein (1988) has studied equilibrium asset prices in a Lucas asset pricing model, where the representative agent follows a Kreps-Porteus utility functional
form. This paper investigated how the household has time preference in risk attitude influence asset prices at equilibrium. Epstein and Zin (1989) studied asset pricing in a consumption-based capital asset pricing model. The problem with asset pricing in these class of models, whether Lucas or CCAPM, is that it only considers a partial equilibrium economy. These models are silent on how the asset pricing dynamics lead to impacts on the aggregate performance of the economy.

Some researchers have considered capital in a production economy to study asset prices (Jermann, 1998; Boldrin, Christiano and Fisher, 2001; Lettau, 2003). They have discussed the role of macroeconomy structure and the importance of real frictions on asset prices and risk premium behaviour. Another study done by Paoli, Scott and Weken (2010) contributed to the understanding of the role of nominal shocks and nominal rigidities on asset prices and the yield curves.

Given the consideration of importance in asset prices with recursive preferences feature and macroeconomy structure, the motivation of this thesis is to incorporate Epstein and Zin preferences to study asset pricing dynamics in a monetary policy DSGE framework. In particular, I investigate how an alternative preferences model matters for asset pricing dynamics and monetary policy transmission mechanism. From this study, we can gain economic intuitions on understanding the importance of the time-varying risk component in asset pricing kernel for monetary policy transmission dynamics, as well as its impact on the aggregate performance of the economy. We can also understand the fundamentals behind the equity risk premium puzzle and its co-movement with the business cycle.
CHAPTER 3

Model setup

3.1. Aspects of the model

I will first describe the general elements of this monetary policy DSGE model. The model is in a closed production NK economy with complete assets market trading one period nominal bond and stock on capital. Agents in this economy lives infinitely long and they are subject to aggregate exogenous shocks that hit the economy. Time is discrete where \( t := \{0, 1, \ldots\} \). State space in this economy is \( s_t := \{\epsilon^A_t, \epsilon^{MP}_t\} \), where it consists the realization of state \( s_t \) at time \( t \) with either total factor productivity shock or monetary policy. Exogenous shocks follow a normal distribution with mean zero and variance. That is, \( \epsilon^A_t \sim N(0, \sigma^2_A) \) and \( \epsilon^{MP}_t \sim N(0, \sigma^2_{MP}) \). The parameters of this model are summarized in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Common discount factor</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Inverse of intertemporal elasticity of consumption substitution</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Elasticity of demand between differentiated goods</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Cost of price adjustment</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Cost of capital adjustment</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Fraction of capital</td>
</tr>
<tr>
<td>( \phi_R )</td>
<td>Degree of smoothing interest rate behavior</td>
</tr>
<tr>
<td>( \phi_Y )</td>
<td>Responsiveness of interest rate changes with respect to potential output</td>
</tr>
<tr>
<td>( \phi_{\Pi} )</td>
<td>Responsiveness of interest rate changes with respect to inflation target</td>
</tr>
</tbody>
</table>

Table 1. Parameters

A representative household has Epstein and Zin preferences. A continuum of firms on \([0, 1]\) participate in a monopolistic competitive goods market. Each of firm, \( i \), produces variety of differentiated goods indexed by \( i \in [0, 1] \). They face a quadratic price adjustment cost (Rotemberg 1982). Also, there is a cost of capital adjustment. Hence, distortion to the competitive equilibrium allocation comes from three sources which are monopolistic goods market, firm’s pricing adjustment cost (e.g., sticky prices) and friction in capital market. The central bank sets the one period nominal interest rate according to a Taylor Rule. This one period nominal
3.2. A GENT’S OPTIMIZATION PROBLEM

3.2. A GENT’S OPTIMIZATION PROBLEM

interest rate is used as a proxy for one period gross return on nominal bond in this economy. This is because the Federal Funds Rate is observed to move quite closely with the 90-day Treasury bond yield as shown on figure 2.

3.2. Agent’s Optimization Problem

3.2.1. Household.

The representative household lives infinitely long and participates in goods, asset, and labour markets. She consumes $C_t$ units of final consumption good, holds one-period state-contingent nominal bond $B_t$, supplies $N_t$ units of labor where $N(i)$ units of labor is rented to each differentiated good firm $i \in [0, 1]$. The household owns capital stock $K_t$ and rents to each firm $i$. The representative household also owns the firms in this economy.

Epstein and Zin Preferences features.

The representative household has Epstein and Zin preferences feature, which is the main focus of this thesis. Her lifetime utility is given by:

$$
\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t U [C_t, N_t] \right\}
$$

(1)

where $\mathbb{E}_t := \mathbb{E} \{ \cdot \mid s_t \}$ is linear expectation conditional on realization of $s_t$ at time $t$.

Assume separable utility function$^1$:

$$
U(C, N) := U(C) - L(N)
$$

$$
U(C) - L(N) = \left[ (1 - \beta) (C_t(s_t))^{1-\gamma} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} (C_{t+1}(s_{t+1})) \right]^{1-\Psi} \right]^{\frac{\gamma}{1-\rho}} - \left[ \frac{N_t^{1+\varphi}(s_t)}{1 + \varphi} \right]
$$

where $\rho, \gamma, \varphi > 0$, $\Psi := \frac{1-\gamma}{1-\rho}$ is an index captures deviation with respect to the benchmark CRRA utility, $J_t(U_{t+1}(C_{t+1}(s_{t+1}))) = \mathbb{E}_t \left( U_{t+1}^{1-\gamma} (C_{t+1}(s_{t+1})) \right)$, $\rho := \frac{1}{\text{IES}}$ is the inverse intertemporal elasticity of substitution, $\gamma$ is the risk aversion, and $\varphi$ is Frisch elasticity of labour supply.

The representative household earns labour income, $W_t(s_t)$, capital income $r^K_k(s_t)$, from supplying labour and capital. She also earns one-period gross return on holding of nominal bond, $R_t(s_t)$. The one period gross return on capital stock is $R^K_k(s_t) :=$

$^1$Separable utility function eliminates income effect. In general equilibrium setting, income effect is an additional indirect effect on consumption in response to labour income. Hence, this separability will not dilute the direct effects of the change in monetary policy rate. This allows us to focus on the importance of the asset pricing kernel with time varying risk component for monetary policy transmission.
3.2. Agent’s Optimization Problem

1 + r^t_k(s_t) - \delta. Since household owns the firm, she also gets total profits from this ownership \Omega_t(s_t).

The household’s lifetime budget constraint is given by:

\[ P_t(s_t)C_t(s_t) + R_t^{-1}(s_t)B_{t+1}(s_{t+1}) + P_t(s_t)I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 \]

\[ = W_t(s_t)N_t(s_t) + B_t(s_t) + r^t_k(s_t)K_t(s_t) + P_t(s_t)\Omega_t(s_t) \quad (2) \]

where the capital cost adjustment is assumed to be the functional form of: \( \Phi \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 \), zero capital cost adjustment when \( \Phi = 0 \). The aggregate nominal price level is \( P_t(s_t) \).

The lifetime budget constraint means that consumption, holding of the nominal bond in future period and investment for the household is financed by the sum of her labour income, capital rent, holding of the bond in current period and profits accruing from ownership of firms.

The law of capital accumulation in this economy is given by:

\[ K_{t+1}(s_{t+1}) = (1 - \delta)K_t(s_t) + I_t(s_t) \quad (2.1) \]

where \( \delta \) is the depreciation rate.

The final consumption good, \( C_t(s_t) \), consists of a continuum of differentiated goods produced by firm \( i \) indexed by \( i \in [0, 1] \) in a monopolistically competitive market such that:

\[ C_t(s_t) = \left[ \int_0^1 [C_{s_t,t}(i)]^{-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\epsilon}}, \epsilon > 1 \]

A household has two optimization problems: (1) expenditure minimization, and (2) utility maximization which will be discussed as follows.

**Problem 1.** Expenditure minimization problem of household for her consumption of final good \( C_t \) at time \( t \) is to choose \( C_t(i) \) for all \( i \in [0, 1] \) such that:

\[ \min_{C_{s_t,t}(i)} \int_0^1 P_{s_t,t}(i)C_{s_t,t}(i)di \]

subject to

\[ \left[ \int_0^1 [C_{s_t,t}(i)]^{-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\epsilon}} = C_t(s_t) \]

The optimal expenditure-minimizing choice (equivalently, the demand function for firm \( i \) product for all \( i \in [0, 1] \)) is given by:

\[ C_{s_t,t}(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t(s_t) \quad (3) \]
Problem 2. The representative household utility maximization problem is to choose consumption $C_t$, capital $K_{t+1}$, bond $B_{t+1}$, investment $I_t$, and labour supply $N_t$ to maximize her life-time utility subject to the life-time budget constraint, equation (2), and the law of capital accumulation, equation (2.1). The problem is as follows:

$$
\max_{(C_t, N_t, K_{t+1}, I_t, B_{t+1})} \sum_{t=0}^{\infty} \beta^t \left[ (1 - \beta) \left( C_t(s_t) \right)^{1-\gamma} \right]^{\frac{1-\gamma}{\delta}} - \frac{\beta}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2
$$

subject to

$$
P_t(s_t)C_t(s_t) + R_t^{-1}(s_t)B_{t+1}(s_{t+1}) + P_t(s_t)I_t(s_t) + \Phi \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 = W_t(s_t)N_t(s_t) + B_t(s_t) + r_t^k(s_t)K_t(s_t) + P_t(s_t)\Omega_t(s_t)
$$

and

$$
K_{t+1}(s_{t+1}) = (1 - \delta)K_t(s_t) + I_t(s_t)
$$

The following equations govern the optimal consumption-labour choice, and optimal risky intertemporal consumption relating to assets returns, for the representative household at equilibrium.

$$
\frac{W_t(s_t)}{P_t(s_t)} = \frac{N_t^\beta(s_t)}{(1 - \beta)C_t^{-\rho}(s_t)} \tag{4}
$$

$$
(C_t(s_t))^{1-\gamma} = R_t(s_t)\beta \mathbb{E}_t \left[ \left( C_{t+1}(s_{t+1}) \right)^{1-\gamma} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right) \right] \left( \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right) \tag{5}
$$

$$
q_t(s_t) \left( C_t(s_t) \right)^{1-\gamma} = \beta \mathbb{E}_t \left[ \left( C_{t+1}(s_{t+1}) \right)^{1-\gamma} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right) \right] \left( \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right) \tag{5.1}
$$

which holds for realization of state $s_t$ for every $t \geq 0$, and $J_t [V_{t+1}(C_{t+1}(s_{t+1}))] = \mathbb{E}_t \left( V_{t+1}(C_{t+1}(s_{t+1}))^{1-\gamma} \right)$. 
3.2. AGENT’S OPTIMIZATION PROBLEM

Tobin’s marginal q ratio in this economy is \( q_t := \frac{Q_t}{\Gamma_t} \), and that is given by:

\[
q_t(s_t) = 1 + \Phi \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)
\]

where \( \Gamma_t \) is the Lagrange multiplier on constraint (2) and \( Q_t \) is the Lagrange multiplier on constraint (2.1). The Tobin’s marginal q ratio measures the the stock value of the firm in terms of consumption good.

An inflation adjusted asset pricing kernel is given by:

\[
M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1+\gamma}{1-\gamma}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right]
\]

\[
\Leftrightarrow M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{1+\gamma}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}} \frac{1}{\Pi_{t+1}(s_{t+1})} \right]
\]

where \( \Pi_t(s_t) := \frac{P_t(s_t)}{P_{t-1}(s_{t-1})} \) is the CPI gross inflation at time \( t \) and realization of state \( s_t \).

In this model, the time varying risk component captured in asset pricing kernel is given by:

\[
\left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}}
\]

which measures the household’s continuation value of future consumption relative to her certainty equivalent value.

Equation (4) captures the optimal consumption-labour choice for household at the point where the marginal benefit (real wage) is equal to the marginal rate of substitution of labour supply and consumption. Equations (5) and (5.1) are stochastic Euler equations relating to nominal bond return, and return on capital stock respectively. These two stochastic Euler equations pin down an optimal risky intertemporal consumption choice for the representative household. At equilibrium, the marginal utility of current consumption must equal to the marginal utility of future consumption, taking into account of the underlying asset return and time varying risk component.
3.2. AGENT’S OPTIMIZATION PROBLEM

The new feature of this monetary policy DSGE model is the time varying risk component as an additional factor captured in household’s stochastic Euler equation, underlying the optimal risky intertemporal consumption choice relating to asset returns. This component is absent under the standard DSGE model with CRRA preferences. I will describe the optimal economic trade-offs for a household has CRRA preferences in the standard DSGE model in the following example.

Example: CRRA Preference when \( \rho = \gamma \) (this serves as a benchmark).

When \( \rho = \gamma \), e.g. \( \Psi = \frac{1 - \gamma}{1 - \gamma} = 1 \), the model is twisted back to the standard NK model with CRRA preferences. In particular, the asset pricing kernel becomes:

\[
M_{t, t+1} = \beta E_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{1-\gamma} \frac{1}{\Pi_{t+1}(s_{t+1})} \right]
\]

where the time varying risk component is equal to 1.

The optimal consumption-labour choice, and intertemporal consumption choice under CRRA preferences is respectively given by:

\[
\frac{W_t(s_t)}{P_t(s_t)} = \frac{N^\rho_t(s_t)}{C^\rho_t(s_t)} \tag{6}
\]

\[
(C_t(s_t))^{\frac{1-\gamma}{\Psi}} - 1 = R_t(s_t) \beta E_t \left[ (C_{t+1}(s_{t+1}))^{\frac{1-\gamma}{\Psi}} - 1 \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right] \tag{7}
\]

\[
q_t(s_t) (C_t(s_t))^{\frac{1-\gamma}{\Psi}} - 1 = \beta E_t \left[ (C_{t+1}(s_{t+1}))^{\frac{1-\gamma}{\Psi}} - 1 \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right]
\]

\[
\left\{ \left( r^k_t - \frac{\Phi}{2} \left( \frac{I_{t+1}(s_{t+1})}{K_{t+1}(s_{t+1})} - \delta \right)^2 + \Phi \frac{I_{t+1}(s_{t+1})}{K_{t+1}(s_{t+1})} - \delta \left( \frac{I_{t+1}(s_{t+1})}{K_{t+1}(s_{t+1})} - \delta \right) \right) + (1 - \delta)q_{t+1}(s_{t+1}) \right\} \tag{7.1}
\]

which holds for realization of state \( s_t \) for every \( t \geq 0 \).

At equilibrium, equation (6), (7) and (7.1) have the same interpretation as (4), (5), and (5.1), except the time varying risk term is now equal to unity.

3.2.2. Firm.

A continuum of firms on \([0, 1]\) participate in a monopolistic competitive goods market. Each of firm, \( i \), produces variety of differentiated goods indexed by \( i \in [0, 1] \). Each firm on \([0, 1]\) faces consumer demand \( C_{t,s_t}(i) \) at any time \( t \) and state \( s_t \) such as:

\[
Y_{t,s_t}(i) = \left( \frac{P_{t,s_t}(i)}{P_{t,s_t}} \right)^{-\epsilon} Y_t(s_t) \tag{8}
\]

This means that the production decision for each of firm \( i \) on \([0, 1]\) is demand determined by the household’s consumption choice in this NK model.
3.2. AGENT’S OPTIMIZATION PROBLEM

The total resources constraint in this economy is given by:

\[ Y_t(s_t) := C_t(s_t) + I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 \]  

Production technology of firms in this economy follows a Cobb-Douglas functional form:

\[ Y_{t,s_t}(i) = A_t(s_t) K_{t,s_t}^{\alpha}(i) N_{t,s_t}^{1-\alpha}(i) \]  

where \( A_t(s_t) \) is the total factor productivity (TFP) subject to an exogenous stochastic productivity shock common to each continuum of firms on \([0, 1]\).

Assume TFP, \( A_t(s_t) \), follows a stationary AR(1) process:

\[ \ln(A_{t,s_t}) = \alpha_A \ln(A_{t-1,s_{t-1}}) + \varepsilon^A_t \]  

where \(|\alpha_A| < 1, \varepsilon^A_t \sim N(0, \sigma^2_A)\).

Firms face convex-price adjustment cost:

\[ \text{AC} \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_{t-1}}(i)}, Y_{t,s_t}(i) \right) := \frac{\theta}{2} \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_{t-1}}(i)} - \Pi_{ss} \right)^2 Y_{t,s_t}(i) \]  

where \( \Pi_{ss} \) is gross CPI inflation in the steady state, i.e., central bank’s inflation target.

This quadratic adjustment cost function implies sticky prices in this economy. In particular, the real cost of price adjustment by the firm is a function of firm’s own price inflation relative to the CPI inflation in steady state. Hence, the average real cost of price adjustment increases substantially when the firm increases their own prices relative to steady state CPI inflation. Convex price adjustment cost, capital cost adjustment and monopolistic competitive goods market all lead to market distortions in equilibrium allocation.

Each of firm \( i \) on \([0, 1]\) have two optimization problems that need to be considered. These are (1) cost-minimization of producing output and (2) expected profit maximization (profits pay out to the household for the ownership of firms).

**Problem 1.** Firm’s cost-minimization problem for output \( Y_{t,s_t}(i) \) at time \( t \) is by choosing \( K_{t,s_t}(i) \) and \( N_{t,s_t}(i) \) for all \( i \in [0, 1] \) such that:

\[ \min_{N_{t,s_t}(i), K_{t,s_t}(i)} W_t(s_t) N_{t,s_t}(i) + r^k_t(s_t) K_{t,s_t}(i) \]

subject to

\[ Y_{t,s_t}(i) = A_t(s_t) K_{t,s_t}^{\alpha}(i) N_{t,s_t}^{1-\alpha}(i) \]

The optimal cost-minimizing choice for production for all firm \( i \in [0, 1] \) is given by:

\[ W_t(s_t) = MC_t(s_t) (1 - \alpha) A_t(s_t) K_{t,s_t}^{\alpha}(i) N_{t,s_t}^{-\alpha}(i) \]  

(12)
The marginal cost of labor is just the wage, and the marginal cost of capital is the account. This holds for both expressions in nominal and real terms at equilibrium.

\[ r^k_t(s_t) = MC_t(s_t)\alpha A_t(s_t)K^{\alpha-1}_{t,s_t}(i)N_{t,s_t}^{1-\alpha}(i) \]  

(12.1) \[ r^k_t(s_t) = MC_t(s_t)\alpha A_t(s_t)K^{\alpha-1}_{t,s_t}(i)N_{t,s_t}^{1-\alpha}(i) \]

where \( A_t \) is the shadow value of Lagrange multiplier on production constraint, hence it is the nominal marginal cost with respect to production \( Y_t(s_t) \), \( A_t = MC_t(s_t) \).

These two first order conditions show that the optimal labor (capital) demand for each firm \( i \) on \([0,1]\) is at the point where marginal cost of labor (capital) is equal to the marginal product of labor (capital), taking marginal cost of production into account. This holds for both expressions in nominal and real term at equilibrium. The marginal cost of labor is just the wage, and the marginal cost of capital is the rental rate.

**Problem 2.** Firm has the same stochastic discount factor as household since household owns firms directly:

\[ M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{\gamma}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\gamma}} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right] \]

where \( J_t[V_{t+1}(C_{t+1}(s_{t+1}))] = \mathbb{E}_t[V_{t+1}^{1-\gamma}(C_{t+1}(s_{t+1}))] \).

Let \( \tilde{M}_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{\gamma}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\gamma}} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \] be the non-inflation adjusted pricing kernel. Firm’s expected profit maximization problem at time \( t \) is by choosing \( P_{t,s_t}(i) \) for all \( i \in [0,1] \) such that:

\[ \Omega_{t,s_t}(i) = \max_{\{P_{t,s_t}(i)\}_{i \in \mathbb{N}}} \mathbb{E}_t \left\{ \sum_{s_{t-1}} \left[ \tilde{M}_{t,t+1} \frac{P_{t,s_t}(i)}{P_t(s_t)} Y_{t,s_t}(i) - \frac{W_t(s_t)}{P_t(s_t)} N_{t,s_t}(i) \right] \right\} \]

subject to

\[ Y_{t,s_t}(i) = A_t(s_t)K_{t,s_t}^{\alpha}(i)N_{t,s_t}^{1-\alpha}(i) \]

At every time \( t \geq 0 \) and state \( s_t \), the firm \( i \) profit maximizing choice (optimal pricing strategy) for all \( i \in [0,1] \) satisfies:

\[ 0 = (1 - \epsilon) \frac{Y_{t,s_t}(i)}{P_{t,s_t}(i)} - \frac{MC_{t,s_t}(i)}{P_t(s_t)} \frac{\partial Y_{t,s_t}(i)}{\partial P_{t,s_t}(i)} - \frac{\partial AC}{\partial P_{t,s_t}(i)} \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_{t-1}}(i)}, Y_{t,s_t}(i) \right) \]
\[ -\mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial AC}{\partial P_{t,s_t}(i)} \left( \frac{P_{t+1,s_{t+1}(i)}}{P_{s_t}(i)}, Y_{t+1,s_{t+1}(i)} \right) \right\} \]  

where (dropping the notation of state \( s_t \) for the following equations) 

\[ \frac{MC_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} = \frac{MC_t(i)}{P_t} \left( -\epsilon \frac{P_t(i)}{P_t} \right)^{-\epsilon-1} Y_t \]

\[ \frac{\partial AC}{\partial P_t(i)} \left( \frac{P_t(i)}{P_{t-1}(i)}, Y_t(i) \right) = \frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{ss} \right)^2 \left( -\epsilon \frac{P_t(i)}{P_t} \right)^{-\epsilon-1} Y_t \]

\[ + \theta \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{ss} \right) \frac{Y_t(i)}{P_{t-1}(i)} \]

and

\[ \frac{\partial AC}{\partial P_t(i)} \left( \frac{P_{t+1}(i)}{P_t(i)}, Y_{t+1}(i) \right) = \theta \left( \frac{P_{t+1}(i)}{P_t(i)} - \Pi_{ss} \right) \left( \frac{P_{t+1}(i)}{(P_t(i))^2} \right) Y_{t+1}(i) \]

Aggregation over all firm’s production for each of the differentiated goods firm on \([0, 1]\) gives total output (total supply in the economy) such that:

\[ Y_t(s_t) = A_t(s_t)k_t(s_t)N_t(s_t) \quad (14) \]

At every time \( t \geq 0 \) and state \( s_t \), the firm \( t \) profit maximizing choice (optimal pricing strategy) for all \( i \in [0, 1] \) satisfies equation (13). This optimal pricing equation captures (1) the current real marginal revenue of firm’s own price variation, (2) real marginal cost of production, (3) current and (4) future expected marginal effects of pricing strategy on firm’s profit through the cost of price adjustment. Given the firm’s optimal choice of labour demand, capital demand and pricing decision, the aggregate output produced by all firms on \([0, 1]\) satisfies (14).

### 3.2.3. Central Bank.

The Central bank’s monetary policy is to set a nominal one-period nominal interest rate, which follows a Taylor rule. The policy rule is as follows:

\[ \frac{R_t(s_t)}{R_{ss}} = \left( \frac{R_{t-1}(s_{t-1})}{R_{ss}} \right)^{\phi_R} \left( \frac{\Pi_t(s_t)}{\Pi_{ss}} \right)^{(1-\phi_R)\phi_H} \left( \frac{Y_t(s_t)}{Y_{ss}} \right)^{(1-\phi_R)\phi_Y} + \epsilon_t^{MP} \quad (15) \]

\( R_{ss} \) is the long term real interest rate at steady state. The parameter \( \phi_R \) measures the degree of central bank’s smoothing interest rate behavior; \((1 - \phi_R)\phi_H \) captures how much the central bank cares about the impact of current inflation, \( \Pi_t := \left( \frac{P_t}{P_{t-1}} \right) \), deviates from the inflation target \( \Pi_{ss} \) in steady state, \( \phi_H \in [0, 1] \); and \((1 - \phi_R)\phi_Y \) captures how much the central bank cares about the impact of current total output, \( Y_t \), deviates from the potential output \( Y_{ss} \) in steady state, \( \phi_Y \geq 0 \).
3.3. Competitive Equilibrium

Agents in this economy also face an aggregate exogenous monetary policy shock captured by $\epsilon_t^{MP}$ where $\epsilon_t^{MP} \sim N(0, \sigma_{MP}^2)$. For example, given an exogenous monetary policy shock that hits the economy, i.e. change in $\epsilon_t^{MP}$, this will cause a shift in the monetary policy curve (15). Ultimately, it impacts the policy rate set by the central bank and therefore the dynamic outcomes of the economy.

In this model, the one-period nominal interest rate (policy rate) set by the central bank is used as a proxy for the one-period gross return on nominal bond. This is because the policy rate moves very closely to the 90-day Treasury Bill rate as shown on figure 1.0.2. Hence, any change in the monetary policy rule will lead to impacts on the dynamics outcomes of the economy. This is known as monetary policy transmission mechanism.

Briefly, the monetary policy transmission mechanism affects the equilibrium outcomes in the model as follows. A change in policy rate implies the return on nominal bond will change. This has a direct influence on the household’s stochastic Euler equation (5). Household will then weight her optimal intertemporal consumption choice relating to change in asset return accordingly. Change in household’s consumption implies firms will change their optimal production accordingly. This is because firm’s production is demand determined. This shows how the monetary policy transmission mechanism interacts with asset prices in this model. Ultimately, this asset pricing dynamics is linked to the aggregate performance of the economy. Details of explanations for this interaction will be discussed in Chapter 4.

3.3. Competitive Equilibrium

3.3.1. Market Clearing.

There are four markets in this closed production economy, which consists of a continuum of labour, capital, differentiated goods, and asset market trading the one-period nominal bond and stock on capital. Each one of these market clearing conditions are respectively explained below.

3.3.2. Labour Market Clearing.

Labour market clears when labour demand equals labour supply such that:

$$mc_t(s_t) (1 - \alpha) A_t(s_t)K_t^\alpha(s_t)N_t^{-\alpha}(s_t) = \frac{N_t^\rho(s_t)}{(1 - \beta)C_t^{\rho}(s_t)}$$

(16)

where $mc_t(s_t) := \frac{MC_t(s_t)}{P_t(s_t)}$ is defined as the real marginal cost of production.

3.3.3. Goods Market Clearing.

Aggregate goods market clears for each of the differentiated goods $i \in [0, 1]$, when consumption demand by household equals firm’s total supply (taking price
and capital adjustment cost into account) such that:

\[
1 - \frac{\theta}{2} \left( \Pi_t - \Pi_{s,t} \right)^2 \] \[Y_t(s_t) = C_t(s_t) + I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2
\]

where total production is \(Y_t(s_t) = A_t(s_t)K_t^\alpha(s_t)N_{t+1}^{1-\alpha}(s_t)\) and law of capital motion is \(K_{t+1}(s_{t+1}) = I_t(s_t) + (1 - \delta)K_t(s_t)\).

Equation (17) shows three sources that contribute to market distortions in equilibrium allocation, which are sticky prices (convex cost of price adjustment), convex cost of capital adjustment and monopolistic goods market. Hence, there is a role for monetary policy intervention to minimize fluctuations over the business cycle.

### 3.3.4. Asset Pricing Condition.

Assuming no profit arbitrage condition and using stochastic Euler equation (5) with one period gross return on nominal bond, the underlying asset pricing condition in equilibrium is as follows:

\[R_t(s_t) = \frac{1}{M_{t,t+1}} \tag{18}\]

and from (5.1) with return on capital stock, the asset pricing condition is:

\[
1 = \left( r^k_t(s_t) + (1 - \delta)q_{t+1}(s_{t+1}) - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) q_t(s_t)
\]

since \(R^k_t := 1 + r^k_t(s_t) - \delta\) is the one period gross return on capital, then it can be written as:

\[
1 = M_{t,t+1} \left( r^k_t(s_t)q_{t+1}(s_{t+1}) - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) q_t(s_t)
\]

using (18) and (19), it yields:

\[
R_t(s_t) = \frac{R^k_t(s_t)q_{t+1}(s_{t+1}) - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}}}{q_t(s_t)} \tag{19.1}
\]

where \(M_{t,t+1} = \beta \mathbb{E}_t \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{1-\gamma/\delta} \left( \frac{V_{t+1}(C_{t+1}(s_{t+1}))^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}(C_{t+1}(s_{t+1}))]} \right)^{-1/\delta} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right), \) and \(q_t(s_t)\) is the Tobin’s marginal \(q\) ratio at time \(t\) and state \(s_t\).

The inflation adjusted asset pricing kernel \(M_{t,t+1}\) captures the time varying risk component in asset pricing in this economy, via the term of \(\left( \frac{V_{t+1}(C_{t+1}(s_{t+1}))^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}(C_{t+1}(s_{t+1}))]} \right)^{-1/\delta} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right). \)

This measures the valuation of next period’s consumption relative to household’s certainty equivalent. Asset pricing conditions clear the asset market as long as it holds for every \(t \geq 0\) and realization of state \(s_t\).
3.3. Capital Market Clearing.
Capital market clearing is governed by the Walra’s law, and the law of capital accumulation and the firm’s capital demand condition.

3.3.6. New Keynesian Phillips Curve (NKPC).
The new Keynesian Phillips curve is derived from the firm’s optimal pricing strategy, equation (13). This is based on the assumption of symmetric pricing strategy, where all firms on \([0, 1]\) will choose the same price to change if they are happened to adjust their prices at each time \(t\) and state \(s_t\). Hence, aggregation over all \(i \in [0, 1]\), then \(P_{t,s_t}(i) = P_t(s_t)\) and CPI gross inflation is \(\Pi_t(s_t) = \frac{P_t(s_t)}{P_{t-1}(s_{t-1})}\).

The firm’s optimal pricing condition (13) implies the New Keynesian Phillips Curve (NKPC) in the equilibrium is of the functional form such as:

\[
\Pi_t(s_t)(\Pi_t(s_t) - \Pi_{ss}) - \frac{\epsilon}{2} (\Pi_t(s_t) - \Pi_{ss})^2
= E_t \left[ M_{t,t+1} (\Pi_{t+1}(s_{t+1}) - \Pi_{ss}) \Pi_{t+1}(s_{t+1}) \frac{Y_{t+1}(s_{t+1})}{Y_t(s_t)} \right]
- \epsilon \frac{\theta}{\theta - 1} \left( mc_t(s_t) + \frac{\epsilon - 1}{\epsilon} \right)
\]  
(20)

where \(\theta\) captures the cost of price adjustment and \(\epsilon\) captures the firm’s own price elasticity of demand for all \(i \in [0, 1]\) differentiated goods.

The NKPC captures impacts of aggregate demand and supply on gross CPI inflation in this economy.

DEFINITION 1. Given monetary policy (15) with Epstein and Zin preferences, a recursive competitive equilibrium in this economy is a system of allocation functions \(s_t \rightarrow \{C_t, N_t, K_t, I_t, Y_t, mc_t\} (s_t)\), and pricing functions \(s_t \rightarrow \{q_t, R^k_t, \Pi_t, M_{t,t+1}\} (s_t)\), such that:

1. Households optimize: (4), (5) and (5.1);
2. Firms optimize: (12), (12.1) and (20);
3. Markets clear: (16), (17) and (19.1);
4. pricing kernel \(M_{t,t+1}\) governs agent’s optimal risky intertemporal consumption trade off;

for each time \(t \geq 0\) and realization of state \(s_t\).

3.3.7. Deterministic Steady State.
The deterministic steady state of this economy can be analytically solved as:

- The real marginal cost: \(mc_{ss} = \frac{\epsilon - 1}{\epsilon}\);
- Inflation target: \(\Pi_{ss}\);
- Aggregate capital: \(\hat{K}_{ss} = \left(\frac{mc_{ss} \alpha A_{ss}}{\beta^\gamma (1 - \delta)}\right)^{\frac{1}{1 - \alpha}}\);
• Aggregate labour: \( \hat{N}_{ss} = (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{mc_{ss}A_{ss}}{\Pi_{ss}^{-\gamma} - (1 - \delta)} \right)^{\frac{1}{1 - \alpha}} \);

• Aggregate investment: \( \hat{I}_{ss} = \delta \hat{K}_{ss} \);

• Aggregate production: \( \hat{Y}_{ss} = \hat{K}_{ss}^\alpha N_{ss}^{1 - \alpha}; \)

• Aggregate consumption \( \hat{C}_{ss} = \hat{Y}_{ss} - \delta \hat{K}_{ss} \).

### 3.4. Solution Method and Parameter Values

A third order approximation (perturbation method) around the steady state is used as a solution method to the nonlinear New Keynesian model with Epstein and Zin preferences. This was done by using Dynare in Matlab. Details of this solution method can be seen from recent studies of solving non-linear DSGE model up to third order approximation by Caldara, Fernandez-Villaverde, Rubio-Ramirez, Yao (2012), and Andreasen, Fernandez-Villaverde, Rubio-Ramirez (2016).

The reason of using this solution method is to capture the higher order moments in asset pricing features. Hence, we can see the contribution of time varying risk component in explaining the link between asset pricing dynamics and monetary policy transmission mechanism.

A set of model’s parameters were chosen as closely as possible from Fernández-Villaverde (2010) and Caldara, Fernandez-Villaverde, Rubio-Ramirez, Yao (2012). The baseline parameters values for numerical computation in this thesis is from table 2. In addition, \( \alpha_a = 0.9 \) captures the persistence of TFP shock, \( \sigma_{TFP}^2 \) is standard deviation of TFP with 25 basic points and \( \sigma_{MP}^2 \) is standard deviation of monetary policy with 25 basic points. Simulated periods are 8000.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Common discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Risk aversion</td>
<td>5</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Inverse of intertemporal elasticity of consumption substitution</td>
<td>2</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Frisch elasticity of labour supply</td>
<td>6</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Elasticity of demand between differentiated goods</td>
<td>5</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
<td>0.0196</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Cost of price adjustment</td>
<td>0.67</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Cost of capital adjustment</td>
<td>1.5</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Fraction of capital</td>
<td>0.3</td>
</tr>
<tr>
<td>( \phi_R )</td>
<td>Degree of smoothing interest rate behavior</td>
<td>0.77</td>
</tr>
<tr>
<td>( \phi_Y )</td>
<td>Responsiveness of interest rate with respect to potential output</td>
<td>0.29</td>
</tr>
<tr>
<td>( \phi_{\Pi} )</td>
<td>Responsiveness of interest rate with respect to inflation target</td>
<td>1.29</td>
</tr>
</tbody>
</table>

**Table 2. Parameters**

A comparison of Epstein and Zin preferences and standard CRRA preferences has been conducted in order to study impacts of time varying risk component on
equity risk premium. For these two cases, all parameter values are the same, except \( \gamma = \rho = 2 \) for the standard CRRA case. It is noticed that the time varying risk component is equal to 1 when \( \gamma = \rho \). The model will then revert back to the standard NK model without Epstein and Zin preferences features.

Details of the numerical results will be discussed in Chapter 5. Also, sensitivity tests are attached in appendix 2 and they include:

1. Increases the level of risk aversion to a relatively higher value, \( \gamma = 8 \), which is 1.5 (4) times higher than the baseline Epstein and Zin (CRRA) case, holding all else constant.

2. Increases the size of the exogenous shock (e.g. standard deviation of total factor productivity, \( \sigma^2_A \)) from 25 basic points to 100 basic points, holding all else constant.

3. Increases the size of the exogenous shock (e.g. standard deviation of monetary policy, \( \sigma^2_{MP} \)) from 25 basic points to 100 basic points, holding all else constant.

4. Increases the cost of capital adjustment, \( \Phi \), from 1.5 to 3, holding all else constant.

It is noticed that for an extremely high level of risk aversion or large size of exogenous shock will move the economy far away from the deterministic steady state. This prevents solving the model at equilibrium. Hence, for the purposes of this study and for the efficacy of the model, a reasonably high level of risk aversion and relatively small exogenous shock has been chosen.
CHAPTER 4

Inspecting the Mechanism

The central bank implements monetary policy as a tool to minimize fluctuations over the business cycle. Thus, it is important to understand how the representative household allocates their consumption across periods and risky states relating to the asset pricing dynamics, as these affect the aggregate performance of the economy in equilibrium. In this chapter, I will explain in four parts the main mechanism that relates Epstein and Zin preferences to asset pricing dynamics and the role of monetary policy in influencing the general equilibrium outcomes.

Firstly, I will explain how the monetary policy transmission is linked with the household's economic trade-offs, asset pricing dynamics and aggregate performance of the economy in the model. Secondly, I will explain how the Epstein and Zin preferences with early resolution of uncertainty can generate countercyclical properties for asset pricing kernel and equity risk premium. Thirdly, I will discuss how the household with early resolution of uncertainty can have a better smoothing consumption than the standard CRRA preference benchmark. Lastly, I will explain the motivation or role of monetary policy intervention in stabilizing fluctuations over the business cycle. The purpose of this chapter is to provide the economic intuitions on understanding the dynamic outcomes of the economy discussed in chapter 5, as well as the puzzle in equity risk premium.

4.1. Implications of Asset Pricing Condition

Example 1: The link between asset pricing dynamics and monetary policy transmission.

In this example, I will explain how the monetary policy transmission is actually linked with household’s optimal intertemporal consumption choice relating to asset returns (prices) in this model. Then, I will provide intuitions on household’s optimal economic trade-offs at equilibrium.

Firstly, the one-period nominal interest rate (e.g. policy rate) set by the central bank is used as a proxy for the nominal bond bond return $R_t (s_t)$ in this model. This is because the Federal Funds Rate moves quite closely with the 90-day Treasury bond yield as shown in figure 2. This suggests that monetary policy transmission directly links with the household’s stochastic Euler equation (5) to pin down her
optimal intertemporal consumption choice. In this model with complete assets market, we will realize that asset prices and asset returns are two sides of the same coin in studying household’s optimal risky intertemporal consumption trade-offs in the economy.

For example, we can price the nominal bond at time \( t \) given realization of state \( s_t \) as:

\[
P^B_t(s_t) = M_{t,t+1} \bar{W}_t(s_t)
\]

where \( M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{\psi}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}} \frac{1}{\Pi_{t+1}(s_{t+1})} \right] \), and \( \bar{W}_t(s_t) \) is the payout of the bond which pays out 1 dollar per unit of the bond for all states.

This means that the price of nominal bond at time \( t \), state \( s_t \), of a claim to the payoff \( \bar{W}_t(s_t) \) is govern by the inflation adjusted asset pricing kernel, \( M_{t,t+1} \). That is,

\[
P^B_t(s_t) = \mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{\psi}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}} \frac{1}{\Pi_{t+1}(s_{t+1})} \right] \bar{W}_t(s_t)
\]

\[
1 = \mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{\frac{1-\gamma}{\psi}} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\psi}} \frac{1}{\Pi_{t+1}(s_{t+1})} \right] \frac{\bar{W}_t(s_t)}{P^B_t(s_t)}
\]

where the \( R_t(s_t) \) is the one period gross return on the asset (nominal bond), and \( M_{t,t+1} \) is the inflation adjusted pricing kernel (i.e. measures of intertemporal marginal rate of substitution of consumption) on this return.

The following shows the break down of the implications on asset price for this state-contingent nominal bond:

\[
R_t(s_t) = \frac{\bar{W}_t(s_t)}{P^B_t(s_t)} = \frac{1}{P^B_t(s_t)}
\]

\[
\iff \frac{1}{R_t(s_t)} = P^B_t(s_t) \quad (4.1.1)
\]

According to equation (4.1.1), the inverse of one period gross return on the asset (nominal bond) is equal to it’s underlying asset price. Both variables are state
contingent because the central bank’s one-period nominal interest rate (e.g., policy rate) is used as a proxy for $R_t(s_t)$, which is subject to exogenous monetary policy shock. Hence, the change in monetary policy rate will affect the nominal bond return, and equivalently it’s underlying asset prices. Given the effect on the change in asset returns, the household’s stochastic Euler equation will pin down her underlying optimal risky intertemporal consumption relating to this asset return accordingly. This shows how the monetary policy transmission is linked with household’s economic trade-offs.

Let us begin with a thought experiment to draw more insights on the relationship between monetary policy transmission mechanism and optimal economic trade-offs for household with recursive preferences. Suppose central bank implements an expansionary monetary policy during recession. From equation (5.2), the asset pricing kernel is higher in response to a lower one period gross return on the nominal bond. This means that the monetary policy rate has direct impact on household’s inflation adjusted asset pricing kernel. According to equation (5), a lower return on nominal bond results in a lower marginal utility of current consumption for household. According to the rule of diminishing marginal utility of consumption, the household’s actual current consumption is higher. This shows how the monetary policy transmission is linked with household’s stochastic Euler equation, which pins down the underlying optimal risky intertemporal consumption relating to asset returns accordingly.

On the other hand, we can also see that lower nominal interest rates set by the central bank implies a positive aggregate demand shock to the economy. Any change in household’s consumption will lead to a change in firm’s production. This is because the firm’s production choice is demand determined. Ultimately, there will be an impact on output in the economy, according to goods market clearing condition, represented by equation (17). In this example, the firm needs to produce more to meet this higher demand as the consumption has now increased. At the same time, this positive aggregate demand shock will lead to higher inflation in the economy, according to the New Keynesian Phillips Curve equation (20). This shows how the transmission of monetary policy links the policy rate to the aggregate performance of the economy.

In terms of asset pricing dynamics with household economic trade-offs, the price of nominal bond increases in response to the lower one-period gross return, according to (4.1.1). The impacts of changing in asset returns directly influences household’s pricing kernel, according to equation (5.2). The household then chooses her optimal intertemporal consumption allocation based on the stochastic Euler equation. An economic intuition on the negative relationship of asset prices and asset returns is that household demand more (less) for holding the nominal bond during recession.
(boom). In this case, an expansionary monetary policy during a recession will result in higher demand for household to hold the nominal bond. This is driving up (down) the bond price (bond return). From this economic trade-off intuition, we can see the spread in equity risk premium will be wider, holding all else constant. Intuitively, the household requires a higher equity risk premium to compensate her for investing the more risky asset (capital stock) during bad times of the economy.

This example helps us understand how the monetary policy transmission has a direct effect on the household’s stochastic Euler equation. In turn, the household’s stochastic Euler equation pins down her optimal intertemporal consumption choice in relation to asset returns. In other words, the link of asset returns (or asset pricing) mechanism in this model is governed by the household’s pricing kernel. The model also shows how the monetary policy transmission influences the policy rate to have a direct impact on the aggregate performance of the economy. I have showed that the asset price of the nominal bond can be derived from the stochastic Euler equation (5) with asset pricing kernel (5.2), e.g. $M_{t,t+1} = \frac{1}{R_t(s_t)}$. The change in asset prices and returns imply the same impacts on the household’s economic trade-offs in this model. Hence, asset prices and asset returns are two sides of the same coin in studying asset pricing dynamics with the household’s underlying intertemporal consumption trade-offs.

**Example 2: Countercyclical properties in asset pricing dynamics with recursive preferences.**

Given that the central bank implements countercyclical monetary policy and a household has Epstein and Zin preferences with $\gamma > \rho$ (prefers early resolution of uncertainty), it implies that:

1. the household’s optimal consumption allocation and investment (e.g. capital accumulation) are both linked with assets return, which has impacts on aggregate performance of the economy,
2. there is a countercyclical asset pricing kernel,
3. there is a countercyclical equity risk premium (e.g. negative co-movement with the business cycle).

Suppose that the economy is experiencing a bad time (recession) and the central bank sets a lower nominal interest rate $R_t(s_t)$ only at time $t = 0$, (e.g. an expansionary monetary policy), holding all else constant.

The lower nominal interest rate will have a direct effect on the household’s intertemporal consumption choice and asset return. This is because the monetary policy transmission links the policy rate with the household’s asset pricing kernel (5.2). At equilibrium, the stochastic Euler equation will pin down the household’s
underlying optimal consumption allocation via asset return in the model. According to equation (5), the household’s marginal utility of consumption in the current period decreases in response to the lower one-period gross return on nominal bond. According to diminishing marginal utility of consumption, the household’s current (future) period consumption is high (low).

An expansionary monetary policy also impacts the investment channel, which influences Tobin’s marginal $q$ ratio and the one-period gross return on capital stock. According to the goods market clearing condition, (17), investment decreases as current consumption increases. This is because the household has to allocate her income in investment and consumption goods, see equation (2). The lower investment leads to a lower Tobin’s marginal $q$ ratio. Since, the $q$ ratio is defined as the stock value of the firm relative to consumption good (details see appendix 1), a low $q$ ratio means investment on capital stock is less attractive to the household in terms of consumption good. Hence, the household accumulates less capital for future consumption. In equilibrium, there must be an increase in the one-period gross return on capital stock due to the rule of diminishing marginal rate of capital. From these two economic trade-offs, we can see the household’s optimal consumption allocation and investment choices move in an different direction to assets return at equilibrium.

In the event that the central bank implements a countercyclical monetary policy, the model can generate a countercyclical asset pricing kernel for a household who prefers an early resolution of uncertainty, $\gamma > \rho$. According to equation (18), the asset pricing kernel, equation (5.2), must increase in response to a fall in nominal interest rate set by the central bank during recession. For a higher asset pricing kernel to hold in equation (5.2), a household that has a preference of early resolution of uncertainty must attach more weight on the low continuation value of future consumption relative to her certainty equivalent value. We can use the time varying risk component to break down this intuition as follows.

The time varying risk component in the asset pricing kernel is:

$$\left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma}}{J_t [V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\frac{1}{\Psi}}$$

For a household prefers early resolution of uncertainty, it means that.

$$[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma} < J_t [V_{t+1}(C_{t+1}(s_{t+1}))],$$

where $\Psi = \frac{1-2}{1-\rho}$ and $\gamma > \rho$ holds.

Intuitively, this inequality means that the household will require a higher certainty equivalent relative to the continuation value of future consumption in order to make her indifferent between the current and future consumption. Due to the
household’s preference of early resolution of uncertainty, she will dislike future consumption risk relatively more. Given this inequality, this means that the time varying risk component will increase since it will be raised to a negative power. We can see how the model can generate a countercyclical asset pricing kernel for a household has Epstein and Zin preferences with early resolution of uncertainty. Countercyclical property in asset pricing dynamics is important to maintain an effective monetary policy transmission mechanism, as it impacts on aggregate performance of the economy in this model. Essentially, the countercyclical asset pricing dynamics can govern a lower monetary policy rate, which will increase the household’s current consumption. This is because the household’s optimal economic trade-offs with asset returns is pinned down by the stochastic Euler equations. Thus, it governs an effective monetary policy transmission mechanism.

Equity risk premium in the model is defined as the difference of returns between a relatively more risky asset (capital stock) and a less risky asset (nominal bond). In response to a lower monetary policy rate set by the central bank during recession, we see there is an increase in return for capital stock and a decrease in return on nominal bond, as already discussed. The spread in equity risk premium is therefore larger. The opposite occurs if the central bank implements a higher policy rate during the boom. This suggests that the equity risk premium also has countercyclical features in this model. In particular, the co-movement of equity risk premium is negative to the business cycle, which explains why the premium is high during recessions and vice versa. The countercyclical asset pricing dynamics in this model captures features on co-movement of equity risk premium with the business cycle, as shown in figure 2.

From the countercyclical property of equity risk premium, we can also gain some economic intuitions on understanding why the historical average spread in the equity risk premium is large (i.e. equity risk premium puzzle). A countercyclical equity risk premium means that household requires a higher (lower) risk premium to compensate herself on investing the capital stock during recession (boom). Intuitively, this implies that the historically large spread in equity risk premium is not only because of the higher potential earning rewards on equity, but is also due to the underlying additional risk compensation required by investors investing in the riskier asset. I will discuss this further in relation to the model statistics in Chapter 5.

In this example, we see that for a household that has Epstein and Zin preferences, especially with early resolution of uncertainty, the model can generate countercyclical properties in asset pricing dynamics. We can gain better economic intuitions on understanding the equity risk premium puzzle and features behind the co-movement of equity risk premium with the business cycle, as shown in figure 2.
Another implication of the Epstein and Zin preference with early resolution of uncertainty, is that the household can also have a better smoothing consumption across periods and risky states, as compared to the household with CRRA preference in the standard NK model. From this example, we can see the time varying risk component in an asset pricing kernel matters for the household self-insurance against the aggregate shock that she faces in the economy. I will illustrate this implication in the following example.

Example 3: Behavior of consumption smoothing under recursive preferences.

In this model, the household with Epstein and Zin preferences who prefer an early resolution of uncertainty, $\gamma > \rho$, can have a better smoothing consumption across periods and risky states, relative to the CRRA preferences, $\gamma = \rho$.

Let us revisit the stochastic Euler equation with nominal bond return, equation (5), again.

$$(C_t(s_t))^{\bar{\gamma}} - 1 = R_t(s_t) \beta \mathbb{E}_t \left[ (C_{t+1}(s_{t+1}))^{\bar{\gamma}} - 1 \right]$$

where $\gamma > \rho$ and $\Psi = \frac{1 - \gamma}{1 - \rho}$ is the index captures deviation with respect to the benchmark CRRA utility, $\rho := \frac{1}{\alpha} ES$, and $\gamma$ is risk aversion.

The stochastic Euler equation reverts to the benchmark CRRA preference when $\gamma = \rho$, such that the household’s risk aversion is equal to the inverse intertemporal elasticity of substitution. Then the index $\Psi = \frac{1 - \gamma}{1 - \rho}$ will result in a unity for the time varying risk component in this case. The stochastic Euler equation for the benchmark CRRA preferences become:

$$(C_t(s_t))^{\bar{\gamma}} - 1 = R_t(s_t) \beta \mathbb{E}_t \left[ (C_{t+1}(s_{t+1}))^{\bar{\gamma}} - 1 \right]$$

Suppose a lower nominal interest rate is set by the central bank during recession. From the above examples, we have seen how the household’s stochastic Euler equation pins down her optimal intertemporal consumption in response to a lower nominal interest rate. However, the magnitude of decreasing marginal utility of current consumption will be smaller for Epstein and Zin preferences with early resolution of uncertainty, relative to the standard CRRA preferences benchmark. This is because the household with early resolution of uncertainty will attach more weight to the low continuation value of future consumption relative to her certainty equivalent. That is,

$$[V_{t+1}(C_{t+1}(s_{t+1}))]^{1-\gamma} < J_t [V_{t+1}(C_{t+1}(s_{t+1}))]$$
4.2. STABILIZING FLUCTUATIONS OVER THE BUSINESS CYCLE

where $\Psi = \frac{1-\gamma}{1-\rho}$ and $\gamma > \rho$.

This makes the time varying risk component increase, as it will be raised to a negative power. The time variation of risk will thus dilute part of the impact of a decrease in nominal interest rate in the stochastic Euler equation. This results in a smaller decrease in the marginal utility of current consumption for a household who prefers an early resolution of uncertainty. Hence, the increase in the household's actual current consumption will be relatively smaller than CRRA preference. According to equation (17), this suggests household will invest more on capital accumulation (saving) for future consumption relative to the household with CRRA preference.

The implication of a household with this type of Epstein and Zin preference is that she has relatively greater incentive to smooth out consumption via asset returns across periods and risky states. This is because she dislikes future consumption risk given that she faces aggregate exogenous shocks in this model. This allows a household that has Epstein and Zin preferences with early resolution of uncertainty to achieve a better smoothing consumption across periods and risky states, via the channel of a time varying risk component in their optimal economic trade-offs.

In these three examples, we have seen how monetary policy transmission is linked with the household's economic trade-offs, and aggregate performance of the economy. We have seen how the household pins down her optimal risky intertemporal consumption trade-offs relating to asset returns in the model. The model can generate countercyclical property in asset pricing dynamics for a household has Epstein and Zin preferences with early resolution of uncertainty. This countercyclical property maintains an effective monetary policy transmission mechanism to have impacts on the dynamic outcomes of the economy. Further, the time varying risk component contributes a better smoothing consumption allocation for a household with recursive preferences relative to the benchmark CRRA preferences.

4.2. Stabilizing Fluctuations Over The Business Cycle

Convex cost of price adjustment implies inflation is costly to the firm and also the economy as a whole. The intuition is as follows. The firm increases their production prices subject to convex cost of price adjustment which reduces their profitability, see equation (13). Since the firm’s own price inflation is costly in their optimal pricing decision, prices therefore tend not to adjust quickly across time. This contributes to sticky prices which is also found in empirical data. Further, there is a convex cost of capital adjustment. Hence, the sticky prices, costly capital adjustment and monopolistic goods market all lead to market distortions on equilibrium allocation.

\footnote{The Cavall and Rigobon (2016) "The Billion Prices Project", big data project on prices and inflation.}
This distortion is costly to the economy as a whole, which suggests that there is a role for monetary policy intervention to stabilize short run fluctuations in the economy. The following example shows that the monetary policy can also be used as a tool to stabilize fluctuation over the business cycle, given that there is an exogenous shock that hits the economy.

Example 4: Mechanism of stabilizing fluctuations over the business cycle.

Given that an exogenous shock hits the economy, lower fluctuations over the business cycle can be achieved via the implementation of the central bank’s monetary policy. Suppose the agent prefers early resolution of uncertainty \( \gamma > \rho \) and there is a positive total factor productivity shock that hits the economy, holding all else constant.

Higher productivity shock (e.g. an increase in level of technology) leads to a higher production, according to equation (14). The rationale behind this is because higher technology lowers the real marginal cost of production, according to equation (12) and (12.1). Hence it is more profitable for firm to produce more goods according to equation (13). Since the firm’s production is demand determined, then the higher production must now be absorbed by higher consumption according to equation (17). Then current period marginal utility of consumption must be lower according to the rule of diminishing marginal utility of consumption. The process of the household shifting consumption forward is governed by a higher pricing kernel, equation (18).

For example, the nominal interest rate is lower in response to the higher pricing kernel. This implies the household’s stochastic Euler equation will then pin down her optimal intertemporal consumption allocation via asset returns accordingly. As a result of a positive total factor productivity shock, there is a higher production and lower real marginal cost in the economy. This is causing an upward inflationary pressure on the economy according to the New Keynesian Phillips Curve, equation (20).

To offset parts of the fluctuations on the economy caused by the technological shock, the central bank needs to raise the nominal interest rate (e.g. a contractionary monetary policy). This is because the inflation adjusted pricing kernel (5.2) will become lower in response to a higher nominal interest rate. Consequently, the lower pricing kernel then governs a process of shifting the household’s consumption to the future period according to equation (5). This is because a lower pricing kernel implies a higher marginal utility of current consumption and therefore decreases current consumption. Considering these two effects, if the central bank raises the nominal interest rate that leads to a lower pricing kernel, it can reduce the effects of the higher pricing kernel caused by technological improvement. This means that there will be minimal change in current consumption because both opposing forces
will partially balance out. The opposite effect occurs if there is a negative total factor productivity shock and the central bank will need to lower the nominal interest rate to minimize fluctuation in this case. Impacts on dynamic outcomes of the economy depending on the sign and size of the shock. This economic intuition also helps us understand how the monetary policy transmission mechanism has an impact on the aggregate performance of the economy, as well as the role of monetary policy intervention in the model.
CHAPTER 5

Experiments and Numerical Results

In this Chapter, I will first discuss the experimental design to study dynamic properties of the model. These experiments allow us to investigate how the asset pricing dynamics and aggregate performance of the economy given an exogenous shock (either total factor productivity or monetary policy) that hits the economy. As a result, it is possible to examine how the asset pricing dynamics is linked with the household’s economic trade-offs to generate an equilibrium outcome. I also compare the NK model with Epstein and Zin preferences ($\gamma > \rho$) to those with the benchmark CRRA preferences ($\gamma = \rho$). That is, if level of risk aversion is equal to inverse intertemporal elasticity of substitution, the model reverts back to the standard NK model without the Epstein and Zin preferences feature. In particular, the time varying risk component is absent in the household’s stochastic Euler equations relating with asset returns. The comparison of two different class of preferences in an NK economy will allow us to understand: (1) the importance of time varying risk component in influencing equity risk premium and (2) the importance of time varying risk component in asset returns dynamics and monetary policy transmission mechanism, subject to total factor productivity (TFP) or monetary policy shock.

5.1. Experiment Design

Results of experiment discussed in section 5.3 and 5.4 considers the response of the NK economy with Epstein and Zin preference (or CRRA preference) to a one-time unexpected TFP or monetary policy shock, holding all else constant. Section 5.3 considers the TFP shock only. The economy is initially at steady state and the exogenous TFP shock has $\sigma_A^2 = 0$. When the experiment is at time $t = 0$, there is an exogenous TFP shock of $\sigma_A = 0.0025$ (25 basic points) that hits the economy.

Section 5.4 considers the monetary policy shock. The economy is initially in a steady state and the exogenous monetary policy shock has $\sigma_{MP}^2 = 0$. When the experiment is at time $t = 0$, there is an exogenous monetary policy shock of $\sigma_{MP}^2 = 0.0025$ (25 basic points) that hits the economy. From these experiments, we can gain intuitions on the household’s optimal risky intertemporal consumption choice relating to asset returns, and hence the dynamic properties of the model.
5.2. Numerical Results

Change in the level of risk aversion has the most significant impacts on equity risk premium and dynamic outcomes of the economy. Hence it is the main focus of discussion in this section. I will first discuss the model statistics for the Epstein and Zin preference with early resolution of uncertainty ($\gamma > \rho$). Then, I will compare this to the CRRA benchmark without Epstein and Zin feature, ($\gamma = \rho$). This gives us a picture about what we will expect to see in section 5.3 and 5.4 regarding the dynamic properties of the model. This will give us insights on the importance of asset pricing dynamics with Epstein and Zin preferences for aggregate performance of the economy, subject to exogenous shocks.

In comparing NK models with Epstein and Zin preferences and benchmark CRRA preferences, all the parameters follow the values found in table 2, except for risk aversion. For the results shown on table 3 and 4, the benchmark CRRA case is uses $\gamma = \rho = 2$. The Epstein and Zin model 1 and 2 follow the values $\gamma = 5, \rho = 2$ and $\gamma = 8, \rho = 2$ respectively.

<table>
<thead>
<tr>
<th>Summary of statistics:</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_k$</td>
<td>0.073661</td>
<td>0.025538</td>
<td>0.805661</td>
<td>0.413923</td>
</tr>
<tr>
<td>$R$</td>
<td>0.053078</td>
<td>0.036737</td>
<td>0.767015</td>
<td>0.904394</td>
</tr>
<tr>
<td>$R_k - R$</td>
<td>0.020580</td>
<td>0.019830</td>
<td>-0.650981</td>
<td>0.216091</td>
</tr>
<tr>
<td>1. Benchmark Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_k$</td>
<td>0.052119</td>
<td>0.000002</td>
<td>0.095161</td>
<td>0.158152</td>
</tr>
<tr>
<td>$R$</td>
<td>0.026374</td>
<td>0.000016</td>
<td>0.000655</td>
<td>-0.007663</td>
</tr>
<tr>
<td>$R_k - R$</td>
<td>0.025745</td>
<td>0.000017</td>
<td>0.010609</td>
<td>-0.020643</td>
</tr>
<tr>
<td>2. Epstein and Zin-Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_k$</td>
<td>0.051508</td>
<td>0.000002</td>
<td>0.130804</td>
<td>0.311550</td>
</tr>
<tr>
<td>$R$</td>
<td>0.024092</td>
<td>0.000010</td>
<td>-0.012731</td>
<td>-0.038653</td>
</tr>
<tr>
<td>$R_k - R$</td>
<td>0.027416</td>
<td>0.000011</td>
<td>0.025892</td>
<td>-0.037417</td>
</tr>
<tr>
<td>3. Epstein and Zin-Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_k$</td>
<td>0.051169</td>
<td>0.000002</td>
<td>0.128918</td>
<td>0.303200</td>
</tr>
<tr>
<td>$R$</td>
<td>0.022818</td>
<td>0.000010</td>
<td>-0.012469</td>
<td>-0.037830</td>
</tr>
<tr>
<td>$R_k - R$</td>
<td>0.028351</td>
<td>0.000011</td>
<td>0.025510</td>
<td>-0.037137</td>
</tr>
<tr>
<td>Premium (2-1)</td>
<td>0.001671</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Premium (3-1)</td>
<td>0.002606</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Asset returns and equity risk premium.

The model statistics in table 3 shows the comparison between the benchmark CRRA preferences against the model with Epstein and Zin preferences. As the level of risk aversion increases from $\gamma = 2$ to $\gamma = 5$, the time varying risk component enlarges the mean of quarterly equity risk premium by approximately 0.17%. Similarly, as the level of risk aversion increases from $\gamma = 2$ to $\gamma = 8$, the time
Summary of model statistics:

1. **Benchmark Model**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.9138</td>
<td>0.000003</td>
<td>0.045658</td>
<td>-0.055219</td>
</tr>
<tr>
<td>$C$</td>
<td>0.8389</td>
<td>0.000024</td>
<td>-0.013993</td>
<td>-0.004704</td>
</tr>
<tr>
<td>$X$</td>
<td>0.0749</td>
<td>0.000021</td>
<td>0.019303</td>
<td>-0.005402</td>
</tr>
<tr>
<td>$K$</td>
<td>3.8226</td>
<td>0.000121</td>
<td>-0.132131</td>
<td>-0.182812</td>
</tr>
<tr>
<td>$mc$</td>
<td>0.8000</td>
<td>0.000006</td>
<td>-0.036618</td>
<td>0.067167</td>
</tr>
<tr>
<td>$N$</td>
<td>0.4949</td>
<td>0.000004</td>
<td>0.007837</td>
<td>0.003554</td>
</tr>
</tbody>
</table>

2. **Epstein and Zin-Model 1**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.9169</td>
<td>0.000007</td>
<td>0.072540</td>
<td>-0.073530</td>
</tr>
<tr>
<td>$C$</td>
<td>0.8411</td>
<td>0.000015</td>
<td>0.047306</td>
<td>-0.036209</td>
</tr>
<tr>
<td>$X$</td>
<td>0.0758</td>
<td>0.000009</td>
<td>-0.025481</td>
<td>-0.019261</td>
</tr>
<tr>
<td>$K$</td>
<td>3.8685</td>
<td>0.000071</td>
<td>-0.143940</td>
<td>-0.215568</td>
</tr>
<tr>
<td>$mc$</td>
<td>0.8000</td>
<td>0.000004</td>
<td>-0.041348</td>
<td>0.068277</td>
</tr>
<tr>
<td>$N$</td>
<td>0.4948</td>
<td>0.000002</td>
<td>-0.042701</td>
<td>-0.009622</td>
</tr>
</tbody>
</table>

Table 4. Aggregate performance of the economy.

Varying risk component enlarges the mean of quarterly equity risk premium by approximately 0.26%. This shows that the household with Epstein and Zin preference who prefers early resolution of uncertainty, $\gamma > \rho$, dislikes future consumption risk relatively more. This means that this type of household attaches more weight on low continuation value of future consumption relative to her certainty equivalent. It implies the asset pricing kernel will increase more (see equation (5.2)) because the time varying risk component increases as the level of risk aversion increases. The fall in nominal bond return must then be larger, according to equation (18). Therefore, the time varying risk component in the asset pricing kernel creates a wider spread between return on capital stock and nominal bond return. The intuitions we gain from Chapter 4 support this finding that higher premiums are required for a more risk adverse household.

Implications of this wider spread in equity risk premium can be explained by the higher order moments in the model statistics. Consider the Epstein and Zin model 1 in table 3. The return on capital stock is moderately skewed to the right, and the return on the nominal bond is symmetric around the mean. This suggests that the capital stock has a relatively higher earning potential than the nominal bond in this model. At the same time, the capital stock is also a riskier asset as compared to the nominal bond. This is because the return on capital stock has an excess kurtosis greater than 0 (i.e. leptokurtic distribution), whereas the return on nominal bond has an excess kurtosis less than 0 (i.e. platykurtic distribution).

A leptokurtic distribution means that the returns on capital stock has a fatter tail and relatively smaller standard deviation. This suggests that small changes on the underlying return on capital stock happens less frequently, and large fluctuations are more likely to be at the fat tails (e.g. extreme and unpredictable event). On the
other hand, a platykurtic distribution means that the returns on nominal bond has a smaller peak and thin tails. This suggests there are less large fluctuations happening at tail than assets (e.g., capital stock) with leptokurtic distribution. In other words, an investment on the nominal bond comes with lower risk in this economy. From these model statistics, we can see there is a trade-off between risks and returns on the underlying assets which implies that the household does not necessarily hold more equity, even if it has higher earnings potential. A more risk averse household would like to hold more of the nominal bond to smooth out her consumption across periods and risky states, as the household will find the investment with platykurtic return distributions (e.g., nominal bond) best suited to her level of risk aversion as compared to a riskier asset (e.g., capital stock).

These model statistics imply investing in assets in this economy. The household enjoys a higher upside potential at the cost of bearing the higher associated downside risk at the tail by investing the capital stock, as compared to the nominal bond. In fact, a more risk averse household dislikes this type of risky distribution of returns. She requires a higher equity risk premium to compensate herself for the additional risk in investing in the riskier asset. This finding is supported by the intuition we gain from Chapter 4. This is because we saw how the household has Epstein and Zin preference with early resolution of uncertainty has greater incentive to smooth out consumption across times and risky states, relative to the benchmark CRRA. Hence, a more risk averse household will hold more of the less risky asset.

According to equation (4.1.1), this increasing demand for holding the nominal bond is driving down the underlying rate of return. This enlarges the difference between returns on capital stock and nominal bond in this model. The asset pricing feature with time varying risk component in this NK Epstein and Zin model enhances a better understanding on the equity risk premium puzzle. That is, the empirically high spread in equity return and treasury bond return is not only because equity has higher upside rewards potential relative to the government bond, but it is also due to the associated risk compensation on investing the equity as required by the investor.

According to table 3, signs and magnitudes of asset returns for skewness and excess kurtosis statistics are almost the same for the household with Epstein and Zin preferences feature and the benchmark CRRA preferences. This means that preferences do not change the fundamental risk aspects of assets. The difference between NK model with Epstein and Zin preference to the benchmark CRRA preference is due to the time varying risk component in the household’s stochastic Euler equations. This suggests that the household’s optimal economic trade-offs will be weighted differently in relation to asset returns depending on the class of preferences. In particular, we can understand the importance of Epstein and Zin preferences with
early resolution of uncertainty in asset pricing dynamics. We can draw some insights on this with the aggregate performance of the economy by looking at volatility of asset returns in these two classes of preferences.

Table 3 also shows that asset returns are less volatile in the NK model with Epstein and Zin feature relative to the CRRA benchmark. This means that the household with Epstein and Zin feature does not require a much larger change of asset returns to pin down the underlying optimal consumption allocation. This is due to the time-varying risk component captured as an additional factor in the household’s stochastic Euler equation. The time varying risk component produces a smaller weight in the optimal economic trade-offs for a household that has Epstein and Zin preferences who prefers an early resolution of uncertainty, \( \gamma > \rho \). This is supported by the economic intuition we discussed in Chapter 4, Example 3. We saw how the time varying risk component dilutes some of the impacts of change in asset returns on marginal utility of current consumption. Given the effect on asset pricing, the actual allocation of consumption should be less volatile for the household because the household who prefers early resolution of uncertainty has a larger incentive for smoothing consumption as she dislikes future consumption risk more. The model statistics in table 4 support this intuition as aggregate consumption is relatively less volatile for the household with Epstein and Zin preferences feature. This suggests that the time-varying risk component contributes to a better smoothing consumption path in the NK Epstein and Zin economy.

There are some weaknesses in studying asset prices in this model. It is well-known that the benchmark NK model with CRRA preferences has already failed at accounting for the additional volatility on asset prices in matching the historical data, see table 3. However, Epstein and Zin preferences with time varying risk component in an NK production economy has its shortcomings as well. In this case, Epstein and Zin preferences make the volatility of the model’s asset returns even smaller, as evident in table 3. Although the time varying risk component can explain the equity risk premium (a mean effect), it is weak in accounting for the asset prices at higher order moments to the data. The weakness is due to the limitation of the Epstein and Zin preferences in a production economy with labour supply, as discussed by Uhlig (2007). Uhlig (2007) showed that for a household that has Epstein and Zin preferences, leisure affects asset pricing in a significant way through the risk-adjusted expectation operator, even when it is separate in the period utility function. Therefore, Epstein and Zin preferences also contribute to the shortcomings in accounting for the asset prices at higher order moments to match the data in a monetary policy DSGE model.
5.3. Impulse Response 1: Total Factor Productivity Shock

**Figure 3.** Impulse response functions of total factor productivity shock for Epstein and Zin preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).

**Figure 4.** Impulse response functions of total factor productivity shock for Epstein and Zin preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).
5.3. IMPULSE RESPONSE 1: TOTAL FACTOR PRODUCTIVITY SHOCK

According to figures 3 and 4 (NK economy with Epstein and Zin preferences, $\gamma = 5 > \rho = 2$) and figures 5 and 6 (NK economy with CRRA preferences, $\gamma = \rho = 2$), these show the impulse responses of the economy subject to a one-time exogenous TFP shock, holding all else constant. Each one of these figures show the level deviation of variable from the deterministic state as a result of the TFP shock. From

**Figure 5.** Impulse response functions of total factor productivity shock for CRRA preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).

**Figure 6.** Impulse response functions of total factor productivity shock for CRRA preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).
this experiment, we can understand the dynamic properties of the model (e.g. asset returns dynamics and aggregate performance of the economy) and thus compare the dynamics with the Epstein and Zin feature, in relation to the CRRA benchmark.

Let us first consider the asset pricing dynamics and aggregate performance of the economy subject to the TFP shock, for the case of Epstein and Zin preferences, as shown in figures 3 and 4. Given a positive exogenous TFP shock, the higher level of technology $A_t(s_t)$ leads to a higher rental rate $r^R_t(s_t)$, see equation (12.1). This higher rental rate leads to a higher return on capital stock, since it is defined as $R^k_t(s_t) = 1 + r^R_t(s_t) - \delta$. According to the rule of diminishing marginal rate of return on capital, lower capital is accumulated in response to the higher net return on capital stock. Intuitively, less capital is needed for production as there is a technological improvement. Hence, there is a decrease in investment.

The lower investment implies that the actual current consumption is higher according to the goods market clearing condition, equation (17). The process of the household increasing current consumption is pinned down by a lower nominal bond return in the stochastic Euler equation (5). This is because the marginal utility of current consumption decreases in response to a lower nominal bond return. This asset returns dynamic links the household’s optimal economic trade-offs to aggregate performance of the economy. As a result of the TFP shock, the net return on capital stock, consumption and output are positively deviating from the deterministic steady state; where capital, investment and return on the nominal bond are negatively deviating from the steady state. Since return on capital stock and return on nominal bond are both going in the opposite direction, there is a positive deviation of equity risk premium from the steady state. The economy eventually converges back to the initial steady state as the lasting effect of the TFP shock fades.

In response to the TFP shock, the sign of the impulse responses in the NK economy with Epstein and Zin preferences are the same as those with CRRA preferences benchmark. The only difference is in the size of the responses for economic variables. Let us first consider the asset pricing dynamics across these two types of preferences in figures 3 and 5. We see that the responses of the asset pricing dynamics with Epstein and Zin preferences feature is relatively smaller, in relation to the benchmark CRRA preferences. For example, the magnitude of responses of return on capital stock and nominal bond are relatively smaller under Epstein and Zin preferences features. Given the effect of the smaller response of asset returns, the impact on the household’s marginal utility of consumption is smaller in her stochastic Euler equation. This lower volatility is due to the impact of the time varying risk component in the household’s stochastic Euler equations underlying the optimal economic trade-offs. Hence, the size of the responses of dynamic outcomes in the economy
with the Epstein and Zin preferences are smaller, as compared to the benchmark CRRA preferences.

Given that the Epstein and Zin feature has a lower volatile asset pricing dynamic, let us compare the aggregate performance of the economy in figures 4 and 6. In response to the TFP shock, the size of the positive deviation of current consumption from the steady state is relatively smaller for Epstein and Zin preferences, in relation to benchmark CRRA preferences. This is supported by the model statistics in table 4, and also the economic intuition we discussed above and in Chapter 4.1 Example 3. In particular, the model statistics show that the aggregate consumption is relatively less volatile for the household with Epstein and Zin preferences feature (e.g. Epstein and Zin preferences with early resolution of uncertainty). This is because the household with this type of preference attaches more weight on low continuation value of future consumption relative to her certainty equivalent. This means that the time varying risk component is higher since it is increasing in the level of risk aversion. The economic implication is that a household has Epstein and Zin preferences with early resolution of uncertainty, \((\gamma > \rho)\), dislikes future consumption risk more. It means that she prefers a more smoothing consumption path across periods and risky states. This suggests that in the household's economic trade-offs, the time varying risk component dilutes some of the impacts of the fall in nominal bond return on marginal utility of current consumption. According to stochastic Euler equation, equation (5), the size of the response of the marginal utility of current consumption is therefore relatively smaller for Epstein and Zin preferences, as compared to the benchmark CRRA preferences. As a result, the size of response in actual consumption is therefore relatively smaller for the Epstein and Zin features.

In response to the higher return on capital stock caused by the TFP shock, the size of the response in falling aggregate capital stock (or investment) is smaller for the household with Epstein and Zin preferences, as compared to the CRRA preferences benchmark. It can be interpreted that the household with Epstein and Zin features saves more for future consumption relative to the benchmark CRRA preferences. This is because the household who prefers early resolution of uncertainty dislikes future consumption risk more. She saves more to ensure a relatively smoother consumption path across periods and risky states. Also, consumption-labour trade-off is relatively smaller for the household with Epstein and Zin features, given that there is a smaller response in asset pricing dynamics. Hence, the size of the response in aggregate labour is relatively smaller. As the TFP shock hits the economy, we can see that there is an increase in the aggregate consumption and a fall in aggregate investment. The size of the response in increasing output is relatively larger under Epstein and Zin preferences, as compared to the benchmark CRRA preferences.
For benchmark CRRA preferences, the magnitude of the fall in investment is larger because the size of an increase in the return on capital stock is larger than that under the Epstein and Zin preferences.

In this experiment, subject to a positive TFP shock, we can see that there are some good dynamic properties in the model with Epstein and Zin features, as compared to the benchmark CRRA preferences. The time varying risk component in stochastic Euler equation results in a relatively more stable (or less fluctuated) asset pricing dynamics. This leads to a relatively smaller response in the household’s underlying optimal economic trade-offs and thus, more stable dynamic outcomes of the economy.

5.4. Impulse Response 2: Monetary Policy Shock

![Figure 7](image_url)

**Figure 7.** Impulse response functions of monetary policy shock for Epstein and Zin preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).
5.4. IMPULSE RESPONSE 2: MONETARY POLICY SHOCK

Figure 8. Impulse response functions of monetary policy shock for Epstein and Zin preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).

Figure 9. Impulse response functions of monetary policy shock for CRRA preference with capital cost adjustment. (x-axis: periods, in quarters and y-axis: level deviation from deterministic steady state).
According to figures 7 and 8 (NK economy with Epstein and Zin preferences, $\gamma = 5 > \rho = 2$) and figures 9 and 10 (NK economy with CRRA preferences, $\gamma = \rho = 2$), these show that the impulse responses of the economy are subject to a one-time exogenous monetary policy shock, holding all else constant.

Let us first consider the asset pricing dynamics and aggregate performance of the economy subject to the monetary policy shock, for the case of Epstein and Zin preferences in figures 7 and 8. Given a negative exogenous monetary policy shock, there is a positive deviation of the nominal interest rate $R_t(s_t)$ from the steady state. This is actually just an exogenous shock to the nominal bond return, since the policy rate is used as a proxy for it in this model. This monetary policy transmission mechanism is linked with the household’s optimal economic trade-offs via a nominal bond return. This results in impacts on the aggregate performance of the economy. In particular, the dynamic outcomes of the economy move in an opposite direction to the example where the monetary policy rate is lower in that case, as discussed in Chapter 4.

In this case, the higher return on nominal bond leads to a higher marginal utility of current consumption, see equation (5). Further, the return on capital stock is higher since marginal utility of current consumption is higher, see equation (5.1). In response to the monetary policy shock, the asset returns dynamics will link the household’s optimal economic trade-offs to have an impact on the aggregate performance of the economy. As a result, consumption, investment and output, are all negatively deviating from the steady state. The size of the response in nominal
bond return is larger than the return on capital stock. This leads to a negative deviation of equity risk premium from the steady state. This is supported by the economic intuition we studied in Chapter 4, where we see that the spread in equity risk premium is small when the monetary policy rate is higher. This implies that the equity risk premium is low if the central bank increases the policy rate during the boom (e.g., implementation of countercyclical monetary policy) or if there is a negative exogenous monetary policy shock. This highlights the co-movement of the equity risk premium and the business cycle as shown in Figure 2.

In response to the monetary policy shock, the sign of the impulse responses in the NK economy with Epstein and Zin preferences are the same as the benchmark CRRA preferences. The only difference is in the size of responses for economic variables. Let us first consider the asset pricing dynamics across these two types of preferences in Figures 7 and 9. We see that the size of the response of the return on capital stock is relatively larger for the household with Epstein and Zin preferences, as compared to the benchmark CRRA preferences. On the other hand, the size of the response of nominal bond return is relatively higher for the household with Epstein and Zin features. Given the net effects of these two assets returns, the size of the response of the equity risk premium is larger under the model with Epstein and Zin features.

The implication of this asset pricing dynamic comes from a household that has Epstein and Zin preferences with early resolution of uncertainty, and her demand for assets holdings. For an Epstein and Zin preference with early resolution of uncertainty, $\gamma > \rho$, it means that the household attaches more weight on the low continuation value of future consumption relative to her certainty equivalent. This is because she dislikes future consumption risk more. A household with this type of preference has a higher time varying risk component since it is increasing in the level of risk aversion. This leads to a higher pricing kernel relative to the standard CRRA preferences benchmark, see equation (5.2). It implies that the nominal bond return will need to fall, see equation (18). The time varying risk component acts as an opposing force to the negative exogenous monetary policy shock (e.g. increase in policy rate). As a result, the time varying risk component dilutes part of the impacts of the higher nominal bond return caused by the shock. Therefore, the size of response in nominal bond return is smaller under the Epstein and Zin preferences feature, as compared to the benchmark CRRA preferences.

Another interpretation is on demand for assets held. This relates to the higher level of risk aversion in the Epstein and Zin preferences model, $\gamma = 5, \rho = 2$, as compared to the CRRA benchmark, $\gamma = \rho = 2$. For a household that has Epstein and Zin preferences with early resolution of uncertainty, it can be interpreted that the household has higher level of risk aversion than the CRRA benchmark, as $\gamma$ is
higher. Although capital stock has a higher upside earnings potential, it also has a larger downside risk at the tail. This is supported by model statistics of assets returns in table 3. A more risk averse household would like to hold more of the nominal bond to smooth out her consumption across periods and risky states. This is because the more risk averse household will find the investment with platykurtic return distributions (e.g., nominal bond) best suited to her level of risk aversion relative to a more risky asset (e.g., capital stock with leptokurtic distribution of returns). Therefore, the more risk averse household will demand more for holding the nominal bond (less risky asset), and less for the capital stock (more risky asset). As a result, the higher demand for the nominal bond drives down its rate of return where lower demand for capital stock drives up the equity return. This economic intuition follows the same logic from the example we discussed in Chapter 4.

Given the effect of asset pricing dynamics, let us compare the aggregate performance of the economy in figures 8 and 10. In response to the monetary policy shock, the size of the negative deviation of current consumption from the steady state is relatively smaller for Epstein and Zin preferences, relative to CRRA preferences benchmark. This is supported by the model statistics in table 4, and also the economic intuition discussed above. In particular, the model statistics show that the aggregate consumption is relatively less volatile for the household with Epstein and Zin preferences feature (e.g. early resolution of uncertainty). This is because the time varying risk component is increasing in the level of risk aversion. The economic implication is that a household has Epstein and Zin preferences with early resolution of uncertainty, \((\gamma > \rho)\), prefers a smoother consumption path across periods and risky states. In her economic trade-offs, the time varying risk component dilutes some of the impacts of the increasing nominal bond return on marginal utility of current consumption. According to stochastic Euler equation, equation (5), the size of the response of marginal utility of current consumption is relatively smaller for Epstein and Zin preferences, as compared to the benchmark CRRA preferences. Given the effect of asset pricing dynamics with time varying risk component, the size of the response in actual consumption is therefore relatively smaller for the Epstein and Zin preferences with early resolution of uncertainty.

There is a relatively larger response to the increase in the capital stock return for the model with Epstein and Zin features. This suggests that the more risk averse household will have a relatively lower demand for the capital stock, as compared to the benchmark CRRA preferences. According to the diminishing marginal rate of return, the size of the response in the falling aggregate capital stock (or investment) is therefore larger for the household with Epstein and Zin preferences. On the other hand, the optimal consumption-labour trade-off is about the same for household with Epstein and Zin preferences features, relative to the benchmark. Given that
there is a negative exogenous monetary policy shock hits the economy, there is a decrease in the aggregate consumption and a fall in aggregate investment. The size of the response in decreasing output is slightly larger under Epstein and Zin preferences than the benchmark CRRA preferences. This is because the magnitude of the fall in investment is larger under Epstein and Zin preferences, given the size of the response in return on capital stock is larger.

In this experiment, subject to a negative monetary policy shock, we understand how Epstein and Zin preferences with early resolution of uncertainty influences asset pricing dynamics in the model. We then gain insights on the importance of the time varying risk component in the household’s stochastic Euler equation, which pins down the underlying optimal economic trade-offs with asset returns. Given this asset pricing dynamic and the household’s economic trade-offs, we see how the monetary policy transmission mechanism is linked with dynamic outcomes of the economy in this model.

5.5. Implications of Monetary Policy

So far, we have studied how the monetary policy transmission mechanism is linked with the household’s economic trade-offs via asset returns, which therefore impacts on the aggregate performance of the economy. Also, we have seen that the time varying risk component contributes to less volatile asset pricing dynamics in the model, subject to a monetary policy shock or a change in policy rate (e.g. countercyclical monetary policy). This can be explained by the time varying risk component which is increasing in the level of risk aversion for a household who prefers an early resolution of uncertainty. A household with this type of preference attaches more weight on low continuation value of future consumption relative to her certainty equivalent value. Ultimately, lower volatile asset pricing dynamics leads to less volatile dynamic outcomes of the economy, as compared to the benchmark model with CRRA preferences.

Dynamic properties of this model suggest that time variation of risk and separability between the household’s risk aversion and intertemporal elasticity of substitution are important elements for the monetary policy transmission mechanism. These features contribute to a lower volatility in the household’s optimal risky intertemporal consumption allocation in relation to asset returns. Hence, the asset pricing kernel, with time variation of risk, governs more stable dynamic outcomes of the economy. This implies that the policy rate set by the central bank does not need to be as large as the benchmark case, in order to stabilize the economy.
CHAPTER 6

Conclusions

In this thesis, I have discussed how the monetary policy transmission is linked with the household's optimal risky intertemporal trade-offs in a New Keynesian economy with Epstein and Zin preferences. The time varying risk component captured in asset pricing kernel matters for an effective monetary policy transmission mechanism, as it has impacts on the aggregate performance of the economy.

The model can generate countercyclical properties in asset pricing dynamics for a representative household that has Epstein and Zin preferences who prefers early resolution of uncertainty. This provides us with better economic intuitions on understanding the equity risk premium puzzle, as well as its co-movement with the business cycle in empirical data.

Results of the model statistics show that returns on capital stock have an asymmetric distribution (positively skewed to the right, i.e. higher upside potential) and it is also associated with larger downside risk at the tail (leptokurtic distribution). For returns on nominal bond, it has a symmetric distribution around the mean and it is associated with smaller downside risk at the tail (platykurtic distribution). This suggests that the household always faces trade-offs between asset returns and the associated risk, in weighing her optimal risky intertemporal consumption choice.

The mean of quarterly equity risk premium increases as the household’s level of risk aversion increases. Intuitively, a more risk averse household does not like an asymmetric returns distribution with high tail risk given by the capital stock in this model. This is because a more risk averse household will demand for more (less) holdings of the nominal bond (capital stock), which drives down (up) the underlying one-period gross return. This enlarges the equity risk premium between the two asset's returns. Hence, the empirically high return on equity is not only because of the underlying higher potential earning rewards but is also due to the extra risk compensation as required by investors. Intuitively, a more risk averse investor with early resolution of uncertainty will require a higher risk premium to compensate her in investing in the riskier asset.

Moreover, the time varying risk component contributes a less volatile asset pricing dynamic with the household’s optimal economic trade-offs in the model. This suggests that the monetary policy transmission mechanism can govern a more stable dynamic outcome of the economy. However, this shows the weaknesses of the model
in accounting for asset prices at higher order moments than the mean. This is due to
the limitation of Epstein and Zin preferences in a production economy. In particular,
labour supply affects the asset prices through the risk-adjusted expectation operator,
even when labour choice is separate in the household’s period utility function.
In a monetary policy DSGE model, the time varying risk component contributes
to explaining the mean effect (e.g., equity risk premium) but it has shortcomings
in matching asset prices at higher order moments to the empirical data. From the
economic intuitions and numerical experiments I have studied in this thesis, we can
see that the time variation of risk and higher order moments are crucial elements
in studying asset pricing but they do not necessarily fit well in a monetary policy
DSGE model. Therefore, I plan to study these shortcomings in my future work.

References.


Bianca De Paoli, Alasdair Scott, Olaf Weeken (2010): “Asset Pricing Implications
of a New Keynesian Model.” Journal of Economic Dynamics & Control, 34 (2010),
2056-2073.

Dario Caldara, Jesús Fernandez-Villaverde, Juan F. Rubio-Ramirez, Wen Yao
(2012): “Computing DSGE models with recursive preferences and stochastic volatility.”

Fumio Hayashi (1982): “Tobin’s Marginal q and Average q: A Neoclassical Inter-

James Tobin (1969): “A General Equilibrium Approach To Monetary Theory.”
Journal of Money, Credit and Banking, Vol. 1, No. 1, 15-29.

Policy Evaluation.” Journal of Economic Perspectives, Journal of Economic Per-
spективes, Vol. 21, No. 4, Fall 2007, 25– 45.

Jesús Fernández-Villaverde (2010). “The Econometrics of DSGE Models.” In-

Review of Economic Studies, 49, 517-531. Risk aversion and asset prices,
Epstein.

of Chicago.


Appendix 1: Detailed derivation of the model

1. Household.

The representative household lives infinitely long and participates in goods, asset and labour market. She consumes $C_t$ units of final consumption good, holds one-period state-contingent nominal bond $B_t$, supplies $N_t$ units of labor where $N_t(i)$ units of labor is rented to each differentiated good firms indexed by $i \in [0, 1]$. The household has Epstein and Zin preference and her lifetime utility is given by:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t U[C_t, N_t] \right\} \quad (A1)$$

where $E_t := E_t \{ \cdot \mid s_t \}$ is linear expectation conditional on realization of $s_t$ at time $t$.

Assume separable utility function:

$$U(C, N) := U(C) - L(N)$$

$$= \left[ (1 - \beta) \left( C_t(s_t) \right)^{1-\gamma} + \beta E_t \left[ U_{t+1}^{1-\gamma}(C_{t+1}(s_{t+1})) \right] \right]^{\frac{\psi}{1-\gamma}} - \left\{ \frac{N_t^{1+\varphi}(s_t)}{1+\varphi} \right\}$$

where $\rho, \gamma, \psi, \varphi > 0$, $\Psi := \frac{1}{1-\rho}$, $J_t(U_{t+1}(C_{t+1}(s_{t+1}))) = E_t \left( U_{t+1}^{1-\gamma}(C_{t+1}(s_{t+1})) \right)$, $\rho$ is the inverse intertemporal elasticity of substitution and $\gamma$ is the risk aversion.

The representative household earns income from supplying labour, capital, holding of financial asset and total profits accruing from ownership of all differentiated good firms indexed by $i \in [0, 1]$. The household’s lifetime budget constraint is given by:

$$P_t(s_t)C_t(s_t) + R_t^{-1}(s_t)B_{t+1}(s_{t+1}) + P_t(s_t)I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2$$

$$= W_t(s_t)N_t(s_t) + B_t(s_t) + r_t^k(s_t)K_t(s_t) + P_t(s_t)\Omega_t(s_t) \quad (A2)$$

and the law of capital accumulation

$$K_{t+1}(s_{t+1}) = (1 - \delta)K_t(s_t) + I_t(s_t) \quad (A3)$$

where $\Phi \frac{I_t(s_t)}{K_t(s_t)} := \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2$ is the functional form of capital cost adjustment.
The final consumption good is consisting of a continuum of differentiated goods produced by a continuum of firm $i$ on $[0, 1]$ in a monopolistically competitive market,

$$C_t(s_t) = \left[ \int_0^1 [C_{s_t,i}(i)]^{\frac{1}{\epsilon-1}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1 \quad (A4)$$

The representative household considers two optimizing problems, (1) expenditure minimization and (2) utility maximization.

**Problem (1):** Household’s expenditure minimization problem for a final consumption good $C_t(s_t)$ at time $t$ is by choosing $C_{s_t,i}(i)$ for all $i \in [0, 1]$ such that.

$$\min_{C_{s_t,i}(i)} \int_0^1 P_{s_t,i}(i)C_{s_t,i}(i) di$$

subject to

$$\left[ \int_0^1 [C_{s_t,i}(i)]^{\frac{1}{\epsilon-1}} di \right]^{\frac{\epsilon}{\epsilon-1}} = C_t(s_t) \quad (A5)$$

Assume locally insatiable, hence constraint will be binding at optimal. Dropping the state $s_t$ notation. Let $\lambda_t$ be the multiplier of Lagrange and take the first order condition with respect to $C_t(i)$:

$$P_t(i) - \lambda_t \left( \int_0^1 [C_t(i)]^{\frac{1}{\epsilon-1}} \right)^{\frac{1}{\epsilon-1}} (C_t(i))^{-\frac{1}{\epsilon}} = 0$$

$$\implies P_t(i) - \lambda_t C_t^{\frac{1}{\epsilon}}(i)C_t^{-\frac{1}{\epsilon}}(i) = 0$$

$$\implies C_t(i) = \left( \frac{P_t(i)}{\lambda_t} \right)^{-\epsilon} C_t \quad (A6)$$

$\lambda_t(i)$:

$$\left[ \int_0^1 [C_t(i)]^{\frac{1}{\epsilon-1}} di \right]^{\frac{\epsilon}{\epsilon-1}} = C_t$$

substitute (A6) into the feasibility constraint (A5) yields:

$$C_t = \left[ \int_0^1 \left( \left( \frac{P_t(i)}{\lambda_t} \right)^{-\epsilon} C_t \right)^{\frac{1}{\epsilon-1}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\implies 1 = \left( \frac{1}{\lambda_t} \right)^{-\epsilon} \left[ \int_0^1 (P_t(i))^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

rearrange and take the power of $-\frac{1}{\epsilon}$ to both sides of the equation,

$$\implies \lambda_t = \left[ \int_0^1 (P_t(i))^{1-\epsilon} di \right]^{\frac{1}{\epsilon}} \equiv P_t \quad (A6.1)$$

and hence (A6.1) can be written as:
\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (A7) \]

Hence, (A7) gives the agent’s optimal choice of cost-minimization for each of the differentiated goods \( i \) for all \( i \in [0, 1] \). (A6.1) gives the marginal cost of purchasing an additional unit of the differentiated good is equal to the aggregate price index for all \( i \in [0, 1] \).

**Problem (2):** Household’s life-time utility maximization problem is by choosing consumption \( C_t \), capital \( K_{t+1} \), bond \( B_{t+1} \), investment \( I_t \) and labour supply \( N_t \) such that.

\[
\max_{(C_t, N_t, K_{t+1}, I_t, B_{t+1}), s_t, t \in \mathbb{N}} \sum_{t=0}^{\infty} \beta^t \left[ \left( 1 - \beta \right) \left( C_t(s_t) \right)^{1+\gamma} \right]^{\frac{1}{\gamma}} - \beta \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} (C_{t+1}(s_{t+1})) \right]^{\frac{1}{\gamma}} - \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{N_t^{1+\varphi}(s_t)}{1+\varphi} \right]
\]

subject to

\[
P_t(s_t)C_t(s_t) + R_t^{-1}(s_t)B_{t+1}(s_{t+1}) + P_t(s_t)I_t(s_t) + \frac{\phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 = W_t(s_t)N_t(s_t) + B_t(s_t) + r^k(t)K_t(s_t) + P_t(s_t)\omega_t(s_t)
\]

and

\[
K_{t+1}(s_{t+1}) = (1 - \delta)K_t(s_t) + I_t(s_t)
\]

Dropping the state \( s_t \) notation. Assume locally insatiable again, hence constraint will be binding at optimal. Let \( \Gamma_t \) and \( Q_t \) be the Lagrange multiplier of A2 and A3, then take the first order condition with respect to:

\( N_t \):

\[-N_t^\varphi = -\Gamma_t W_t \quad (A7)\]

\( B_{t+1} \):

\[
\frac{\Gamma_t}{R_t} = \beta \mathbb{E}_t \left[ \Gamma_{t+1} \right] \quad (A8)
\]

\( I_t \):

\[
\Gamma_t \left( 1 + \varphi \left( \frac{I_t}{K_t} - \delta \right) \right) = Q_t \quad (A9)
\]

\( K_{t+1} \):
\[ Q_t = \beta \mathbb{E}_t \left[ \Gamma_{t+1} \left\{ \left( r_t^k(s_t) - \Phi \left( \frac{I_{t+1}}{K_{t+1}} \right) + \Phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right) \right\} + (1 - \delta)Q_{t+1} \right] \]

\[ Q_t = \beta \mathbb{E}_t \left[ \Gamma_{t+1} \left\{ \left( r_t^k - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right\} + (1 - \delta)Q_{t+1} \right] \quad (A9') \]

Tobin’s marginal \( q \) ratio is defined as \( q_t = \frac{Q_t}{r_t \gamma} \), e.g., the stock value of firm in terms of consumption good, hence (A9) and (A9') can be written as:

\[ q_t = 1 + \Phi \left( \frac{I_t}{K_t} - \delta \right) \quad (A9*) \]

\[ q_t = \beta \mathbb{E}_t \left[ \frac{\Gamma_{t+1}}{\Gamma_t} \left\{ \left( r_t^k - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right\} + (1 - \delta)q_{t+1} \right] \quad (A10) \]

For optimal consumption \( C_t \), rewrite the problem as follows:

\[ V_t(C_t) = \left[ (1 - \beta) \left( C_t \right)^{\frac{1-\gamma}{\delta} - 1} + \beta \mathbb{E}_t \left[ V_{t+1}^{1-\gamma}(C_{t+1}) \right] \right]^{\frac{1}{\delta} - 1} \]  

subject to A2 and A3.

Define the pricing kernel of the economy as:

\[ M_{t+1} = \frac{\partial V_t}{\partial C_t} \]

Taking the first order condition of right hand side of (A11):

\[ \frac{\partial V_t}{\partial C_t} = (1 - \beta) t^{\frac{1-\gamma}{\delta} - 1} t^{1 - \frac{1-\gamma}{\delta}} = \Gamma_t P_t \quad (A12) \]

\[ \frac{\partial V_t}{\partial C_{t+1}} = \beta V_t^{1 - \frac{1-\gamma}{\delta}} \mathbb{E}_t \left( V_{t+1}^{1-\gamma} \right)^{\frac{1}{\delta} - 1} \mathbb{E}_t \left[ V_{t+1}^{1-\gamma}(1 - \beta) V_{t+1}^{1-\gamma} C_{t+1}^{\frac{1-\gamma}{\delta} - 1} \right] \]

\[ = \Gamma_{t+1} P_{t+1} \quad (A13) \]

\[ \Rightarrow \frac{\partial V_t}{\partial C_{t+1}} = \beta V_t^{1 - \frac{1-\gamma}{\delta}} \mathbb{E}_t \left( V_{t+1}^{1-\gamma} \right)^{\frac{1}{\delta} - 1} \mathbb{E}_t \left[ (1 - \beta) V_{t+1}^{1-\gamma} C_{t+1}^{\frac{1-\gamma}{\delta} - 1} \right] = \Gamma_{t+1} P_{t+1} \]

\[ \Rightarrow \frac{\partial V_t}{\partial C_{t+1}} = \beta V_t^{1 - \frac{1-\gamma}{\delta}} (1 - \beta) \mathbb{E}_t \left[ \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t(V_{t+1}^{1-\gamma})} \right)^{1 - \frac{1}{\delta}} C_{t+1}^{\frac{1-\gamma}{\delta} - 1} \right] = \Gamma_{t+1} P_{t+1} \]
noted that: Envelope theorem has applied in the last step of substitution, it has updated (A12) one period forward, \( \frac{\partial V_{t+1}}{\partial C_{t+1}} = (1 - \beta) V_{t+1}^{1-\frac{1}{\gamma}} C_{t+1}^{\frac{1-\gamma}{\gamma}} \), in order to get (A13).

Hence, use definition of pricing kernel and canceling terms, it yields:

\[
M_{t,t+1} = \frac{\partial V_t}{\partial C_t} \frac{\partial V_t}{\partial C_t+1} = \Gamma_{t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{1-\frac{1}{\psi}} \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t (V_{t+1}^{1-\gamma})} \right)^{1-\frac{1}{\psi}} \frac{P_t}{P_{t+1}} \right] \tag{A14}
\]

where \( \Psi = \frac{1-\gamma}{1-\rho} \), this pricing kernel shows time-varying risk component via the term of \( \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t (V_{t+1}^{1-\gamma})} \right)^{1-\frac{1}{\psi}} \), which measures the valuation of next period’s consumption relative to agent’s certainty equivalent.

Optimal choice for consumption and labour trade off (e.g. the labour supply):

\[
\frac{W_t}{P_t} = \frac{N_t}{(1 - \beta)C_t^{1-\rho}} \tag{A15}
\]

where (A15) says that the optimal choice of labour and consumption is at the point when real wage equals to the marginal rate of substitution for consumption \( C_t \) and labour \( N_t \).

Because of no arbitrage profit, using (A8) and (A14), so it means \( \mathbb{E}_t [M_{t,t+1} R_t] = 1 \iff R_t = \frac{1}{\mathbb{E}_t [M_{t,t+1}]} \), yields the stochastic Euler equation relating to nominal bond return as follows:

\[
1 = \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{1-\frac{1}{\psi}} \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t (V_{t+1}^{1-\gamma})} \right)^{1-\frac{1}{\psi}} \frac{P_t}{P_{t+1}} \right] R_t
\]

\[
\iff (C_t)^{1-\frac{1}{\psi}} = \left( V_{t+1}^{1-\gamma} \right)^{\frac{1-\gamma}{\psi}} R_t \beta \mathbb{E}_t \left[ (C_{t+1})^{1-\frac{1}{\psi}} \right] \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t (V_{t+1}^{1-\gamma})} \right)^{1-\frac{1}{\psi}} \frac{P_t}{P_{t+1}} \right] \tag{A16}
\]

Then using (A10) and (A14), it yields the stochastic Euler equation relating to stock return on capital:
which holds for every $t \geq 0$ and state $s_t$.

(A16) and (A17) gives the stochastic Euler equation that pin down optimal risky intertemporal consumption choice, relating to return on nominal bond and stock return on capital respectively. At equilibrium, both assets, nominal bond and stock on capital, yield the same marginal utility of consumption to household across periods. The inflation adjusted pricing kernel \((A14)\) is the pricing mechanism satisfies the agent’s optimal intertemporal consumption trade off decision in the case of Epstein and Zin preference. In this case, the inflation adjusted pricing kernel captures (i) intertemporal consumption choice and (ii) time varying risk component.

Implication of asset pricing.

Special case with Constant Relative Risk Aversion (CRRA) preference when $\rho = \gamma$.

For CRRA preference, problem (1) is the same as the expenditure-minimization with Epstein and Zin utility, hence omitted here. The only difference between CRRA and Epstein and Zin preference can be seen in problem (2) where the time-varying risk component becomes unity. Since in special case, $\Psi = \frac{1-\gamma}{1-\rho} = 1$, then the time-varying risk component in (A14) becomes unity and the pricing kernel (A14) is as follows:

\[
M_{t,t+1} = \frac{\partial V_t}{\partial C_t} = \frac{\Gamma_{t+1}}{\Gamma_t} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} -1 \left( \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t (V_{t+1}^{1-\gamma})} \right)^{1-\frac{1}{\gamma}} \frac{P_t}{P_{t+1}} \right]
\]

\[
M_{t,t+1} = \frac{\partial V_t}{\partial C_t} = \frac{\Gamma_{t+1}}{\Gamma_t} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} -1 \left( \frac{P_t}{P_{t+1}} \right) \right]
\]

which is same as the pricing kernel in the standard case of CRRA preference.

Also, because of asset market with no arbitrage profit, so we have $\mathbb{E}_t [M_{t,t+1}, R_t] = 1 \iff R_t = \frac{1}{\mathbb{E}_t[M_{t,t+1}]}$, yields:

\[
1 = \mathbb{E}_t \left[ \beta \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \frac{P_t}{P_{t+1}} \right] R_t
\]

\[
\implies C_t^{-\rho} = \beta R_t \mathbb{E}_t \left[ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \frac{P_t}{P_{t+1}} \right]
\]

which holds for every $t \geq 0$ and state $s_t$. 

Using (B7) and (B9), it yields:

\[ q_t C_t^{-\rho} = \left( V_{t+1}^{1-\gamma} \right)^{\frac{1}{\gamma-1}} \beta E_t \left[ C_{t+1}^{-\rho} \frac{P_t}{P_{t+1}} \right] \times \left\{ \left( r^{h_t} - \frac{\Phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \Phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) + (1 - \delta) q_{t+1} \right\} \]

(B3)

which holds for every \( t \geq 0 \) and state \( s_t \).

(B2) and (B3) give the stochastic Euler equation that pin down optimal risky intertemporal consumption choice, relating to return on nominal bond and stock return on capital respectively. At equilibrium, both assets, nominal bond and stock on capital, yield the same marginal utility of consumption to household across periods.

The inflation adjusted pricing kernel (B1) is the pricing mechanism satisfies the agent’s optimal intertemporal consumption trade off decision in the case of CRRA preference. However, the time varying risk component drops out when \( \rho = \gamma \). In this case, the inflation adjusted pricing kernel only captures (i) intertemporal consumption choice.

2. Firm.

A continuum of firms on \([0, 1]\) participate in monopolistic competitive market and each of differentiated goods firm \( i \), \( i \in [0, 1] \) faces consumer demand \( C_t(i) \) at any \( t, s \in \mathbb{N} \). Also, given that there is a final consumption good consists of \( i \) differentiated goods for all \( i \in [0, 1] \) produced by firms, so we have \( C_{t,s}(i) = C_{t,s} = Y_{t,s_i} = Y_{t,s}(i) \) (\( * \)). Hence, using the relationship of (\( * \)) with equation (A7), the firm faces demand:

\[ Y_{t,s_i}(i) = \left( \frac{P_{t,s}(i)}{P_{t,s}} \right)^{-\epsilon} Y_{t,s}, \quad Y_{t,s} := C_{t,s} \]  

(C1)

Each of the firm’s production technology follows a Cobb-Douglas function in capital \( K_{t,s_i}(i) \) and labour \( N_{t}(i) \):

\[ Y_{t,s_i}(i) = A_t(s_t) K_{t,s_i}(i)^\alpha N_{t,s_i}^{1-\alpha}(i) \]  

(C2)

Assume technology \( A_t \) follows a stationary AR(1) process:

\[ \ln(A_t) = \alpha_A \ln(A_{t-1}) + \epsilon_{t}^A \]  

(C3)

where \( |\alpha_A| < 1, \epsilon_{t}^A \sim N(0, \sigma_A^2) \).

The firm also faces convex-price adjustment cost:

\[ AC \left( \frac{P_{t,s}(i)}{P_{t,s-1}(i)}, Y_{t,s}(i) \right) := \frac{\theta}{2} \left( \frac{P_{t,s}(i)}{P_{t,s-1}(i)} - \Pi_{ss} \right)^2 Y_{t}(i) \]  

(C4)

where \( \Pi \) is gross CPI inflation in the steady state, i.e. central bank’s inflation target. This adjustment cost function means that the real cost of price adjustment by firms
is a function of firm’s price inflation relative to the CPI inflation in steady state. This quadratic cost function means that the average real cost of price adjustment increases substantially when firm increases prices (own firm price inflation) relative to steady state CPI inflation.

Firm has two optimization problems need to consider such as (1) cost minimization of producing output and (2) expected profit maximization (profits pay out to the household for the ownership of firms).

**Problem (1):** Firm’s cost-minimization problem for output $Y_{t,s_t}(i)$ at time $t$ is by choosing $K_{t,s_t}(i)$ and $N_{t,s_t}(i)$ for all $i \in [0, 1]$ such that:

$$
\min_{N_{t,s_t}(i),K_{t,s_t}(i)} W_t(s_t)N_{t,s_t}(i) + r^k_t(s_t)K_{t,s_t}(i)
$$

subject to

$$
Y_{t,s_t}(i) = A_t(s_t)K_{t,s_t}^\alpha(i)N_{t,s_t}^{1-\alpha}(i)
$$

Let $A_t$ be the shadow value of Lagrange multiplier on production constraint, hence it is the nominal marginal cost with respect to production $Y_t(s_t)$, $A_t = MC_t(s_t)$. Taking the first order conditions, it yields the optimal cost-minimizing choice for production for all $i \in [0, 1]$ as follows:

$$
\frac{W_t(s_t)}{P_t(s_t)} = \frac{MC_t(s_t)(1-\alpha)}{P_t(s_t)} A_t(s_t)K_{t,s_t}^\alpha(i)N_{t,s_t}^{1-\alpha}(i)
$$

if dividing both sides by $P_t(s_t)$, it can be written as:

$$
\iff \frac{W_t(s_t)}{P_t(s_t)} = \frac{MC_t(s_t)(1-\alpha)}{P_t(s_t)} A_t(s_t)K_{t,s_t}^\alpha(i)N_{t,s_t}^{1-\alpha}(i)
$$

and

$$
\frac{r^k_t(s_t)}{P_t(s_t)} = \frac{MC_t(s_t)}{P_t(s_t)} A_t(s_t)K_{t,s_t}^{-1}(i)N_{t,s_t}^{1-\alpha}(i)
$$

(C5) gives the firm’s optimal demand for labour at the point where real wage is equal to marginal product of labour, taking the real marginal cost of production into account. (C6) gives the firm’s optimal demand for capital at the point where real rental rate is equal to marginal product of capital, taking the real marginal cost of production into account.

**Problem (2):** Firm has same stochastic discount factor as household since household owns firms directly:

$$
M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{1-\gamma} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\gamma} \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right]
$$

and let $\widetilde{M}_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}(s_{t+1})}{C_t(s_t)} \right)^{1-\gamma} \left( \frac{[V_{t+1}(C_{t+1}(s_{t+1}))]}{J_t[V_{t+1}(C_{t+1}(s_{t+1}))]} \right)^{1-\gamma} \right]$ be the non-inflation adjusted pricing kernel.
Firm’s expected profit maximization problem at time $t$ is by choosing $P_{t,s_t}(i)$ for all $i \in [0,1]$ such that:

$$
\Omega_{t,s_t}(i) = \max_{\{P_{t,s_t}(i)\}_{i \in N}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \tilde{M}_{t,t+1} \left[ \frac{P_{t,s_t}(i)}{P_t(s_t)} Y_{t,s_t}(i) - \frac{W_t(s_t)}{P_t(s_t)} N_{t,s_t}(i) - r^k_t(s_t) K_{t,s_t}(i) - AC \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_t-1}(i)} Y_{t,s_t}(i) \right) \right] \right\}
$$

subject to

$$
Y_{t,s_t}(i) = A_t(s_t) K_{t,s_t}(i) N_{t,s_t}^{1-\alpha}(i)
$$

where

$$
AC \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_t-1}(i)} Y_{t,s_t}(i) \right) := \frac{\theta}{2} \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_t-1}(i)} - \Pi_{ss} \right)^2 Y_{t,s_t}(i)
$$

Know, $Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$, in terms of the first two periods, $t = 0$ and $t = 1$, and dropping the notation of state $s_t$, the Lagrange multiplier is $mc_t(i) := \frac{MC_t(i)}{P_t(i)}$ the real marginal cost with production, and taking the first order condition with respect to $P_t(i)$, it yields:

$$
\frac{\partial}{\partial P_t(i)} Y_t(i) + \frac{P_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} - \left( \frac{\partial \left( \frac{MC_t(i)}{P_t(i)} \right)}{\partial P_t(i)} Y_t(i) + \frac{MC_t(i) \partial Y_t(i)}{P_t} \right)
$$

$$
- \frac{\partial AC \left( \frac{P_t(i)}{P_{t-1}(i)} Y_t(i) \right)}{\partial P_t(i)} + \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial \left( \frac{P_{t+1}(i)}{P_{t+1}(i)} Y_{t+1}(i) \right)}{\partial P_t(i)} \right\}
$$

$$
- \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial \left( MC_{t+1}(i) Y_{t+1}(i) \right)}{\partial P_t(i)} \right\} - \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial AC \left( \frac{P_{t+1}(i)}{P_{t+1}(i)} Y_{t+1}(i) \right)}{\partial P_t(i)} \right\}
$$

It is recognized that the first partial derivative with respect to $P_t(i)$, the following terms are zero:

$$
\frac{\partial MC_t(i)}{\partial P_t(i)} Y_t(i) = 0, \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial \left( \frac{P_{t+1}(i)}{P_{t+1}(i)} Y_{t+1}(i) \right)}{\partial P_t(i)} \right\} = 0 \text{ and } \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial AC \left( \frac{P_{t+1}(i)}{P_{t+1}(i)} Y_{t+1}(i) \right)}{\partial P_t(i)} \right\} = 0
$$

Hence, the first order condition of (B7) with respect to $P_t(i)$ yields:

$$
\frac{\partial}{\partial P_t(i)} Y_t(i) + \frac{P_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} - \frac{MC_t(i) \partial Y_t(i)}{P_t} \frac{\partial P_t(i)}{\partial P_t(i)} - \frac{\partial AC \left( \frac{P_t(i)}{P_{t-1}(i)} Y_t(i) \right)}{\partial P_t(i)} - \mathbb{E}_t \left\{ \tilde{M}_{t,t+1} \frac{\partial AC \left( \frac{P_{t+1}(i)}{P_{t+1}(i)} Y_{t+1}(i) \right)}{\partial P_t(i)} \right\}
$$
Denote first term, second term, third term, forth term and fifth term as (1), (2), (3), (4) and (5) respectively, so

(1):

\[
\frac{\partial}{\partial P_t(i)} \frac{P_t(i)}{P_t} Y_t(i) = \frac{Y_t(i)}{P_t(i)}
\]

(2):

\[
P_t(i) \frac{\partial Y_t(i)}{\partial P_t(i)} = P_t(i) \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon + 1} Y_t = \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{Y_t(i)}{P_t} = -\epsilon \frac{Y_t(i)}{P_t}
\]

(3):

\[
\frac{MC_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} = \frac{MC_t(i)}{P_t} \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon + 1} \frac{Y_t(i)}{P_t}
\]

(4):

\[
\frac{\partial AC}{\partial P_t(i)} \left( \frac{P_t(i)}{P_{t-1}(i)}, Y_t(i) \right) = -\frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi \right)^2 \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon + 1} \frac{Y_t(i)}{P_t}
\]

(5):

\[
\frac{\partial AC}{\partial P_t(i)} \left( \frac{P_{t+1}(i)}{P_t(i)}, Y_{t+1}(i) \right) = \frac{\theta}{2} \left( \frac{P_t + 1(i)}{P_t(i)} - \Pi_{ss} \right) \left( \frac{P_{t+1}(i)}{P_t(i)} \right) Y_{t+1}(i)
\]

At every time \( t \geq 0 \) and state \( s_t \), the firm \( i \) profit maximizing choice (optimal pricing strategy) for all \( i \in [0, 1] \) satisfies:

\[
0 = \left( 1 - \epsilon \right) \frac{Y_t(i)}{P_t(i)} - \frac{MC_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} - \frac{\partial AC}{\partial P_t(i)} \left( \frac{P_t(i)}{P_{t-1}(i)}, Y_t(i) \right)
\]

\[
-\mathbb{E}_t \left\{ \widetilde{M}_{t,t+1} \frac{\partial AC}{\partial P_t(i)} \left( \frac{P_{t+1}(i)}{P_t(i)}, Y_{t+1}(i) \right) \right\} \quad (C8)
\]

where

\[
\frac{MC_t(i)}{P_t} \frac{\partial Y_t(i)}{\partial P_t(i)} = \frac{MC_t(i)}{P_t} \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon + 1} \frac{Y_t(i)}{P_t}
\]

\[
\frac{\partial AC}{\partial P_t(i)} \left( \frac{P_t(i)}{P_{t-1}(i)}, Y_t(i) \right) = \frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{ss} \right)^2 \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon + 1} \frac{Y_t(i)}{P_t} + \theta \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{ss} \right) \frac{Y_t(i)}{P_{t-1}(i)} = 0
\]

and
\[
\frac{\partial AC}{\partial P_t(i)} \left( \frac{P_{t+1}(i)}{P_t(i)}, Y_{t+1}(i) \right) = \theta \left( \frac{P_{t+1}(i)}{P_t(i)} - \Pi_{ss} \right) \left( \frac{P_{t+1}(i)}{(P_t(i))^2} \right) Y_{t+1}(i) = 0
\]

Aggregation over all firm \(i\)'s production technology for each of the differentiated goods \(i \in [0, 1]\) gives total output (supply side) such that:

\[
Y_t(s_t) = A_t(s_t)K^n_t(s_t)N_k^{1-\alpha}(s_t) \quad (C9)
\]

At every time \(t \in N\) and state \(s_t\), the firm \(i\) profit maximizing choice (optimal pricing strategy) for all \(i \in [0, 1]\) satisfies equation (C8). This first order condition of firm’s pricing strategy will be used to derive the new Keynesian Phillips curve later. Given the firm’s optimal choice of labour, capital demand and pricing decision, the aggregate output produced by firms satisfies (C9).


Goods market.

Goods market clear for each differentiated good \(i \in [0, 1]\) is given by:

\[
Y_{t,s_t}(i) = A_t(s_t)K^n_{t,s_t}(i)N_k^{1-\alpha}(i) = C_{t,s_t}(i) + I_{t,s_t}(i) + \frac{\Phi}{2} \left( \frac{I_{t,s_t}(i)}{K_{t,s_t}(i)} - \delta \right)^2
\]

This means that the aggregate resources feasibility must hold in equilibrium such that supply is equal demand.

Taking the cost of price adjustment from firms into account such that:

\[
Y_{t,s_t}(i) - AC \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_t-1}(i)}, Y_{t,s_t}(i) \right) = \left[ 1 - \frac{\theta}{2} \left( \frac{P_{t,s_t}(i)}{P_{t-1,s_t-1}(i)} - \Pi_{ss} \right)^2 \right] Y_{t,s_t}(i)
\]

\[
= C_{t,s_t}(i) + I_{t,s_t}(i) = \int_0^1 \left( \frac{P_{t,s_t}(i)}{P_{t}(s_t)} \right)^{-\epsilon} C_t + I_{t,s_t}(i) + \frac{\Phi}{2} \left( \frac{I_{t,s_t}(i)}{K_{t,s_t}(i)} - \delta \right)^2
\]

where \(\Pi_t := \frac{P_t}{P_{t-1}}\) and the capital cost adjustment is taking into account.

Aggregation over supply side and demand side for each of the differentiated goods \(i \in [0, 1]\), the resources feasibility constraint, gives aggregate goods market clearing condition such that.

\[
Y_t(s_t) = A_t(s_t)K^n_t(s_t)N_k^{1-\alpha}(s_t) = C_t(s_t) + I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2
\]

Using the above relation and the fact that firms use symmetric pricing strategy, it yields the aggregate goods market clearing condition:

\[
\left[ 1 - \frac{\theta}{2} (\Pi_t - \Pi_{ss})^2 \right] Y_t = C_t(s_t) + I_t(s_t)
\]

\[
= C_t(s_t) \int_0^1 \left( \frac{P_{t,s_t}(i)}{P_t(s_t)} \right)^{-\epsilon} I_t(s_t) + \frac{\Phi}{2} \left( \frac{I_t(s_t)}{K_t(s_t)} - \delta \right)^2 \quad (D1)
\]
where total production is $Y_t(s_t) = A_t(s_t)K_t^\alpha(s_t)N_t^{1-\alpha}(s_t)$ and law of capital motion $K_{t+1}(s_{t+1}) = I_t(s_t) + (1 - \delta)K_t(s_t)$.

**Labour market clearing.**

Labour market clears at equilibrium if labour supply equals labour demand: labour supply (A12) equals labour demand (C5):

$$\frac{W_t(s_t)}{P_t(s_t)} = \frac{N_t^\rho(s_t)}{(1 - \beta)C_t}(A12)$$

$$\frac{W_t(s_t)}{P_t(s_t)} = \frac{MC_t(s_t)}{P_t(s_t)} (1 - \alpha) A_t(s_t) K_{t,s_t}^\alpha(i) N_{t,s_t}^{-\alpha}(i) \quad (C5)$$

Sub (C5) into (A12), it yields:

$$\implies mc_t(1 - \alpha) A_t(s_t) K_t^\alpha(s_t) N_t^{-\alpha}(s_t) = \frac{C_t^\rho N_t^\rho}{(1 - \beta)} \quad (D2)$$

where $\rho$ is representing the inverse of intertemporal elasticity of substitution, $mc_t := \frac{MC_t}{P_t}$.

**Asset pricing condition.**

Assuming no profit arbitrage condition and using stochastic Euler equations (A16) and (A17), asset pricing condition in equilibrium is derived as follows:

$$1 = R_t(s_t)M_{t,t+1}$$

$$\frac{1}{R_t(s_t)} = M_{t,t+1} \quad (D3)$$

$$1 = \frac{M_{t,t+1}}{q_t(s_t)} \left( r_t^k(s_t) + (1 - \delta)q_{t+1}(s_{t+1}) - \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \left( \frac{I_{t+1}}{K_{t+1}} \right) \right) \quad (D4)$$

since $R_t^k := 1 + r_t^k(s_t) - \delta$, is the gross period nominal stock return on capital, hence (D4) can be written as:

$$1 = \frac{R_t^k(s_t)q_{t+1}(s_{t+1}) - \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \left( \frac{I_{t+1}}{K_{t+1}} \right)}{q_t(s_t)} \quad (D5)$$

Using (D3) and (D5), it yields:

$$R_t(s_t) = \frac{R_t^k(s_t)q_{t+1}(s_{t+1}) - \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \left( \frac{I_{t+1}}{K_{t+1}} \right)}{q_t(s_t)} \quad (D6)$$

(D6) says in equilibrium, the return on nominal bond is equal to the stock return on capital, taking the firm’s stock value and cost of capital adjustment into account.

**Capital market.**

Capital market clearing is governing by Walra’s law, with the law of capital accumulation $K_{t+1}(s_t) = (1 - \delta)K_t(s_t) + I_t(s_t)$.

APPENDIX 1: DETAILED DERIVATION OF THE MODEL

Denote \( \Pi_{t,s_t} := \frac{P_t(s_t)}{P_{t-1}(s_t)} \), \( mc_{t,s_t}(i) := \frac{MC_{t,s_t}(i)}{P_t(s_t)} \). Dropping the notation of state \( s_t \). Then using the assumption of firm’s symmetric pricing adjustment decision \( P_t(i) = P_t \), \( mc_t(i) = mc_t \), and (C8), summing over all \( i \in [0,1] \) yields:

\[
0 = (1 - \epsilon) \frac{Y_t(i)}{P_t(i)} - \frac{MC_t(i)}{P_t} \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon - 1} \frac{Y_t(i)}{P_t} \\
- \left( -\frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{s,s} \right)^2 \right) \left( -\frac{P_t(i)}{P_t} \right)^{-\epsilon - 1} \frac{Y_t(i)}{P_t} \\
- \left[ \mathbb{E}_t \tilde{\Pi}_{t+1} - \Pi_{s,s} \right] \left( \frac{P_{t+1}(i)}{P_t(i) Y_{t+1}(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)}
\]

\[
\Rightarrow 0 = (1 - \epsilon) \frac{Y_t(i)}{P_t(i)} + \epsilon mc_t \left( \frac{Y_t(i)}{P_t(i)} \right) - \frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \Pi_{s,s} \right)^2 \frac{Y_t(i)}{P_t(i)} \\
+ \theta \left( \Pi_t - \Pi \right) \left( \frac{Y_t(i)}{P_{t-1}(i)} \right) - \theta \mathbb{E}_t \tilde{\Pi}_{t+1} \left( \frac{Y_{t+1}(i)}{Y_t(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)}
\]

\[
\Rightarrow (1 - \epsilon) + \epsilon mc_t + \theta \left( \Pi_t - \Pi_{s,s} \right) \Pi - \frac{\theta}{2} \left( \Pi_t - \Pi \right)^2 \\
= \theta \mathbb{E}_t \left[ \tilde{\Pi}_{t+1} \left( \frac{Y_{t+1}(i)}{Y_t(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)} \right] - \epsilon mc_t - (1 - \epsilon)
\]

\[
\Rightarrow \theta \left( \Pi_t - \Pi \right) \Pi_t - \frac{\theta}{2} \left( \Pi_t - \Pi_{s,s} \right)^2 \\
= \theta \mathbb{E}_t \left[ \tilde{\Pi}_{t+1} \left( \frac{Y_{t+1}(i)}{Y_t(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)} \right] - \epsilon \left( \frac{\epsilon - 1}{\epsilon} \right) - (1 - \epsilon)
\]

\[
\Rightarrow (\Pi_t - \Pi_{s,s}) \Pi_t - \frac{\epsilon}{2} (\Pi_t - \Pi_{s,s})^2 \\
= \mathbb{E}_t \left[ \tilde{\Pi}_{t+1} \left( \frac{Y_{t+1}(i)}{Y_t(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)} \right] - \frac{\epsilon}{\theta} \left( \frac{\epsilon - 1}{\epsilon} \right) - \frac{1}{\theta} (1 - \epsilon)
\]

\[
\Rightarrow (\Pi_t - \Pi_{s,s}) \Pi_t - \frac{\epsilon}{2} (\Pi_t - \Pi_{s,s})^2 \\
= \mathbb{E}_t \left[ \tilde{\Pi}_{t+1} \left( \frac{Y_{t+1}(i)}{Y_t(i)} \right) \frac{Y_{t+1}(i)}{Y_t(i)} \right] - \frac{\epsilon}{\theta} \left( \frac{\epsilon - 1}{\epsilon} \right) - \frac{1}{\theta} \left( \frac{\epsilon - \epsilon^2}{\epsilon^2} \right)
\]
\[
\Rightarrow (\Pi_t - \Pi_{ss}) \Pi_t - \frac{\epsilon}{2} (\Pi_t - \Pi_{ss})^2
\]
\[
= \mathbb{E}_t \left[ \tilde{M}_{t,t+1} (\Pi_{t+1} - \Pi_{ss}) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] - \frac{\epsilon}{\theta} \left( \frac{\epsilon - 1}{\epsilon} \right) + \frac{\epsilon}{\theta} \left( \frac{\epsilon - 1}{\epsilon} \right)
\]
\[
(\Pi_t - \Pi_{ss}) \Pi_t - \frac{\epsilon}{2} (\Pi_t - \Pi_{ss})^2
\]
\[
= \mathbb{E}_t \left[ \tilde{M}_{t,t+1} (\Pi_{t+1} - \Pi_{ss}) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] + \frac{\epsilon}{\theta} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right) \quad (E1)
\]

where \( M_{t,t+1} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1(s_{t+1})}}{C_t(s_t)} \right)^{\frac{1-\gamma}{1-\rho}} \left( \frac{V_t^{1-\gamma}}{E_t \left( V_t^{1-\gamma} \right)} \right)^{1-\frac{1}{\varphi}} \frac{1}{\Pi_{ss}} \right] \) is the stochastic discount factor, and \( \Psi = \frac{1-\gamma}{1-\rho} \).

5. **Deterministic steady state.** When the economy is at the steady state, it must be:

\[\{X_t = X_{t+1} = X_{ss}\}\]

where variable \( X := \{C, N, K, Y, mc, R^k, r^k, R, \Pi\} \).

From the representative household’s Euler equation (A16), at the steady state, we have:

\[
(C_{ss})^{\frac{1-\gamma}{1-\varphi}} - 1 = R_{ss} / \beta \mathbb{E}_t \left[ (C_{ss})^{\frac{1-\gamma}{1-\varphi}} - 1 \left( \frac{V_{ss}^{1-\gamma}}{E_t \left( V_{ss}^{1-\gamma} \right)} \right)^{1-\frac{1}{\varphi}} \frac{1}{\Pi_{ss}} \right]
\]

\[
\Rightarrow R_{ss} = \frac{\Pi_{ss}}{\beta} \quad (F1)
\]

where \( R_{ss} \) is the one-period gross return on the bond at the steady state, and \( \Pi_{ss} \) is the gross inflation target.

From the law of capital motion \( K_{ss} = I_{ss} + (1 - \delta)K_{ss} \), it gives:

\[I_{ss} = \delta K_{ss}\]

From (A17) at the steady state, it implies:

\[
q_{ss} (C_{ss})^{\frac{1-\gamma}{1-\varphi}} - 1 = \beta \mathbb{E}_t \left[ \left( C_{ss} \right)^{\frac{1-\gamma}{1-\varphi}} - 1 \left( \frac{V_{ss}^{1-\gamma}}{E_t \left( V_{ss}^{1-\gamma} \right)} \right)^{1-\frac{1}{\varphi}} \frac{1}{\Pi_{ss}} \right]
\]

\[
\left\{ \left( r_{ss}^k - \frac{\Phi}{2} \left( \frac{I_{ss}}{K_{ss}} - \delta \right)^2 + \Phi \left( \frac{I_{ss}}{K_{ss}} - \delta \right) \frac{I_{ss}}{K_{ss}} \right) \left( 1 - \delta \right) q_{ss} \right\}
\]

From the law of capital motion \( K_{ss} = I_{ss} + (1 - \delta)K_{ss} \), it gives:

\[I_{ss} = \delta K_{ss}\]

hence it implies:

\[
q_{ss} (C_{ss})^{\frac{1-\gamma}{1-\varphi}} - 1 = \beta \mathbb{E}_t \left[ \left( C_{ss} \right)^{\frac{1-\gamma}{1-\varphi}} - 1 \left( \frac{V_{ss}^{1-\gamma}}{E_t \left( V_{ss}^{1-\gamma} \right)} \right)^{1-\frac{1}{\varphi}} \frac{1}{\Pi_{ss}} \right] \left( r_{ss}^k + (1 - \delta)q_{ss} \right)
\]
From (A9*) at the steady state, 
\[ q_{ss} = 1 + \Phi \left( \frac{I_{ss}}{K_{ss}} - \delta \right) = 1 \]
and \( R_{ss}^k := r_{ss}^k + (1 - \delta) \), so using all these conditions at steady state, (A17) gives:
\[ \frac{\Pi_{ss}}{\beta} = R_{ss}^k \]  
\( (F2) \)
From (C6), \( r_{ss}^k = mc_{ss} \alpha A_{ss} K_{ss}^{\alpha - 1} N_{ss}^{1 - \alpha} \) is the rental rate at the steady state, and by definition of \( R_{ss}^k := r_{ss}^k + (1 - \delta) \), the one-period gross stock return on capital, so we have:
\[ r_{ss}^k + (1 - \delta) = \frac{\Pi_{ss}}{\beta} \]
\[ r_{ss}^k = \frac{\Pi_{ss}}{\beta} - (1 - \delta) \]
\[ mc_{ss} \alpha A_{ss} K_{ss}^{\alpha - 1} N_{ss}^{1 - \alpha} = \frac{\Pi_{ss}}{\beta} - (1 - \delta) \]
\[ \hat{K}_{ss} = \frac{K_{ss}}{N_{ss}} = \left( \frac{mc_{ss} \alpha A_{ss}}{\frac{\Pi_{ss}}{\beta} - (1 - \delta)} \right)^{\frac{1}{1 - \alpha}} \]  
\( (F3) \)
From the new Keynesian Phillips curve (E1), at the steady state, we have:
\[ (\Pi_{ss} - \Pi_{ss}) \Pi_{ss} - \frac{\epsilon}{2} (\Pi_{ss} - \Pi_{ss})^2 \]
\[ = \mathbb{E}_t \left[ \tilde{M}_{t,t+1} (\Pi_{ss} - \Pi_{ss}) \Pi_{ss} \frac{Y_{ss}}{Y_{ss}} \right] + \frac{\epsilon}{\theta} \left( mc_{ss} - \frac{\epsilon - 1}{\epsilon} \right) \]
\[ 0 = \frac{\epsilon}{\theta} \left( mc_{ss} - \frac{\epsilon - 1}{\epsilon} \right) \]  
\[ mc_{ss} = \frac{\epsilon - 1}{\epsilon} \]  
\( (F4) \)
The real marginal cost at the steady state is equal to the static markup which depends on the firm’s own price elastic of demand. Since \( w_{ss} = mc_{ss} \) at steady state, and use (C5) \( w_{ss} = mc_{ss} (1 - \alpha) A_{ss} K_{ss}^\alpha N_{ss}^{-\alpha} \), it gives:
\[ \frac{N_{ss}}{K_{ss}} = (1 - \alpha)^{\frac{1}{\alpha}} \]
multiply LHS by \( \frac{N_{ss}}{N_{ss}} \), it gives:
\[ \frac{N_{ss} N_{ss}}{N_{ss} K_{ss}} = (1 - \alpha)^{\frac{1}{\alpha}} \]
\[ \hat{N}_{ss} = \frac{N_{ss}}{N_{ss}} = (1 - \alpha)^{\frac{1}{\alpha}} \frac{K_{ss}}{N_{ss}} = (1 - \alpha)^{\frac{1}{\alpha}} \hat{K}_{ss} \]  
\( (F5) \)
sub (F3) into (F5), it gives:
\[ \hat{N}_{ss} = (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{mc_{ss} \alpha A_{ss}}{\frac{\Pi_{ss}}{\beta} - (1 - \delta)} \right)^{\frac{1}{1 - \alpha}} \]  
\( (F6) \)
Since, technology $A_t$ follows a stationary AR (1) process as shown in (C3),
\[ \ln(A_t) = \alpha \ln(A_{t-1}) + \varepsilon_t^A \]
where $|\alpha| < 1$, $\varepsilon_t^A \sim N(0, \sigma_t^2)$ and taking exponential to both sides,
\[ \iff \quad A_t = A_{t-1} \exp \{ \varepsilon_t^A \} \]
So, we have technology $A_t$ at the steady state as:
\[ E[A_{ss}] = E[A_{ss} \exp \{ \varepsilon_{ss} \}] \]
\[ \iff \quad E[A_{ss}] = E[\exp \{ \varepsilon_{ss} \}] \]
\[ A_{ss} = 1^{1-\alpha} = 1 \quad (i) \]
Since $Y_{ss} = A_{ss} K_{ss}^\alpha N_{ss}^{1-\alpha}$, divide both sides by $N_{ss}$, it gives:
\[ \hat{Y}_{ss} = \frac{Y_{ss}}{N_{ss}} = \hat{K}_{ss}^\alpha \hat{N}_{ss}^{1-\alpha} \]
such that the output per capita can be pin down after (F3) and (F6) is computed.
From the goods market clearing condition (D1) and with aggregate production, at the steady state, we have:
\[ \left[ 1 - \frac{\theta}{2} (\Pi_{ss} - \Pi_{ss})^2 \right] Y_{ss} = C_{ss} \left( \frac{P_{ss}}{\hat{P}_{ss}} \right)^{-\varepsilon} + I_{ss} + \frac{\phi}{2} \left( \frac{I_{ss}}{\hat{K}_{ss}} - \delta \right)^2 \]
where $Y_{ss} = A_{ss} K_{ss}^\alpha N_{ss}^{1-\alpha}$ and $I_{ss} = K_{ss} - (1 - \delta) K_{ss} = \delta K_{ss}$
\[ \iff \quad C_{ss} = Y_{ss} - \delta \hat{K}_{ss} \]
divide both sides by $N_{ss}$, it gives:
\[ \hat{C}_{ss} = \hat{Y}_{ss} - \delta \hat{K}_{ss} \]
where $\hat{Y}_{ss} = \hat{K}_{ss}^\alpha \hat{N}_{ss}^{1-\alpha}$.

In summary, the derived deterministic steady state in this economy is as follows:

- The real marginal cost: $mc_{ss} = \frac{-1}{\varepsilon}$;
- Inflation target: $\Pi_{ss}$;
- Aggregate capital: $\hat{K}_{ss} = \left( \frac{mc_{ss} A_{ss}}{\mu_{ss} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}$;
- Aggregate labour: $\hat{N}_{ss} = (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{mc_{ss} A_{ss}}{\mu_{ss} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}$;
- Aggregate investment $\hat{I}_{ss} = \delta \hat{K}_{ss}$;
- Aggregate production: $\hat{Y}_{ss} = \hat{K}_{ss}^\alpha \hat{N}_{ss}^{1-\alpha}$;
- Aggregate consumption $\hat{C}_{ss} = \hat{Y}_{ss} - \delta \hat{K}_{ss}$. 
## Appendix 2: Sensitivity tests

### Summary of simulated statistics:

<table>
<thead>
<tr>
<th>Table</th>
<th>Model Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Benchmark Model</td>
<td>Benchmark Model</td>
<td>0.052118</td>
<td>0.000002</td>
<td>0.095848</td>
<td>0.161146</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.026373</td>
<td>0.000016</td>
<td>0.001371</td>
<td>-0.007082</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.025745</td>
<td>0.000017</td>
<td>0.010020</td>
<td>-0.020482</td>
</tr>
<tr>
<td>2. Epstein and Zin Model 1</td>
<td>Epstein and Zin Model 1</td>
<td>0.052673</td>
<td>0.000016</td>
<td>0.001371</td>
<td>-0.007082</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.025745</td>
<td>0.000017</td>
<td>0.010020</td>
<td>-0.020482</td>
</tr>
<tr>
<td>3. Epstein and Zin Model 2</td>
<td>Epstein and Zin Model 2</td>
<td>0.051508</td>
<td>0.000002</td>
<td>0.131926</td>
<td>0.316709</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.024090</td>
<td>0.000010</td>
<td>-0.012485</td>
<td>-0.038903</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.027418</td>
<td>0.000011</td>
<td>0.025725</td>
<td>-0.037732</td>
</tr>
</tbody>
</table>

**Table 1.** Increase capital adjustment cost from $\Phi = 1.5$, to $\Phi = 3$, holding all else constant.

<table>
<thead>
<tr>
<th>Table</th>
<th>Model Description</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Benchmark Model</td>
<td>Benchmark Model</td>
<td>0.051501</td>
<td>0.000009</td>
<td>-0.051538</td>
<td>0.074963</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.024071</td>
<td>0.000065</td>
<td>0.037109</td>
<td>0.042231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.027430</td>
<td>0.000069</td>
<td>-0.038581</td>
<td>0.029411</td>
</tr>
<tr>
<td>2. Epstein and Zin Model 1</td>
<td>Epstein and Zin Model 1</td>
<td>0.051520</td>
<td>0.000006</td>
<td>-0.008043</td>
<td>0.178294</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.023083</td>
<td>0.000041</td>
<td>0.084014</td>
<td>0.032853</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.028157</td>
<td>0.000046</td>
<td>-0.076021</td>
<td>0.013757</td>
</tr>
<tr>
<td>3. Epstein and Zin Model 2</td>
<td>Epstein and Zin Model 2</td>
<td>0.047195</td>
<td>0.000006</td>
<td>0.077337</td>
<td>0.118760</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.007900</td>
<td>0.000045</td>
<td>0.093354</td>
<td>0.023846</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.039295</td>
<td>0.000049</td>
<td>-0.072852</td>
<td>0.009630</td>
</tr>
</tbody>
</table>

**Table 2.** Increase size of TFP shock from $\sigma_{TFP}^2 = 0.0025$, to $\sigma_{TFP}^2 = 0.01$, holding all else constant.
### Summary of simulated statistics:

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Benchmark Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^k$</td>
<td>0.051475</td>
<td>0.000000</td>
<td>0.107669</td>
<td>0.047772</td>
</tr>
<tr>
<td>$R$</td>
<td>0.023975</td>
<td>0.000003</td>
<td>0.034699</td>
<td>0.052115</td>
</tr>
<tr>
<td>$R^k - R$</td>
<td>0.027499</td>
<td>0.000003</td>
<td>-0.033916</td>
<td>0.051813</td>
</tr>
<tr>
<td>2. Epstein and Zin Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^k$</td>
<td>0.051127</td>
<td>0.000000</td>
<td>-0.045321</td>
<td>0.003396</td>
</tr>
<tr>
<td>$R$</td>
<td>0.022661</td>
<td>0.000002</td>
<td>0.019348</td>
<td>0.050263</td>
</tr>
<tr>
<td>$R^k - R$</td>
<td>0.028466</td>
<td>0.000001</td>
<td>-0.024576</td>
<td>0.054084</td>
</tr>
<tr>
<td>3. Epstein and Zin Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^k$</td>
<td>0.047156</td>
<td>0.000000</td>
<td>-0.039013</td>
<td>0.029003</td>
</tr>
<tr>
<td>$R$</td>
<td>0.007751</td>
<td>0.000003</td>
<td>0.037665</td>
<td>0.060619</td>
</tr>
<tr>
<td>$R^k - R$</td>
<td>0.039405</td>
<td>0.000003</td>
<td>-0.036960</td>
<td>0.059225</td>
</tr>
</tbody>
</table>

| Premium (2-1)                 | 0.000967 |
| Premium (3-1)                 | 0.011906 |

**Table 3.** Increase size of monetary policy shock from $\sigma_{MP}^2 = 0.0025$, to $\sigma_{MP}^2 = 0.01$, holding all else constant.