Tax evasion, social norms and economic growth

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Abstract

This paper proposes a theoretical model to account for the most relevant micro- and macroeconomic empirical facts in the tax evasion literature. To do so, we integrate tax morale into a dynamic overlapping generations model of tax evasion. Tax morale is modeled as a social norm for tax compliance. It is shown that accounting for such nonpecuniary costs of evasion may not only explain (i) why some taxpayers never evade even if the gamble is profitable, and (ii) how a higher tax rate can increase evasion, but also that (iii) the share of evaded taxes over GDP decreases when countries grow and (iv) that tax morale is positively correlated with the level of GDP per capita, as suggested by recent empirical evidence. Finally, a higher tax rate increases aggregate evasion as well as the number of evaders in the economy when taxpayers’ decisions are interdependent. Simulations highlight the quantitative importance of our findings.
Keywords tax evasion, social norms, economic growth, overlapping generations

JEL-Classification H24, H26, D91
1 Introduction

Tax evasion is one of the main problems faced by fiscal authorities. For example, Slemrod (2007) estimates that the U.S. income tax gap in 2001 amounts to a total of $345 billion—more than 15% of the estimated actual (paid plus unpaid) tax liability. However, tax evasion is not a particular phenomenon in developed countries.\footnote{See Fuest and Riedel (2009) for a survey of tax evasion in developing countries. Alm et al. (1991), e.g., put the figure for tax evasion (and avoidance) at 46% of the Jamaican income tax in 1983.} As estimated by Cobham (2005), for instance, the overall level of tax revenue lost due to tax evasion in developing countries is equal to $285 billion per year. Thus, explaining the patterns of tax evasion and identifying tools to reduce it is an important concern in all economies.

The theoretical analysis of tax evasion starts with the seminal papers by Allingham and Sandmo (1972) and Yitzhaki (1974) which model tax evasion as a static portfolio selection issue.\footnote{Andreoni et al. (1998), Slemrod and Yitzhaki (2002) and Slemrod (2007) are recent surveys reviewing the main literature on the nature and determinants of tax evasion.} Subsequent empirical and experimental findings, however, have revealed important inconsistencies between theory and evidence.\footnote{See Alm et al. (1992) and Frey and Feld (2002) among others.} Specifically, the literature has identified four main puzzles first, the finding that tax evasion increases with the tax rate (Clotfelter, 1983, Poterba, 1987, Joulfaian and Rider, 1996) while theory predicts the opposite. Second, the finding of low levels of tax evasion in many countries compared to the high level predicted by theory given the low levels of deterrence (see Alm (1999) and Torgler (2002) for a review). Third, the finding that some taxpayers never evade, even if evasion is the profitable
option (Baldry, 1986, Alm, 1999). Fourth, the finding that the level of tax evasion and taxpayers’ attitudes towards evasion are related to the behavior of other taxpayers (Gaechter, 2006). This paper sets up a dynamic model of tax evasion in order to reconcile theory with empirical evidence.

While many studies have extended the basic portfolio selection model to explain these puzzles in a similar static framework, only a few recent papers analyze tax evasion in a dynamic context (Lin and Yang, 2001, Chen, 2003, Dzhumashev and Gahramanov, 2011, Levaggi and Menoncin, 2012, 2013, ?). However, as the main focus of these papers is on how tax evasion affects the relationship between income taxation and economic growth in the long-run, with the exception of ?, none of these papers studies the effects of tax evasion behavior in the short-run and in particular when countries are growing throughout the development process. To close this gap in the literature is the aim of the present paper.

We set up a general equilibrium model of tax evasion. More precisely, we integrate the Allingham-Sandmo framework into a dynamic two period overlapping-generations setting with production. Our model departs from a simple version with amoral agents and subsequently, in order to address the

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4Most of the existing literature has focused on explaining the positive relationship between the tax rate and evasion. Among these papers, Cowell and Gordon (1988) poses a framework where taxpayers consider the provision of public goods, Lee (2001) considers the possibility of self-insurance against possible penalties and Bayer (2006) endogenizes the probability of audit. A more recent literature also incorporates techniques of behavioral economics to address the puzzling tax effect (see Hashimzade et al. (2013)). Moreover, Kleven et al. (2011) emphasize the role of information available to the tax authority as a limiting factor of actual evasion opportunities to explain the low levels of observed tax evasion and the fact that some taxpayers never evade.

5See e.g. Boadway and Keen (1998) for a related approach. They study the role of capital income tax evasion in alleviating welfare losses due to time inconsistent taxation within an open economy model.

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inconsistencies between theory and evidence, considers a more sophisticated version with tax morale (namely, taxpayers’ attitudes, perceptions and moral values) and heterogeneous agents. The model is focused on of capital income tax evasion which is the most extended and easily implementable type of evasion. However, we also consider different ways to extend the model, like the possibility of individuals may evade labor income taxes, in order to see its robustness.

The importance of tax morale for individual evasion decisions has recently been emphasized by many empirical studies, see Luttmer and Singhal (2014) for a recent overview. According to these findings, the decision to evade taxes is not only determined by rational cost-benefit considerations but also by social and morale influences which are, in turn, shaped by the (perceived) compliance of other taxpayers: Individuals consider tax evasion to be a less serious wrongdoing the more widespread they presume it to be (Frey and Torgler, 2007). In line with this evidence, we define tax morale as an internalized social norm for tax compliance (Elster, 1989) which expands the cost incurred by evaders to include not only the fines payable upon detection, but also certain non-pecuniary considerations. Specifically, as in Gordon (1989) and Traxler (2010), the strength of the norm depends on an individual specific degree of norm internalization and the endogenously determined share of evaders in the economy (with a higher share inducing a weaker norm). Hence, individual evasion decisions depend on the behavior of others implying that individuals act conditionally cooperative (Gaechter, 2006).

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6See e.g. (Poterba, 1987, Sandmo, 2012) and Slemrod (2007).
As in Yitzhaki (1974), taxpayers are audited with a positive probability and, if caught, have to pay a penalty on the amount of evaded taxes. Tax revenue is used to finance a public consumption good which increases individual utility. Aggregate savings of utility maximizing agents determine the dynamics of the economy. Given a neoclassical technology, per capita capital increases throughout the transition towards the steady state, which in turn decreases the rate of return and therefore the incentives to evade taxes. In such a framework, the amount of undeclared taxes may increase both when countries are growing and in the steady state when the tax rate rises and individuals care about morality. More specifically, an increase in the tax rate generates two competing effects: a negative income effect that provides incentives to evade less and a net increase in the benefit of being dishonest which encourages taxpayers to evade. Thus, for a reasonable set of parameters the second effect prevails. Moreover, it is shown that increases in the strength of the norm to honestly pay what is owed as well as in the audit probability produce low levels of tax evasion in the long-run and throughout the transition towards the new steady state. Consequently, policies aimed at deterring tax evasion may not only alter economic incentives but also have long-lasting and amplifying effects through changes in the formation of social norms.

The main contribution of this paper is to present a dynamic model of tax evasion which accounts simultaneously for well known micro empirical findings as well as for the latest macro-dynamic observations. Specifically, our model allows us to derive several new results in the literature on tax evasion which are consistent with existing empirical evidence.
First, our model predicts that the share of tax evasion is declining at the extent that countries are growing and accumulating capital. In this respect, Crane and Nourzad (1986) who focus on aggregate tax evasion in the United States over the period 1947-81, show that despite tax evasion increases in absolute terms, it has fallen in relative terms when income has risen. This result is further supported by evidence in Schneider et al. (2011). According to their findings, the relative size of the shadow economy\textsuperscript{7} for 162 countries over the period 1999 to 2007 has decreased whereas the unweighted average of GDP per capita for the same set of countries and over the same time horizon has increased.\textsuperscript{8}

Second, neoclassical growth theory which describes the dynamics of the economies in the model, predicts a negative relationship between economic growth and tax evasion, i.e., that countries with low levels of per capita GDP (per capita capital) display high levels of tax evasion. By contrast, high-income countries (high levels of per capita capital) show low levels of tax evasion, for the same size of tax rates and similar technologies and preferences. Gordon and Li (2009), for example, document sharp differences in the ability to generate tax revenue among developed and developing countries:

\textsuperscript{7}Tax evasion can be considered to be an integral part of the shadow economy. Though it is difficult to have reliable information about the exact size of tax evasion, since it is an illegal activity and individuals have strong incentives to conceal their cheating, and though the shadow economy is clearly not synonymous with tax evasion, many researchers (Schneider (2005) and Alm and Embaye (2013) among others) frequently use shadow economy estimates as an indicator for the size of tax evasion. See Alm (2012) for a detailed discussion about the measuring of tax evasion.

\textsuperscript{8}Specifically, this unweighted average of GDP per capita rose from 5200 US$ to 8400 US$ over the whole period, while the unweighted average size of the shadow economies of all of these 162 countries decreased from 34.0\% of official GDP in 1999 to 31.2\% of official GDP in 2007. Data on GDP per capita are taken from the World Bank, see http://data.worldbank.org/.
though statutory tax rates are fairly similar across countries, effective tax rates differ widely given the lower fraction of GDP collected by these taxes among poorer countries. For instance, the maximum personal income tax rate in developed countries is on average 1.23 times higher than in developed countries, whereas income tax revenue over GDP is 2.47 times larger in developed countries. A similar pattern is also demonstrated by Easterly and Rebelo (1993a) and Easterly and Rebelo (1993b). According to their findings, income tax evasion is an important phenomenon, in particular for developing countries. Consistent with these observations, our model predicts that the size of tax evasion decreases insofar countries accumulate capital and reach higher levels of income.

Finally, we find a positive relationship between per capita GDP and tax morale. More precisely, our model predicts that countries with high levels of per capita income display low levels of evasion and a low share of evaders in the economy. A low share of evaders, in turn, implies larger moral costs of evading since the majority of population pays what they owe and this is perceived as the right behavior. Thus, the more other taxpayers are perceived to be honest, the more willing individuals are to pay their own taxes and reduce evasion. In this respect, Weck (1983) and Torgler (2003) document the existence of a positive relationship between tax evasion and tax morale for a wide sample of countries. Their findings support the hypothesis that the behavior of a taxpayer is influenced strongly by his perception of the behavior of other taxpayers. Moreover, Frey and Torgler (2007) and Torgler and

\footnote{Also, the cross sectional findings about the relative size of the informal economy by Friedman et al. (2000) indicate that informality is (on average) a more severe problem in countries with low GDP per capita, especially in Latin American countries.}
Schneider (2007) find that countries which display higher rates of tax evasion are characterized by low quality institutions or weak direct democratic rights\textsuperscript{10}, and Acemoglu et al. (2005) and Bethencourt (2013) among others, show that these countries are typically developing countries with low levels of per capita income. Thus, the empirical facts support the finding that high income countries exhibit high quality institutions, high levels of tax morale and so, low levels of evasion.

Our work relates to the literature analyzing the effects of morality, customs and stigma on tax evasion behavior, see Gordon (1989), Myles and Naylor (1996), Kim (2003), Fortin et al. (2007) and Traxler (2010). While these papers demonstrate how such non-pecuniary considerations may account for some of the tax evasion puzzles within a static framework\textsuperscript{11}, our contribution relative to these studies lies in modeling the dynamics of per capita capital and linking the size of tax evasion to the state of economic development.

Our work also relates to papers studying dynamic models of tax evasion, see e.g. Lin and Yang (2001), Chen (2003), Dzhumashev and Gahramanov (2011) and Levaggi and Menoncin (2012, 2013). Relative to these papers, however, we do not focus on the tax evasion-growth nexus in the steady state (or balance growth path), but rather on the evolution of tax evasion throughout the transition towards the steady state. This, in turn, allows us to document not only cross country variations in levels of tax evasion but

\textsuperscript{10}These results are further supported by evidence in Friedman et al. (2000) suggesting that weak economic institutions imply a large unofficial economy.

\textsuperscript{11}Note that we refer to these models as static insofar as they do not allow for income dynamics. However, as in the present paper, the share of evaders may well change over time.
also to account for the development of these levels over time. The paper more related to ours is [?], which incorporates a social norm in an OLG model and analyzes the evolution of tax evasion along the transition. However, unlike us, first, he considers that the amount of concealed income is exogenous, that is, that individuals’ decision is about evading or not evading taxes. Second, taxation and the possibility of evading taxes is only for young agents with respect of their labor income, capital income tax evasion is not considered.

The remainder is organized as follows. Section 2 describes the basic model without morality. Section 3 extends the basic framework to account for nonpecuniary costs of evasion and presents numerical results from Monte Carlo simulations to illustrate our man findings and explore their robustness against several alternative modeling assumptions. Section 4 presents some extensions of the model. Section 5 concludes. All proofs and technical considerations are included in the appendix.

2 The Basic Model

The basic framework is a two period overlapping-generations model in the tradition of Diamond (1965). The size of each generation is assumed to be constant and normalized to one. Non-altruistic individuals are endowed with one unit of labour time when young, and are retired during old age. Markets are competitive.

The government collects a proportional tax on capital income which may, however, be evaded by individuals. The reason we focus on capital income tax evasion is twofold. First, the probability of detection is much lower than
for other income sources and therefore the opportunities for hiding true income from the tax collector are substantially higher than for example in the case of labor (Poterba, 1987, Sandmo, 2012). Second, capital income tax evasion is indeed a serious problem in many countries (see e.g., Slemrod (2007) for the US). Also, as pointed out by Sandmo (2012), ‘in the theoretical literature, the evasion of taxes on labour income has received considerably more attention than the evasion of taxes on capital. It is not obvious why this should be so; as already noted, it is difficult to argue that capital income evasion is of less empirical importance.\(^{12}\)

2.1 Firms

On the production side of the model, perfect competition between a large number of identical firms is assumed. A representative firm in period \(t\) produces a homogenous output good according to a Cobb–Douglas production function with capital \(K_t\) and homogeneous labour \(L_t\) as inputs:

\[
Y_t = AK_t^\alpha L_t^{1-\alpha},
\]

where \(1 > \alpha > 0\) is the share parameter of capital.

Each firm maximizes profits under perfect competition, implying that, in equilibrium, production factors are paid their marginal products:

\[
\omega_t = (1 - \alpha)AK_t^\alpha L_t^{-\alpha} = (1 - \alpha)\lambda_k^\alpha
\]  

\(^{12}\)In order to check the robustness of our results we extend model to include labor income tax evasion in section 4.
and

\[ r_t = aAK_t^{a-1}L_t^{1-a} = aAk_t^{a-1} \]  \hspace{1cm} (3) \]

where \( k_t = K_t/L_t \) is the capital intensity.

### 2.2 Consumers

Each generation consists of agents whose life has two periods of equal lengths: the young adult age during which each agent inelastically supplies one unit of labour time to work and raises one offspring, and the old age spent in retirement. Since each young adult produces one offspring, the population remains constant in every generation and is normalized to one. When adult, working individuals receive the wage \( w_t \). Income is spent on consumption \( c_t \) and savings \( s_t \):

\[ w_t = c_t + s_t. \]  \hspace{1cm} (4) \]

When old, each individual consumes the return to his savings and may evade a fraction \( e_{t+1} \in [0,1] \) of this return. The declared income gets taxed with a proportional income tax at rate \( r \). With a fixed probability \( p \) the evasion gets detected. In this case, the tax evader has to pay the full taxes and a penalty \( \gamma \) which is proportional to the taxes evaded Yitzhaki (1974). With probability \( 1 - p \) the evasion remains undetected and the evader only pays taxes on the declared income. The corresponding levels of second period consumption for state \( u \) - escaping undetected - or state \( d \) - getting detected - are given by

\[ d_{t+1}^u = R_{t+1}^u s_t \]  \hspace{1cm} (5) \]
\[ d^d_{t+1} = R^d_{t+1} s_t \]  

(6)

where \( R^u_{t+1} = 1 - \delta + r_{t+1}(1 - \tau + \tau e_{t+1}) \) and \( R^d_{t+1} = 1 - \delta + r_{t+1}(1 - \gamma \tau e_{t+1}) \) denote the respective after tax interest factors (or the private total returns on capital)\(^{13}\) and \( \delta \in [0, 1] \) is the depreciation rate of capital. Finally, we consider that both young and old individuals also consume a public good, \( g_t \), which is financed with tax collection.] The life-cycle utility function of an individual born in \( t \) is

\[
E[U(c_t, d^u_{t+1}, d^d_{t+1})] = u(c_t) + v(g_t) + (1 - p)\beta u(d^u_{t+1}) + p \beta u(d^d_{t+1}) + \beta v(g_{t+1}) \quad (7)
\]

where \( \beta > 0 \) is a discount factor [\([v' > 0]\)]. For reasons of tractability we will assume that the expected utility representation \( u \) is logarithmic, i.e., \( u(x) = \ln(x) \). Each individual maximizes the utility (7), subject to the constraints (4), (5) and (6), by choosing \( c_t, s_t, e_{t+1}, d^u_{t+1} \) and \( d^d_{t+1} \). With logarithmic preferences, it is straightforward to show that

\[
s_t = \frac{\beta}{1 + \beta} w_t \quad (8)
\]

and that the decision on \( e_{t+1} \) does not depend on \( s_t \). The first and second order conditions with respect to the choice of \( e_{t+1} \) are then given by

\[
E[U(e_{t+1})]' = \beta \tau r_{t+1} \left[ (1 - p)u'(R^u_{t+1}) - \gamma pu'(R^d_{t+1}) \right] = 0 \quad (9)
\]

\[
E[U(e_{t+1})]'' = \beta(\tau r_{t+1})^2 \left[ (1 - p)u''(R^u_{t+1}) + p \gamma^2 u''(R^d_{t+1}) \right] < 0 \quad (10)
\]

\(^{13}\)Note that the corresponding after tax interest rates per period are \( i^u_{t+1} = R^u_{t+1} - 1 \) and \( i^d_{t+1} = R^d_{t+1} - 1 \). See, e.g., Uhlig and Yanagawa (1996) for a similar modeling approach and a more extensive treatment of the basic overlapping generations model.
Equation (9) characterizes $e^*_{t+1}$, the optimal fraction of income concealed. We will assume in the following that such an interior solution $e^*_{t+1} \in (0, 1)$ always exists.\footnote{For the basic model and with the assumption of logarithmic preferences such a solution will always exist. This is not necessarily true, however, for the extended model that will be considered in section 3.}

The fraction of evasion decreases as tax enforcement becomes stricter, i.e. $de^*_{t+1}/dp < 0$ and $de^*_{t+1}/d\gamma < 0$. This can be seen by implicitly differentiating the first order condition (9). Furthermore, a marginal increase in the tax rate reduces the optimal share of evasion:

$$\frac{de^*_{t+1}}{d\tau} = \frac{(1-p)\beta \rho'_{t+1}u'(R^u_{t+1})[\rho(R^u_{t+1})(1-e_{t+1}) - \rho(R^d_{t+1})(1+\gamma e_{t+1})]}{-E[U(e_{t+1})]''} < 0$$

(11)

where $\rho(x) = -u''(x)/u'(x)$ is the Arrow-Pratt measure of absolute risk aversion, which satisfies $\rho'(x) \leq 0$ and thereby $\rho(R^d_{t+1}) \geq \rho(R^u_{t+1})$ for non-increasing absolute risk aversion\footnote{Note that this assumption is always satisfied for our logarithmic preference representation.}. Intuitively, a higher tax rate reduces taxpayers’ income and makes them less willing to take risks. There is no substitution effect as the penalty is assumed to be levied on the share of income evaded, implying that marginal gains and marginal costs from evasion exactly offset each other. In addition to the standard model which considers evasion in levels, however, the above result predicts a decreasing share of evasion instead of a decreasing level.

Similarly, for $\rho(R^d_{t+1}) \geq \rho(R^u_{t+1})$, an increase in the interest rate (capital
income) lowers the optimal share of evasion:

\[
\frac{de^*_t}{dr_{t+1}} = \frac{(1-p)\tau \beta r_{t+1} u'(R_{t+1}^u)\rho(R_{t+1}^d)(1-\tau-\gamma e_{t+1}) - \rho(R_{t+1}^u)(1-\tau+\gamma e_{t+1})}{E[U(e_{t+1})]^\prime} \leq 0
\]

(12)

Thus, our model predicts that the percentage of evasion over total income decreases. The intuition is similar to the one coming from an increase in the tax rate. First, there is no substitution effect as the penalty is assumed to be levied on the share of income evaded, implying that marginal gains and marginal costs from evasion exactly offset each other. Second, a higher interest rate increases taxpayers' income and makes them more willing to evade income\textsuperscript{16}. However, given that concealed income has an income elasticity of demand less than one, the percentage of evaded taxes decreases.

2.3 Dynamics and Steady State

We are now able to define the intertemporal equilibrium of the economy. Given a fiscal policy (parameters \(\tau, p\) and \(\gamma\)) and an initial value of the capital stock \(k_0 = s_{-1}\), a perfect-foresight intertemporal equilibrium is characterized by a sequence of quantities and prices:

\[
\{c_t, d_t, k_t, s_t, e_t; w_t, r_t\}_{t \geq 0}.
\]

[The government provides the public good and balances its budget, i.e.,

\[
g_{t+1} = \tau[1 - e_{t+1}(1 - p(1 + \gamma))] r_{t+1} s_t
\]

(13)

\textsuperscript{16}It is straightforward to show that the amount of concealed income, \(e_{t+1} r_{t+1}\), increases in \(r_{t+1}\).
Individuals maximize utility, firms maximize profits, factor markets are competitive, and all markets clear. The market-clearing conditions for the labour and capital markets are

\begin{align*}
L_t &= 1, \quad \text{(14)} \\
K_t &= s_{t-1}. \quad \text{(15)}
\end{align*}

The dynamics of the basic model are characterized by the following first order difference equation (using (2), (8), (14) and (15)):

\begin{equation}
k_{t+1} = \frac{\beta}{1 + \beta}(1 - \alpha)Ak_t^a \quad \text{(16)}
\end{equation}

which monotonically converges towards a unique steady state $k^*$. [Countries with an initial per capita capital lower than the steady state level, $k_0 < k^*$, accumulate capital till they reach the steady state.] Clearly, the tax rate does not affect the dynamics so that the share of evasion increases throughout the transition towards the steady state, i.e. $de_{t+1}/dk_t > 0$. [Therefore, insofar countries are growing they are also experiencing increases in tax evasions, i.e., the economic growth process is accompanied by an increase in the share of evasion over GDP].

Therefore, the basic model turns out to be inconsistent with a wide range of empirical findings: First, it predicts a decrease of the share of evasion as a response to a tax increase whereas empirical studies point to an increase of evasion at the individual level (see e.g. Clotfelter (1983) and Joulfaian and Rider (1996)) as well as at the aggregate level (Poterba, 1987). Second,
in this model agents will always evade taxes as long as this is the profitable option while empirical evidence shows that there are individuals that never evade (see for example, Baldry (1986) and Alm (1999)). Third, recent empirical literature shows that taxpayers’ attitudes towards evasion are related to the behavior of other taxpayers in the society (see e.g. Gaechter (2006)). Still, in the basic framework taxpayers behavior is absolutely independent of others. Moreover, cross sectional data and longitudinal data suggest that the share of evasion over GDP decreases with the stage of economic development (see for instance, Besley and Persson (2014) and Gordon and Li (2009) for the cross-sectional case and, Crane and Nourzad (1986) and Schneider et al. (2011) for the longitudinal one). By contrast, the results of the basic model imply an increase of the share of evasion along the transitional path of an economy.

In order to reconcile theory with empirical evidence, the next section introduces moral concerns into the basic model.

3 Morality

In this section we introduce morality and reputation concerns along the lines of Gordon (1989). Accordingly, tax morale is modeled as an internalized social norm for tax compliance. The strength of this norm is assumed to be endogenous and depends on the number of individuals in society adhering to it (Akerlof, 1980, Lindbeck et al., 1999). Hence, the more individuals evade

17Similar approaches have extended the classical portfolio choice model to allow for a social custom for tax compliance (Myles and Naylor, 1996) or a stigmatization effect (Kim, 2003). See Traxler (2010) for a discussion of these different approaches and a generalization of the model by Gordon (1989).
taxes, the weaker the social norm, as it becomes easier for the individual taxpayer to justify his own wrongdoing to himself, the more other people violate the societies’ code of conduct. Preferences therefore do not only depend on consumption levels but also on the ‘moral costs’ of tax evasion. Consequently, the life-cycle utility function of an individual $i$ born in period $t$ can be represented by the following additive preference structure:

$$U_i(c_t, d^u_{t+1}, d^d_{t+1}, e_{t+1}) = E[U(c_t, d^u_{t+1}, d^d_{t+1})] - e_{t+1}(\theta_i + \mu(1 - n_t))$$ (17)

where the expression $(\theta_i + \mu(1 - n_t))e_{t+1}$ captures the moral costs of tax evasion. These costs are linearly increasing in the individual degree of norm internalization $\theta_i \geq 0$, which has distribution function $F(\theta_i)$ and support $[0, \bar{\theta}]$. Furthermore, moral costs depend on individually fixed (marginal) reputation costs $\mu > 0$ and on the share of evaders in society $n_t$.\(^\text{18}\) Equivalent assumptions are made in static models by Gordon (1989), Fortin et al. (2007) and Traxler (2010). Moreover, for reasons of analytical tractability, we assume that moral costs are linearly increasing in the share rather than the absolute amount of evaded income. This latter assumption captures the idea that the extent of guilt or shame incurred by evaders is evaluated relative to the amount of taxes that could have been evaded. For example, moral concerns should be more pronounced for a taxpayer evading almost all of his income as compared to one evading only a small fraction, given that both have the same amount of income (see Bosco and Mittone (1997) and Levaggi

\(^{18}\)Our main results would equally hold for $\mu = 0$. In this case, however, the dynamics of the model would be completely determined by the evolution of the capital stock and not depend on the share of evaders in society.
and Menoncin (2013) for a similar approach).\footnote{In the appendix, however, we show, by means of Monte Carlo simulations, that our main results readily carry over to the alternative framework where moral costs depend on the level of evaded income (as in Gordon (1989)).}

Individuals maximize (44) subject to (8), (5) and (6) taking prices and the number of evaders $n_t$ as given. The first order condition for an interior solution with respect to $e_{t+1}$ is

$$E[U(.)]' \equiv \beta t r_{t+1} \left[ (1 - p)u'(R^u_{t+1}) - \gamma p u'(R^d_{t+1}) \right] = \theta_i + \mu(1 - n_t)$$

(18)

while the second order condition is the same as (10). Norm guided taxpayers will choose a share of evasion such that the marginal expected utility $E[U(.)]'$ equals $\theta_i + \mu(1 - n_t)$, the marginal moral costs from concealing income. An interior solution requires the evasion gamble to be better than fair\footnote{This requires $1 - p(1 + \gamma) > 0$ or equivalently $\gamma < (1 - p)/p$. The opposite case, in which $1 - p(1 + \gamma) < 0$ is negative, is of little interest, since tax evasion would never take place.}, i.e.

$$z(r_{t+1}) \equiv E[U(0)]' = \frac{(1 - p(1 + \gamma)) \beta t r_{t+1}}{1 - \delta + (1 - \tau)r_{t+1}} > 0$$

(19)

From equations (18) and (10) it follows that taxpayers with $\theta_i + \mu(1 - n_t) > z(r_{t+1})$ do not conceal any income. This implies the threshold\footnote{Note that the threshold value of the individual tax morale parameter $\hat{\theta}$ results from the combination of the standard concave utility function with the linear functional form of the tax morale term.}

$$\hat{\theta}(n_t, r_{t+1}) \equiv z(r_{t+1}) - \mu(1 - n_t)$$

(20)

which allows us to characterize the optimal individual evasion behavior $e_{t+1}^{*,i}$.
for a given level of $n_t$ and $r_{t+1}$:

$$e^{*,i}_{t+1} = \begin{cases} 0 & \text{for } \theta_i \geq \hat{\theta}(n_t, r_{t+1}) \\ e^{*,i}_{t+1} & \text{for } \theta_i < \hat{\theta}(n_t, r_{t+1}) \end{cases}$$

Those individuals with $\theta_i < \hat{\theta}(n_t, r_{t+1})$ will choose an intermediate share of evasion, $e^{*,i}_{t+1} \in [0, e^{*,i}_t]$, whereas those with $\theta_i \geq \hat{\theta}(n_t, r_{t+1})$ do not evade as compliance to the norm is the best policy.

Similar to the basic model without morality, evasion decreases when $p$ or $\gamma$ increase for those individuals with $\theta_i < \hat{\theta}$. Moreover, $\hat{\theta}$ falls in both cases, so that the number of individuals choosing $e^{*,i}_{t+1} = 0$ increases. Hence, aggregate evasion must fall. The effects of a change in $\tau$ and $r_{t+1}$ are described by the following proposition:

**Proposition 1** There exists some $\bar{\theta}(n_t, r_{t+1}) < \hat{\theta}(n_t, r_{t+1})$ such that $\partial e^{*,i}_{t+1} / \partial \tau \leq 0$, $\partial e^{*,i}_{t+1} / \partial r_{t+1} \leq 0$ if $\theta_i < \bar{\theta}(n_t, r_{t+1})$ and $\partial e^{*,i}_{t+1} / \partial \tau > 0$, $\partial e^{*,i}_{t+1} / \partial r_{t+1} > 0$ if $\theta_i > \bar{\theta}(n_t, r_{t+1})$ for all $k_t$ and $n_t$.

**Proof:** See Appendix.

The effect of a change in the tax rate on tax evasion has been demonstrated before by Gordon (1989) and Traxler (2010) in a static framework. It is shown here how such a result carries over to a dynamic framework and that it holds along the complete transitional path of the economy. The basic intuition is the following: a higher tax rate increases the marginal benefits, as well as the marginal costs (associated with higher expected fines and with morality concerns). In the model of the previous section (without morality) we show that this marginal gains and marginal costs from evasion exactly
offset each other, implying no substitution effect and only a negative income effect which encourages taxpayers to take less risks and so, to reduce tax evasion. However, in this version of the model, given that moral costs of evasion are assumed to depend on the share of income concealed rather than on taxes evaded, costs are not affected by a tax change. As a result, marginal benefits from concealing exceed marginal expected costs implying a substitution effect that provides an incentive to increase evasion. We prove that, for those with $\theta_i < \tilde{\theta}$, the negative income effect dominates and tax evasion reduces as taxes rises, whereas for those with $\theta_i > \tilde{\theta}$ the substitution effect prevails and tax evasion increases.

An increase in the interest rate produces similar effects to those of the tax rate. It increases the marginal benefits as well as the marginal costs of evasion. However, as moral costs of evasion are assumed to depend on the share of income concealed, the increase in marginal benefits from concealing is above the increase in marginal expected costs, producing a substitution effect which provides incentives to increase tax evasion. Moreover, similar to the model without morality, there is a positive income effect which increases the total amount of evaded taxes. However, given that tax evasion has an income elasticity of demand less than one, the percentage of evaded taxes decreases. Thus, the resulting effect depends on a positive income effect as in the basic model and a substitution effect working into the opposite direction. It will be negative for those individuals with $\theta_i < \tilde{\theta}$ and positive otherwise.
Finally, differentiation of (20) yields

$$\frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial \tau} > 0 \quad \text{and} \quad \frac{\partial \hat{\theta}(n_t, r_{t+1})}{\partial r_{t+1}} > 0.$$ (22)

Therefore, the emergence of new evaders tends to increase the share of aggregate evasion. Only those evaders with a sufficiently small degree of norm internalization act according to the standard portfolio approach and reduce their holdings of the risky asset whereas the more honest individuals evade more the higher the tax rate.

### 3.1 Dynamics and Steady State

The definition of an intertemporal equilibrium is analogous to the basic model without morality. Nevertheless, apart from a fiscal policy (parameters $\tau$, $p$ and $\gamma$) and an initial value of the capital stock $k_0 > 0$, an additional initial value of the share of evaders in society is required, $n_0 \geq 0$. Thus, a perfect-foresight intertemporal equilibrium is characterized by a sequence of quantities and prices:

$$\{c_t, d_t, k_t, s_t, e_t, n_t; w_t, r_t\}_{t \geq 0}$$

such that [[the government provides the public good and balances its budget,]] individuals maximize utility, firms maximize profits, factor markets are competitive, and all markets clear.

The capital stock in period $t + 1$ results from individuals’ savings in the
preceding period, i.e. $k_{t+1} = s_t$ which implies (using (2) and (8)):

$$k_{t+1} = \frac{\beta}{1+\beta}(1-a)Ak_t^a$$

(23)

The equilibrium share of evaders evolves according to the following dynamic equation:

$$n_{t+1} = F\left(\hat{\theta}(n_t,r_{t+1})\right)$$

(24)

Clearly, the share of evaders in period $t+1$ is a positive function of the tax rate, the share of evaders in the preceding period and the interest rate. Just note that $F' = f(\theta_i) > 0$ and recall equations (20) and (22).

The aggregate (average) share of evasion in period $t+1$ is given by

$$\bar{e}_{t+1} = \int_0^{\hat{\theta}(n_t,r_{t+1})} e_{t+1} f(\theta_i)d\theta_i = \int_0^{\hat{\theta}(n_t,r_{t+1})} e_{t+1}^* f(\theta_i)d\theta_i.$$  

(25)

The derivatives of $\bar{e}_{t+1}$ with respect to $k_t$ and $\tau$ can be written as follows:

$$\frac{\partial \bar{e}_{t+1}}{\partial k_t} = \left[ \int_0^{\hat{\theta}(n_t,r_{t+1})} \frac{\partial e_{t+1}^*}{\partial r_{t+1}} f(\theta_i)d\theta_i + e_{t+1}^* \right] \frac{\partial \hat{\theta}(n_t,r_{t+1})}{\partial k_t} \frac{\partial r_{t+1}}{\partial k_t}$$

(26)

and

$$\frac{\partial \bar{e}_{t+1}}{\partial \tau} = \int_0^{\hat{\theta}(n_t,r_{t+1})} \frac{\partial e_{t+1}^*}{\partial \tau} f(\theta_i)d\theta_i + e_{t+1}^* \frac{\partial \hat{\theta}(n_t,r_{t+1})}{\partial \tau} \frac{\partial r_{t+1}}{\partial \tau}$$

(27)

The second summand in both equations describes the change of aggregate evasion due to the emergence of new evaders in the society. Since $\partial \hat{\theta}/\partial \tau > 0$ and $\partial \hat{\theta}/\partial r_{t+1} > 0$, an increase of the share of evaders should increase the
share of tax evasion. However, at the margin, this effect is equal to zero and the second summand vanishes as $e^{\ast,i}_{t+1}$ evaluated at $\theta_i = \hat{\theta}$ is equal to zero according to its definition. The first summand describes the response of existing evaders and can be decomposed into a negative effect for those individuals with $\theta_i < \hat{\theta}(n_t, r_{t+1})$ and a positive effect for those with $\theta_i > \hat{\theta}(n_t, r_{t+1})$, as has been demonstrated in proposition 1. The overall effect thus critically depends on the distribution function $F(\theta_i)$.

As has been demonstrated in Gordon (1989), a sufficient condition for an interior steady state $n_* \in (0, 1)$ is given by

$$\max\{\bar{\theta}, \mu\} > \hat{\theta}(k_*, 1) > \min\{\bar{\theta}, \mu\}$$ (28)

where $k_* = (\beta/(1 + \beta)(1 - \alpha)A)^{1/(1 - \alpha)}$ is the steady state solution of equation (23). Moreover, in the appendix it is shown that a sufficient condition for a stable steady state $(n_*, k_*)$ of the dynamic system defined by equations (23) and (24) is

$$\frac{1}{F'(\hat{\theta}(n_*, r_*))} > \beta\alpha(1 - \alpha)Ak_\alpha^a\left(\frac{\mu}{1 + \beta} + \frac{\tau(1 - \delta)(1 - p(1 + \gamma))}{k_*(1 - \delta + (1 - \tau)r_*)^2}\right)$$ (29)

The existence of multiple steady states clearly depends on the functional form of $F$.\textsuperscript{23} For an uniform distribution, however, there is a unique steady state $(n_*, k_*)$ and the stability condition boils down to assuming that $\bar{\theta}$ is sufficiently large.\textsuperscript{24} As a consequence, in this case it is also possible to explicitly

\textsuperscript{23}See Kim (2003) or Traxler (2010) for an analysis of multiple steady states.

\textsuperscript{24}It is important to note that our main results are robust to alternative functional forms which however preclude analytical solutions. Section 3.2 presents Monte Carlo simulations with alternative functional forms to illustrate this point.
determine the signs of equations (26) and (27). More precisely, we get

\[
\text{sign} \left( \frac{\partial \tilde{e}_{t+1}}{\partial k_t} \right) = - \text{sign} \left( \frac{\partial \tilde{e}_{t+1}}{\partial \tau} \right) = \mu(1 - n_t)(\gamma - 1)R_{t+1} - \beta \tau \gamma r_{t+1} + \bar{m}
\]

(30)

where \( R_{t+1} = 1 - \delta + (1 - \tau)r_{t+1} \) and

\[
\bar{m} = \sqrt{(\beta \tau \gamma r_{t+1} - \mu(1 - n_t)(\gamma + 1)R_{t+1})^2 + 4p\beta \tau \gamma r_{t+1} \mu(1 - n_t)(\gamma + 1)R_{t+1}}.
\]

(31)

Straight forward calculations show that

\[
- \text{sign} \left( \frac{\partial \tilde{e}_{t+1}}{\partial k_t} \right) = \text{sign} \left( \frac{\partial \tilde{e}_{t+1}}{\partial \tau} \right) > 0 \Leftrightarrow \hat{\theta}(n_t, r_{t+1}) > 0.
\]

(33)

These findings are summarized in the following propositions:

**Proposition 2** Assume that \( \theta_i \) is uniformly distributed with support \([0, \bar{\theta}]\).

Then, [[the aggregate share of evasion decreases insofar countries are accumulating capital throughout the development process,]] ((a higher level of per capita capital decreases the aggregate share of evasion throughout the transition towards the steady state,)) i.e. \( \frac{\partial \tilde{e}_{t+1}}{\partial k_t} < 0 \).

**Proof:** See Appendix.

When the initial amount of per capita capital is below its steady state level \([, k_0 < k^\ast ,]\) the rate of return to investing into the capital stock is high. [[Countries monotonically accumulate capital until they reach steady level.]] Capital accumulation however decreases the interest rate and thus the threshold determining the number of evaders in the economy (see equation (22)).
A lower interest rate reduces the tax base which in turn renders tax evasion less profitable so that the number of tax evaders declines throughout the transition towards the steady state. However, as has been established in proposition 1, the behavioral responses of the remaining evaders are generally ambiguous and depend on the individual specific degree of honesty. If \( \theta_i \) is uniformly distributed, proposition 4 demonstrates that the overall effect is negative, implying that both the share of evaded taxes and the number of evaders decline (throughout the transition) [throughout the capital accumulation process]. The decrease in the number of evaders increases the moral costs of tax evasion for the remaining evaders in each of the following periods, thereby reinforcing the decline in the share of tax evasion. [Thus, a negative relationship between economic growth and tax evasion is established.] As a result, economies with low levels of per capita capital and an initial stage of economic development are characterized by high levels of tax evasion and a large number of evaders, whereas economies with high levels of per capita capital are characterized by the opposite.

These results are consistent with the empirical findings of Crane and Nourzad (1986) who analyze aggregate tax evasion in the US over the period 1947-81. Furthermore, interpreting each point along the dynamic path of the economy as a specific set of countries, our model accounts for the observation that tax evasion is a more severe problem in developing countries as compared to developed ones (see e.g. Gordon and Li (2009) and Besley and Persson (2014)). In particular, our simple model predicts that the share of evasion decreases as economies converge towards the (unique) steady state. Moreover, as a consequence, the average moral cost of tax evasion increases.
with per capita income, which is consistent with empirical results (see Torgler and Schneider (2007) and Bethencourt (2013)).

The next proposition summarizes the effect of income taxation on the aggregate share of evasion:

**Proposition 3** Assume that $\theta_i$ is uniformly distributed with support $[0, \bar{\theta}]$. Then, a higher tax rate increases the aggregate share of evasion, i.e. $\partial \bar{e}_{t+1}/\partial \tau > 0$.

**Proof:** See Appendix.

To understand the effect of the tax rate suppose that the economy is initially in steady state. An increase in the tax rate then implies a larger threshold which determines the number of evaders in the economy (according to equation (22)): Tax evasion becomes more profitable and therefore the subset of tax evading individuals increases. At the same time, proposition 1 establishes that only those individuals with sufficiently high moral concerns expand their evasion activities. The overall change in the share of tax evasion will thus depend on the relative size of the decrease in tax evasion for individuals with low moral concerns. Similarly to the preceding proposition, if $\theta_i$ is uniformly distributed, the first effect dominates, and so the share of evaded taxes increases. In each of the following periods, the larger number of evaders will reduce the moral cost of evading taxes which will in turn increase tax evasion and the number of evaders even more. The economy will then converge to a new steady state with a larger number of evaders and a higher share of tax evasion.

The extended version of our model therefore allows us to account for exis-
tent ((puzzles)) [[findings]] in the literature of tax evasion which the basic model without morality could not explain. In fact, predictions of the model are consistent with the following empirical findings: a positive relationship between tax evasion and tax rates at the individual and aggregate level; the fact that there exists taxpayers that never evade as long as this is the profitable option and the observation that taxpayers’ evasion decisions are interdependent.

Summarizing, the model is able to account simultaneously for well known micro empirical findings as well as for the latest macro-dynamic observations which have been recently documented in the empirical literature on tax evasion.

3.2 Monte Carlo simulations

The aim of this section is twofold. First, it serves to illustrate how the aggregate share of evasion and the share of evaders in the economy react to an increase in the tax rate and how these variables evolve along the transition towards the steady state. Second, and importantly, it shows that our main results are robust against alternative functional forms of the underlying distribution function $F(.)$.

In order to simulate the model, we use the following parameter configuration: $\alpha = 0.3$, a standard value in the literature, $p = 0.05$, $\gamma = 2$ which implies $1 - p(1 + \gamma) = 0.85$ and therefore corresponds to the average value implied by the fiscal systems of most countries (see Kim (2003)), and $A = 8$ and $\delta = 0.9$ as in Rivas (2003). Finally, in order to generate plausible values for...
the share of evaded income and the number of evaders in the economy, we set $\beta = 0.7$ and $\mu = 0.1$. We consider three alternative distribution functions with support on the positive real axis: the uniform distribution (on $[0,1]$), the log-normal distribution (with parameters $\text{LN}(0,1)$) and the Weibull distribution (with parameters $\text{WBL}(1,1.5)$).

For a given distribution function $F(.)$, the model is solved using Monte Carlo simulations. Specifically, we take $N = 100000$ draws from the corresponding density function and solve the first order conditions for each draw numerically. The individual choices are then averaged to obtain the aggregate stock of capital and the aggregate amount of evaded income in each period $t$. The threshold level $\hat{\theta}$ which, together with the underlying distribution function, determines the evolution of the number of evaders in society (see equation (24)) is obtained by taking the maximum value of $\theta_i$ for which $e_{i,t+1}^*\geq 0$.

Table 1 summarizes the share of evaders and the share of evaded income in steady state for varying levels of $\tau$ and the different functional forms of $F(.)$. In particular, we solve the economy for $\tau = \{0.2, 0.3, 0.4\}$. Notice that for the characterization of the economy we propose, in almost all cases the share of evaded income ranges from 1.7-7.9%. These numbers seem to be consistent with recent evidence. More precisely, estimations for the US show that the share of evaded taxes is about 2-3% of GDP (Andreoni et al., 1998, Slemrod, 2007), whereas Lang et al. (1997) find a tax gap for Germany in 1983 which corresponds to about 7% of GDP.

Clearly, an increase in the capital income tax rate by ten percentage points raises the share of evaders in society as well as the share of taxes
evaded. For example, increasing \( \tau \) from 0.2 to 0.3 raises the share of evaders by 4.6 percentage points from 2.2\% to 6.7\% while the aggregate share of evaded income increases by 2.2 percentage points from 1.6\% to 3.8\% (for the log-normal distribution).

[Insert table 1 around here.]

Figures 1-3 present the dynamics of \( \bar{e}_{t+1} \) and \( n_{t+1} \) for alternative levels of \( \tau \) and the different functional forms of \( F(.) \). More precisely, in each figure, \( \tau = 0.2 \) corresponds to the lowest transitional paths whereas the highest ones correspond to \( \tau = 0.4 \). Two findings are noteworthy. First, the underlying distribution function does indeed affect the quantitative implications of our model (as expected). For example, the drop in the share of evaded income throughout the transitional path seems to be more pronounced for the uniform as compared to the log-normal or the Weibull distribution. Second, the qualitative results, however, are not sensitive to alternative functional forms of \( F(.) \). Specifically, both the share of evaded income as well as the share of evaders in society monotonically decrease as capital accumulates in line with our theoretical predictions.

[Insert Figures 1-3 around here.]
4 Including labour income tax evasion

The basic framework we have considered in previous sections is focused on capital income tax evasion. Both theoretical and empirical reasons allowed us justify this assumption (see section 2). However, even though, we think that it would be interesting analyzing implications of considering evasion of taxes on labour income and, checking if main results we have obtained remain stable. Thus, the objective of this section is extending the framework to include the labour income tax evasion. We start with a simple version without morality and then we include it.

4.1 Labor income tax evasion without morality

We keep the same structure that the basic framework, except for the fact that working individuals have to pay taxes and they may evade a fraction $e_t$ of their labour income. As in the old individuals case, the declared income gets taxed with the proportional income tax rate $\tau$. Moreover, evasion is detected with a fixed probability $p$ and evaders would have to pay the full taxes and the penalty $\gamma$. In this model, the corresponding levels of first period consumption for both states undetected and getting detected are given by

$$c^u_t = w_t(1 - \tau + \tau e_t) - s_t$$

$$c^d_t = w_t(1 - \tau - \gamma \tau e_t) - s_t$$

In a simple version without morality, the life-cycle utility function of an
individual born in $t$ is

$$E[U(c^u_t,c^d_t,d^u_{t+1},d^d_{t+1})] = (1 - p)u(c^u_t) + pu(c^d_t) + v(g_t) + (1 - p)\beta u(d^u_{t+1}) + p\beta u(d^d_{t+1}) + \beta v(g_{t+1})$$  \hspace{1cm} (36)$$

First order conditions with respect to $s_t$, $e_t$ and $e_{t+1}$ for an interior solution are:

$$E[U(.)]' \equiv -\left[(1 - p)u'(c^u_t) + pu'(c^d_t)\right] + \frac{\beta}{s_t} = 0 \hspace{1cm} (37)$$

$$E[U(.)]' \equiv \tau w_t \left[(1 - p)u'(c^u_t) - \gamma pu'(c^d_t)\right] = 0 \hspace{1cm} (38)$$

$$E[U(.)]' \equiv \beta \tau r_{t+1} \left[(1 - p)u'(R^u_{t+1}) - \gamma pu'(R^d_{t+1})\right] = 0 \hspace{1cm} (39)$$

Notice that first order condition on $e_{t+1}$ is exactly the same we obtain in section 2 (equation 9). Therefore, as in the basic model, decision on $e_{t+1}$ does not depend on $s_t$ and $e_t$ and so, all results we obtained is section 2 hold.

Combining equations (37) and (38) we obtain

$$e_t = \frac{1 - \tau}{\tau \gamma (1 + \beta)} (1 - p(1 + \gamma)) \hspace{1cm} (40)$$

$$s_t = \frac{\beta}{1 + \beta} (1 - \tau) w_t \hspace{1cm} (41)$$

Likewise capital income tax evasion, $e_{t+1}$, an interior solution for $e_t$ requires the evasion gamble to be better than fair, that is, $1 - p(1 + \gamma) > 0$. Analogously, the fraction of labor income tax evasion decreases as tax enforcement becomes stricter, i.e. $de^*_t/dp < 0$ and $de^*_t/d\gamma < 0$ and, a marginal increase in the tax rate reduces it, $de^*_t/d\tau < 0$. However, whereas that $e_{t+1}$ is decreas-
ing in the interest rate (capital income), $e_t$ remains stable to changes in $w_t$, i.e., $de_t^*/dw_t = 0$. The intuition is as follows: first, there is no substitution effect as the penalty is assumed to be levied on the share of income evaded, implying that marginal gains and marginal costs from evasion exactly offset each other; second, a higher wage increases taxpayers’ income and makes them more willing to evade. However, given that the income elasticity of the concealed income is unitary, the percentage of evaded taxes remains unchanged.

The definition of an intertemporal equilibrium is analogous to the basic model: a sequence of quantities and prices such that the government provides the public good and balances its budget, individuals maximize utility, firms maximize profits, factor markets are competitive, and all markets clear. Notice that level of public revenues increases since there is tax collection on labor income. The amount of public good provided the government is now defined by

$$g_{t+1} = \tau(1 - e_{t+1}(1 - p(1 + \gamma))(r_{t+1}s_t + w_{t+1}))$$ (42)

The dynamics of the economy is now characterized by the equation

$$k_{t+1} = \frac{\beta}{1 + \beta}(1 - \tau)(1 - \alpha)Ak^g_t$$ (43)

which monotonically converges towards a unique steady state $k^\ast$. We observe that the dynamics of the model is exactly the same like the basic model is section 2. The unique the difference is that now the per capita capital level is reduced in the fraction $(1 - \tau)$ in both throughout the capital accumulation process and the steady state. Clearly, the share of evasion on labor income
remains unchanged since increases in per capita capital and so in wages do not affect it, i.e., \( \frac{de_t}{dk_t} = 0 \), whereas the share of evasion of capital income increases when the economy accumulates capital, i.e. \( \frac{de_{t+1}}{dk_t} > 0 \). Altogether, we conclude the total evasion increases throughout the development process and, hence, that there exists a positive relationship between economic growth and tax evasion. Thus, we obtain the same counterfactual result as in section 2 which forces to introduce morality to account for the existent empirical evidence.

### 4.2 Labor income tax evasion with morality

As in section 3, tax morale is formalized as an endogenous social norm for tax compliance: the more agents evade taxes, the weaker the social norm and so, the easier for taxpayer to justify her own evasion. The unique difference with respect to section 3 is that individuals’ consider moral costs not only for capital income tax evasion but for labor income tax evasion as well. Thus, the life-cycle utility function of an individual \( i \) born in period \( t \) with a degree of norm internalization \( \theta_i \leq 0 \) can be represented by the following equation:

\[
U_i(c_t, d^u_{t+1}, d^d_{t+1}, e_{t+1}) = E[U(c^u_t, c^d_t, d^u_{t+1}, d^d_{t+1})] - (e_t + e_{t+1})(\theta_i + \mu(1-n_t)) \tag{44}
\]

where the expression \( (e_t + e_{t+1})(\theta_i + \mu(1-n_t)) \) captures the moral costs of capital and labor income tax evasion and \( \theta \) has distribution function \( F(\theta_i) \) and support \( [0, \tilde{\theta}] \).

First order conditions (FOCs) with respect to \( s_t \), \( e_t \) and \( e_{t+1} \) for an inte-
rior solution are:

\[
E[U(.)]' \equiv - \left[(1 - p)u'(c_t^u) + pu'(c_t^d)\right] + \frac{\beta}{s_t} = 0 \tag{45}
\]

\[
E[U(.)]' \equiv \tau w_t \left[(1 - p)u'(c_t^p) - \gamma pu'(c_t^d)\right] = \theta_i + \mu(1 - n_t) \tag{46}
\]

\[
E[U(.)]' \equiv \beta \tau r_{t+1} \left[(1 - p)u'(R_{t+1}^u) - \gamma pu'(R_{t+1}^d)\right] = \theta_i + \mu(1 - n_t) \tag{47}
\]

Notice that the FOC with respect to \( s_t \) is not affected by morality. The FOC with respect to \( e_{t+1} \) is exactly the same as the one we get in basic model with morality in section 3 and so, all features we derived in proposition 1 remain unchanged. Moreover, the first order condition with respect to \( e_t \) has the same structure that the one of \( e_{t+1} \). In fact, an interior solution, \( e_t^* > 0 \), requires

\[
z(w_t) \equiv E[U(e_t = 0)]' = \frac{(1 - p)(1 + \gamma)\tau(1 + \beta)}{(1 - \tau)} > 0 \tag{48}
\]

that is, we need to guarantee that \( 1 - p(1 + \gamma) > 0 \) which is the same assumption we need for having \( e_{t+1}^* > 0 \). However, while \( z(r_{t+1}) \) (eq. 19) depends on \( r_{t+1} \) and so, the per capita capital of the economy, \( z(w_t) \) is independent of \( k_t \) since it is constant along time. Taxpayers with \( \theta_i + \mu(1 - n_t) > z(w_t) \) do not conceal any income. This implies the threshold

\[
\hat{\theta}^w(n_t) \equiv z(w_t) - \mu(1 - n_t) \tag{49}
\]

which allows us to characterize the optimal individual evasion behavior \( e_t^{*,i} \).
for a given level of $n_t$:

$$e_{t}^{*i} = \begin{cases} 
0 & \text{for } \theta_i \geq \hat{\theta}^w(n_t) \\
e_{t}^{*i} & \text{for } \theta_i < \hat{\theta}^w(n_t)
\end{cases}$$

(50)

The effects of a change in $w_t$ and $\tau$ are described by the following proposition:

**Proposition 4** $\partial e_{t}^{*i} / \partial w_t = 0$ and there exists some $\hat{\theta}^w(n_t, w_t) > 0$ such that $\partial e_{t}^{*i} / \partial \tau \leq 0$ if $\theta_i < \hat{\theta}^w(n_t, w_t)$ and $\partial e_{t}^{*i} / \partial \tau > 0$ if $\theta_i > \hat{\theta}^w(n_t, w_t)$ for all $k_t$ and $n_t$.

**Proof:** See Appendix.

The effect of a change in the tax rate on labor income tax evasion is the same that the one we obtained in capital income tax evasion (section 3): there is a substitution effect that encourages to increase evasion and there is a negative income effect that discourages it. We show that, for those with $\theta_i < \hat{\theta}^w$, the negative income effect dominates and tax evasion reduces as taxes rises, whereas for those with $\theta_i > \hat{\theta}^w$ the substitution effect prevails and tax evasion increases. An increase in the labor income produces similar effects to those we describe in subsection 4.1 in the model without morality. First, it increases the marginal benefits as well as the marginal costs of evasion. However, given that moral costs of evasion are assumed to depend on the share of evaded income, the increase in marginal benefits is above the increase in marginal costs, producing a substitution effect that encourages taxpayers to increase tax evasion. Second, there is a positive income effect which increases the total amount of evaded taxes. However, given that labor
income tax evasion has an income elasticity of demand less than one, the percentage of evaded taxes remains stable.

Finally, equivalently to $\hat{\theta}(n_t, r_{t+1})$, differentiation of (49) yields $\frac{\partial \hat{\theta}(n_t)}{\partial \tau} > 0$. This implies that emergence of new labor income evaders in rises the share of aggregate evasion. Only those evaders with a sufficiently small degree of norm internalization reduce their evasion of labor income whereas the more honest individuals evade more in the first period the higher the tax rate.

The definition of an intertemporal equilibrium is analogous to the basic model with morality. The capital stock in period $t + 1$ is determined by the size of the aggregated savings in the proceeding period, i.e.,

$$ k_{t+1} = \int_0^{\bar{\theta}} s^*_t f(\theta_i) d\theta_i = \frac{\beta}{1+\beta} (1-r)(1-\alpha)w_t + \int_0^{\hat{\theta}(n_t)} s^*_t f(\theta_i) d\theta_i $$

(51)

with $f(\theta_i) = F' > 0$. We observe that the dynamics of the model differs with respect to model (without morality) in the previous section. Whereas in previous section all taxpayers evade taxes, in this model individuals with relatively high levels of tax morale, $\bar{\theta} \geq \theta_i \geq \hat{\theta}^w$, do not evade taxes. The fact that the level of tax morale depends on the total numbers of evaders in society, $n_t$, implies that aggregate savings depends on it as well. An increase in $n_t$ reduces the marginal cost of labor income tax evasion, encouraging a rise on it. When evasion increases, marginal cost of savings changes but the sign is not clear since increases the income when evasion is undetected and decreases the income when evasion is undetected. It is easy to prove that for those with $\theta_i < \hat{\theta}^w$, the rise on evasion is small, implying an increase in the marginal cost of savings and thus, a reduction in savings, whereas for those with $\theta_i > \hat{\theta}^w$ the opposite is true. Consequently, the relationship between the aggregate savings and the shares of evaders in the economy will depend on the on the functional form of $F$. Equation (58) makes explicit this result.
to prove\textsuperscript{26} that
\[
\frac{\partial s^{s,i}_t}{\partial w_t} = s^{s,i}_t \frac{w_t}{w_t}
\] (52)
which allow us rewrite savings as \(s^{s,i}_t = s^{s,i}(n_t)w_t\). Then, using equation (2), the dynamic of the capital (equation 51) results as

\[
k_{t+1} = s(n_t)(1 - \alpha)Ak_t^\alpha
\] (53)

where \(s(n_t) = \left[\frac{\beta}{1 + p}(1 - \tau)(1 - \alpha) + \int_0^{\hat{\theta}(n_t)} s^{s,i}(n_t)f(\theta_i)d\theta_i\right].\)

Individuals have the possibility to evade taxes with respect to both their labor income and/or to their capital income. However, for obtaining the number of evaders for period \(t + 1\), it is only relevant to compute the total number of individuals that have decide to evade at time \(t\) whatever the type of income (labor or capital income). It is straight forward to show that \(\hat{\theta}(n_t) > \hat{\theta}(n_t, r_{t+1})\) which implies that in this model all taxpayers that decide to conceal capital income are also concealing labor income but the opposite is not true.\textsuperscript{27} Consequently, the dynamic of the number of evaders evolves according to the following equation:

\[
n_{t+1} = F\left(\hat{\theta}(n_t)\right)
\] (54)

\textsuperscript{26}See Appendix for the proof.

\textsuperscript{27}The model predicts that the number of labor income evaders is higher that the capital income evaders. Apparently, this result is running against the empirical evidence (see section 2) which suggests that the size of capital income tax evasion is larger than the of labor income tax evasion. However, in our model we are assuming that penalties and probabilities of detection are the same in both cases and this is not true. For example, (Poterba, 1987, Sandmo, 2012) argue that higher opportunities to hide capital income are due to the relatively low probability of detection compared with other income sources. Thus, increasing sufficiently the parameter \(p\) for labor income would make the model consistent with the empirical evidence.
where the share of evaders in period \( t + 1 \) depends positively on the share of evaders in period \( t \).

The aggregate share of capital income evasion in period \( t + 1 \), \( \tilde{e}_{t+1} \), is given by equation (55) and, analogously, the aggregate share of labor income evasion in period \( t \), is given by

\[
\tilde{e}_t = \int_0^\theta e^{*,i}_t f(\theta_i) d\theta_i = \int_0^{\delta(n_t, r_{t+1})} e^{*,i}_t f(\theta_i) d\theta_i.
\]

The derivative of \( \tilde{e}_{t+1} \) with respect to \( k_t \) is given by equation (26) and the derivative of \( \tilde{e}_t \) with respect to \( k_t \) results being

\[
\frac{\partial \tilde{e}_t}{\partial k_t} = \left[ \int_0^{\delta(n_t)} \frac{\partial e^{*,i}_t}{\partial w_t} f(\theta_i) d\theta_i \right] \frac{\partial w_t}{\partial k_t} = 0
\]

This result indicates that the share of evaded labor income does not depend on the amount of capital in the economy. The reason is that that share of concealed labor income independent on the size of labor income\(^{28}\), i.e., \( \frac{\partial e^{*,i}_t}{\partial w_t} = 0 \). Consequently, the relationship between the aggregate share of evasion (labor and capital income evasion) and the level of capital in the economy is determined exclusively by the relationship between the capital income tax evasion and the level of capital (equation 26) that we analyzed in section 3. We then can conclude that the extended framework with labor income tax evasion leaves unaffected the relation between capital accumulation and tax evasion that we find in the model with only capital income tax evasion.

As in section 3, a sufficient condition for an interior steady state \( n_* \in

\(^{28}\)See proposition 4.
(0, 1) is given by
\[ \max \{ \bar{\theta}, \mu \} > \hat{\theta}^{\mu} > \min \{ \bar{\theta}, \mu \} \]  
(57)

Next, we obtain \( k_* = (s(n_*)(1 - \alpha)A)^{1/(1 - \alpha)} \) as the steady state solution of equation (53). Equivalently to section 3, a sufficient condition for a stable steady state \((n_*, k_*)\) of the dynamic system defined by equations (53) and (54) is
\[ 1 > \alpha \mu F'(\hat{\theta}(n_*)) + \left| \frac{s'(n_*) k_*}{s(n_*)} \right| \]  
(58)

The existence of multiple steady states clearly depends on the functional form of \( F \). In section 3 we prove that for an uniform distribution there is a unique steady state \((n_*, k_*)\) and the stability condition boils down when \( \bar{\theta} \) is sufficiently large. For equivalent assumptions, this extended version of the model would feature similar properties and consequently, main results would remain. In particular, the capital accumulation process would be characterized by the declining of the share of aggregated evaded taxes, i.e. \( \partial \hat{e}_{t+1}/\partial k_t < 0 \) and a negative relationship between economic growth and tax evasion is established. In other words, the model would predict that economies with low levels of per capita capital and an initial stage of economic development are characterized by high levels of tax evasion, whereas economies with high levels of per capita capital are characterized by the opposite.
5 Conclusions

This paper integrates non-pecuniary costs of evasion into a dynamic overlapping generations model of (capital income) tax evasion to explain the empirical observation that evasion is a more severe phenomenon among developing countries as compared to developed countries. It is shown that morale concerns may not only explain why some taxpayers never evade even if the gamble is better than fair, and how a higher tax rate can increase evasion but also that the share of evaded taxes over GDP decreases with the stage of economic development as per capita income increases. By contrast, tax morale increases with per capita income as the number of evaders in society declines. Moreover, an increase in the tax rate increases aggregate evasion as well as the number of evaders in the economy when taxpayers decisions are interdependent. Hence, an important insight emerging from our analysis is that policies aimed at deterring tax evasion may have long-lasting and amplifying effects by altering both economic incentives and the formation of social norms throughout an economy’s transitional path and in steady state.

Our findings complement the existing literature on evasion in important ways by demonstrating how the size of tax evasion evolves along the (transitional path) [economic growth process] of an economy, whereas previous studies either consider a static environment without production (see e.g. Gordon (1989) or Kim (2003)) or focus on the relationship between tax evasion and economic growth on a balanced growth path (see e.g. Dzhumashev and Gahramanov (2011) or Levaggi and Menoncin (2012)). Furthermore, the present paper documents a positive relationship between per capita income
and tax morale consistent with recent empirical evidence (see Torgler and Schneider (2007) and Bethencourt (2013)).

Our theoretical model thus emphasizes the role of social norms in determining the dynamics of tax evasion. A complementary explanation of these dynamics, however, is related to the quality of institutions. Indeed, as has been shown, e.g., by Torgler and Schneider (2009), both a higher tax morale and a higher quality of institutions imply a smaller size of the shadow economy. Given that the institutions of advanced economies have on average a higher quality and these countries are thus better at collecting taxes (e.g. due to better information available to tax authorities as has been emphasized by Kleven et al. (2011)), tax moral and the quality of institutions tend to reinforce each other if a share of tax collections is devoted to quality improvements. To disentangle the relative importance of both mechanisms in shaping the relationship between tax evasion and economic development, however, is ultimately a quantitative question.

Our analysis could be extended in various ways. First, incorporating productive public spending (and endogenous growth) or redistributive transfers would be valuable. This complicates the analysis but should not change our results qualitatively. For example, if tax revenue is redistributed lump sum to the young households, an increase in the share of evaders in the economy exerts a direct negative effect on per capita income as individuals save less due to a smaller transfer. Furthermore, if public spending increases productivity and thus wages, the level of savings would increase, which in turn accelerates the process of capital accumulation and therefore the decline in the rate of return, implying a lower share of evaded
taxes and a lower share of evaders in the economy. In such a framework, however, poverty traps may arise if the level of tax evasion is sufficiently large and tax collections are thus too low to generate productive spending and the accompanying increases in productivity.

(Second, even though the main focus of our analysis is on capital income tax evasion, one could introduce evasion activities from alternative income sources, as e.g. wage income. The possibility to evade wage income taxes would increase the potential to save and therefore foster capital accumulation. This, in turn, would imply a sharper decline of the interest rate throughout the economy’s transitional path and thus lower the incentive to evade taxes on capital income. Quantitatively, however, the aggregate amount of tax evasion might then decrease more moderately since tax evasion on wages should increase with capital accumulation.\footnote{We have experimented with simulations for such an extended model. The results of these simulations are available upon request. They suggest that our conjecture is warranted.} (Look at above, this is not "exactly" true ... ;)) (Look at above, this is not "exactly" true ... ;))

Finally, more elaborated numerical simulations may be used to address the welfare implications of capital income tax evasion in a dynamic context (see Boadway and Keen (1998) for a related static analysis).

References


Appendix

Proof of proposition 1:

In order to prove proposition 1 we need to obtain the derivatives of $e_t^{i,*}$ with respect to $r_{t+1}$ and $\tau$. The derivation of these expressions relies on the explicit solution of equation (18). More precisely, solving for $e_t^{i,*}$ gives

$$e_t^{i,*} = \beta r_{t+1} \gamma \tau - R_{t+1}(\gamma - 1)(\theta_i + \mu(1 - n_t)) - \tilde{m}$$

(A.1)

$^{30}$Note that there are two solutions. However, one of them can be excluded from the analysis due to economic reasoning as such a solution is positive for all values of $\theta_i$. 

50
where $R_{t+1} = 1 - \delta + (1 - \tau)r_{t+1}$ and

$$
\tilde{m} = \sqrt{\left(\beta \tau \gamma r_{t+1} - (\theta_i + \mu(1 - n_t))(\gamma + 1)R_{t+1}\right)^2 + 4p \beta \tau \gamma r_{t+1}(\theta_i + \mu(1 - n_t))(\gamma + 1)R_{t+1}}.
$$

(A.2)

Derivation of the above expression gives:

$$
\frac{\partial e_{t+1}^i}{\partial r_{t+1}} = (1 - \delta)(\gamma - 1)\tilde{m} + (\gamma + 1)(R_{t+1}(\gamma + 1)(\theta_i + \mu(1 - n_t)) - \beta r_{t+1}\gamma \tau(1 - 2p)))
$$

$$
2r_{t+1}^2\gamma \tau \tilde{m}
$$

(A.3)

and

$$
\frac{\partial e_{t+1}^i}{\partial \tau} = (1 - \delta + r_{t+1})(\gamma - 1)\tilde{m} - (\gamma + 1)(R_{t+1}(\gamma + 1)(\theta_i + \mu(1 - n_t)) + \beta r_{t+1}\gamma \tau(1 - 2p)))
$$

$$
2r_{t+1}^2\gamma \tau \tilde{m}
$$

(A.4)

Solving the inequalities $\frac{\partial e_{t+1}^i}{\partial r_{t+1}} \geq 0$ and $\frac{\partial e_{t+1}^i}{\partial \tau} \geq 0$ for $\theta_i$ yields:

$$
\frac{\partial e_{t+1}^i}{\partial \tau} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} 
\end{cases}
$$

(A.5)

and, similarly,

$$
\frac{\partial e_{t+1}^i}{\partial r_{t+1}} \begin{cases} 
> 0 & \text{if } \theta_i > \tilde{\theta} \\
\leq 0 & \text{if } \theta_i \leq \tilde{\theta} 
\end{cases}
$$

(A.6)

with

$$
\tilde{\theta} = \frac{\beta \tau r_{t+1} \sqrt{p(1-p)(\gamma - 1) - \sqrt{p(1-2p)}}}{(1 + \gamma)R_{t+1}} - \mu(1 - n_t)
$$

(A.7)

Finally, a comparison of $\tilde{\theta}$ and $\hat{\theta}$ (see equation (20) yields

$$
\tilde{\theta} < \hat{\theta} \iff \gamma < \frac{1 - p}{p}.
$$

(A.8)
This condition is assumed to hold throughout the paper as it ensures the existence of an interior solution (see also equation (19)).

**Proof of equation (29):**

A steady state \((k_*, n_*)\) of the dynamic system

\[
k_{t+1} = \Phi(k_t) \tag{A.9}
\]
\[
n_{t+1} = \Psi(n_t, k_t) \tag{A.10}
\]

is locally stable if the following conditions are met (see de la Croix and Michel (2002))

\[
|1 + D| > |T| \quad \text{and} \quad |D| < 1 \tag{A.11}
\]

where \(T = \Psi_{k_1}(n_*, k_*) = F'(n_*, k_*) \frac{\partial \theta}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial k_t}\) is the trace of the Jacobian matrix \(G\) derived from a first order Taylor expansion of the dynamic system around a steady state, i.e.

\[
G = \begin{pmatrix}
0 & \Phi_{k_1}(k_*) \\
\Psi_{r_1}(n_*, k_*) & \Psi_{k_1}(n_*, k_*)
\end{pmatrix} \tag{A.12}
\]

and \(D = -\alpha \frac{\beta}{1 + \beta}(1 - \alpha)Ak_*^{-1}F'(n_*, k_*)\mu\). The condition \(|D| < 1\) is equivalent to

\[
\frac{1}{F'(n_*, k_*)} > \frac{\beta}{1 + \beta} \alpha(1 - \alpha)Ak_*^{-1} \mu. \tag{A.13}
\]
Similarly, the condition $|1+D| > |T|$ is equivalent to (note that $1+D > 0$ since $|D| < 1$)

$$\frac{1}{F'(n_+,k_+)} > \beta \alpha (1 - \alpha) A k_*^{a-1} \left( \frac{\mu}{1 + \beta} + \frac{\tau (1 - \delta)(1 - p(1 + \gamma))}{k_*(1 - \delta + (1 - \tau) r_*)^2} \right). \quad (A.14)$$

This proves equation (29).

**Proof of propositions 2 and 3:**

In order to prove propositions 2 and 3 we need to derive the derivatives of $\tilde{e}_{t+1}$ with respect to $r_{t+1}$ and $\tau$. As in the proof of proposition 1, the derivation of these expressions relies on the explicit solution of $e_{t+1}^\star$, see equation (A.1). The derivative of $e_{t+1}^\star$ with respect to $r_{t+1}$ and $\tau$ are given by equations (A.3) and (A.4). Integrating these equations over the relevant range $([0, \hat{\theta}])$ and assuming an uniform distribution yields:

$$\frac{\partial \tilde{e}_{t+1}}{\partial r_{t+1}} = \frac{(1 - \delta) (\beta r_{t+1} \gamma \tau - R_{t+1}(\gamma - 1)\mu(1 - n_t) - \tilde{m})}{2R_{t+1} r_{t+1}^2 \gamma \tau} \quad (A.15)$$

and

$$\frac{\partial \tilde{e}_{t+1}}{\partial \tau} = \frac{(1 - \delta + r_{t+1}) (\beta r_{t+1} \gamma \tau - R_{t+1}(\gamma - 1)\mu(1 - n_t) - \tilde{m})}{2R_{t+1} r_{t+1} \gamma \tau^2} \quad (A.16)$$

The sign of these derivatives is determined by the sign of the expression in curly brackets. The proof of propositions 2 and 3 follow immediately by noting that $dr_{t+1}/dk_t < 0$ and by recalling equation (33).
Proof of proposition 4:

In order to prove proposition 4 we need to obtain the derivatives of $e_t^{i,*}$ with respect to $r_{t+1}$ and $\tau$. The derivation of these expressions relies on the solution of the 2 equation system defined by equations (45) and (46). Given that we are not able to obtain a closed solution of it, we redefine it as a 2 equation system of implicit equations:

\[ F = \left[ (1 - p)u'(c_t^u) + pu'(c_t^d) \right] - \frac{\beta}{s_t} \tag{A.17} \]
\[ G = \tau w_t \left[ (1 - p)u'(c_t^u) - \gamma pu'(c_t^d) \right] - (\theta_i + \mu(1 - n_t)) \tag{A.18} \]

We then obtain the Jacobian matrix of the system, that is,

\[ \frac{\partial(F,G)}{\partial(e_t,s_t)} = \begin{pmatrix} F_s' & F_e' \\ G_s' & G_e' \end{pmatrix} \tag{A.19} \]

\[ F_s' = \frac{1 - p}{(c_t^u)^2} + \frac{p}{(c_t^d)^2} + \frac{\beta}{s_t^2} > 0 \tag{A.20} \]
\[ F_e' = -\frac{1 - p}{(c_t^u)^2} + \frac{p\gamma}{(c_t^d)^2} \left(\tau w_t\right) \tag{A.21} \]
\[ G_s' = -F_e' \tag{A.22} \]
\[ G_e' = -\frac{1 - p}{(c_t^u)^2} + \frac{p\gamma^2}{(c_t^d)^2} \left(\tau w_t\right)^2 < 0 \tag{A.23} \]

It is straightforward to see that the sign of the determinant of the Jacobian matrix is negative, i.e., $|J| = F_s'G_e' - F_e'G_s' < 0$, since $|F_s'G_e'| > |F_e'G_s'|$. We then
apply the Cramer rule for obtaining the derivatives:

\[
\frac{\partial e_i^*}{\partial w_t} = \left| \begin{array}{cc}
F'_s & -F'_{w_t} \\
G'_s & -G'_{w_t}
\end{array} \right| \frac{F'_w G'_s - F'_s G'_{w_t}}{|J|} = \frac{F'_w G'_s - F'_s G'_{w_t}}{|J|} \tag{A.24}
\]

\[
F'_{w_t} = -\left[\frac{(1-p)(1-\tau + \tau e_t)}{(e_t^u)^2} + \frac{p(1-\gamma \tau e_t)}{(e_t^d)^2}\right] < 0 \tag{A.25}
\]

\[
G'_{w_t} = -\left(\frac{s_t}{w_t}\right) G'_s \tag{A.26}
\]

since \( F'_{w_t} G'_s - F'_s G'_{w_t} = 0 \) then \( \frac{\partial e_i^*}{\partial w_t} = 0 \). Similarly,

\[
\frac{\partial e_i^*}{\partial \tau} = \left| \begin{array}{cc}
F'_s & -F'_{\tau} \\
G'_s & -G'_{\tau}
\end{array} \right| \frac{F'_\tau G'_s - F'_s G'_{\tau}}{|J|} = \frac{F'_\tau G'_s - F'_s G'_{\tau}}{|J|} \tag{A.27}
\]

\[
F'_{\tau} = \left[\frac{(1-p)(1-e_t)}{(e_t^u)^2} + \frac{p(1+\gamma e_t)}{(e_t^d)^2}\right] w_t > 0 \tag{A.28}
\]

\[
G'_{\tau} = \left(\frac{w_t - st}{\tau}\right) G'_s \tag{A.29}
\]

since \( |F'_{\tau} G'_s| \leq |F'_s G'_t| \) the sign of the numerator of expression (A.27) depends on the sign of \( G'_{\tau} \), that is, the sign of \( G'_s \). Therefore,

\[
\frac{\partial e_i^*}{\partial \tau} \begin{cases} 
> 0 & \text{if } G'_s > 0 \Rightarrow \theta_i > \tilde{\theta}^w \\
\leq 0 & \text{if } G'_s \leq 0 \Rightarrow \theta_i \leq \tilde{\theta}
\end{cases} \tag{A.30}
\]
with
\[ \bar{\theta}^w = \frac{1 - p(1 + \gamma)}{\gamma} \tau w_t \] (A.31)

**Proof of equation (52):**

From the 2 equation system of implicit equations defined in the proof of proposition 4 and using the Cramer rule we obtain:

\[ \frac{\partial s^i,\ast_t}{\partial w_t} = \frac{\begin{vmatrix} -F'_{w_t} & F'_e \\ -G'_{w_t} & G'_e \end{vmatrix}}{|J|} = \frac{G'_{w_t}F'_e - G'_e F'_w}{F'_sG'_e - F'_eG'_s} \] (A.32)

Again, using definitions of partial derivatives of \( F \) and \( G \) defined in the proof of proposition 4, we rewrite the above expression as

\[ \frac{\partial s^i,\ast_t}{\partial w_t} = \left( \frac{\partial s^i,\ast_t}{\partial w_t} \right) = \frac{G'_{w_t}F'_e - G'_e F'_w}{F'_sG'_e - (G'_s)^2} \] (A.33)

Since \( \frac{\partial s^i,\ast_t}{\partial w_t} = F'_s \) then \( \frac{\partial s^i,\ast_t}{\partial w_t} = \frac{G'_{w_t}F'_e - (G'_s)^2}{F'_sG'_e - (G'_s)^2} = 1 \) and so \( \frac{\partial s^i,\ast_t}{\partial w_t} = \frac{s^i,\ast_t}{w_t} \).

**An alternative framework: Tax evasion in levels**

The aim of this section is to illustrate that our main results do not depend on the assumption that moral costs are linear in the share of evaded income. To do so, we consider the following alternative specification of the individ-
ual’s maximization problem [[of section 3]] where moral costs are linear in the amount of evaded income (as in Gordon (1989)):

\[
U_i = \ln(c_t) + \beta [p \ln(d_{t+1}^d) + (1 - p) \ln(d_{t+1}^u)] - \tilde{e}_{t+1}(\theta_i + \mu(1 - n_t))
\]  
(A.34)

s.t. \[ w_t = c_t + s_t \]  
(A.35)

\[
d_{t+1}^u = (1 - \delta + r_t(1 - \tau))s_t + \tau \tilde{e}_{t+1}
\]  
(A.36)

\[
d_{t+1}^d = (1 - \delta + r_t(1 - \tau))s_t - \gamma \tau \tilde{e}_{t+1}
\]  
(A.37)

where \( \tilde{e}_{t+1} \) is the individual amount of evaded income. Using equations (2) and (3), the first order conditions with respect to \( s_t \) and \( \tilde{e}_{t+1} \) can be written as

\[
\frac{\partial U_i}{\partial s_t} : 0 = -\frac{1}{(1 - \alpha)AK_t^{a-1} - s_t} + \frac{\beta p(1 - \delta + (1 - \tau)AK_t^{a-1})}{(1 - \delta + (1 - \tau)AK_t^{a-1})s_t - \tau \gamma \tilde{e}_{t+1}} + \frac{\beta(1 - p)(1 - \delta + (1 - \tau)AK_t^{a-1})}{(1 - \delta + (1 - \tau)AK_t^{a-1})s_t + \tau \tilde{e}_{t+1}}
\]  
(A.38)

and

\[
\frac{\partial U_i}{\partial \tilde{e}_{t+1}} : 0 = -\theta_i - \mu(1 - n_t) - \frac{\beta p \gamma}{(1 - \delta + (1 - \tau)AK_t^{a-1})s_t - \tau \gamma \tilde{e}_{t+1}} + \frac{\beta(1 - p)\tau}{(1 - \delta + (1 - \tau)AK_t^{a-1})s_t + \tau \tilde{e}_{t+1}}
\]  
(A.39)

As in section 3.2, the model is solved using Monte Carlo simulations with \( N = 100000 \) draws from a given distribution function \( F(.) \) (either the uniform, the log-normal or the Weibull distribution). The numerical solutions of
equations (A.38) and (A.39) for individual choices \( s_t \) and \( \tilde{e}_{t+1} \) and each of the corresponding draws are then averaged which in turn yields the aggregate stock of capital and the aggregate amount of evaded income in each period \( t \), given the capital market clearing condition. The parameters are basically the same as in section 3.2. The only difference is that we set \( \mu = 0.01 \) in order to obtain plausible solutions for the size of evaded income and the number of evaders in society.\(^{31}\) The threshold level \( \hat{\theta} \) which, together with the underlying distribution function, determines the evolution of the number of evaders in society (see equation (24)) is obtained by taking the maximum value of \( \theta_i \) for which \( \tilde{e}_{t+1}^{i,j} \) is larger or equal to zero.

Similar to the presentation of the results in the basic model, table 2 summarizes the share of evaders and the share of evaded income in steady state for varying levels of \( \tau \) and different functional forms of \( F(\cdot) \). Clearly, an increase in the capital income tax rate by ten percentage points raises the share of evaders in society as well as the share of taxes evaded. This holds independent of the underlying distribution function. For example, with a Weibull distribution (WBL(1,1.5)), increasing \( \tau \) from 0.2 to 0.3 raises the share of evaders by 4.8 percentage points from 3.6% to 8.4% while the aggregate share of evaded income increases by 3.3 percentage points from 1.7% to 4.97%. Figure 4 illustrates the dynamics of the share of evaded income,\(^{31}\) Note that in the baseline model the moral costs of evasion are naturally smaller as they are assumed to depend on the share of evaded income rather than on the absolute amount. In order to reproduce a similar economy with similar levels of morality, however, the size of \( \mu \) is reduced accordingly.
i.e. \( \bar{e}_{t+1}/(r_{t+1}k_{t+1}) \), and the numbers of evaders in society \( n_{t+1} \) for a uniform distribution while figures 5 and 6 present the results with a log-normal and a Weibull distribution, respectively. The lowest transitional path in each figure corresponds to the case \( \tau = 0.2 \) whereas the highest one corresponds to \( \tau = 0.4 \). Clearly, the qualitative results are the same as in section 3. Hence, our simulations suggest that the main findings are robust against assuming moral costs to be linear in the share rather than the level of evaded income.

[Insert figures 4-6 around here.]

### Tables and Figures

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<td>( n_* )</td>
<td>0.0472</td>
<td>0.1007</td>
<td>0.1812</td>
</tr>
<tr>
<td></td>
<td>( \bar{e}_* )</td>
<td>0.0372</td>
<td>0.0653</td>
<td>0.0797</td>
</tr>
</tbody>
</table>

Table 1: Predicted steady state shares of evaded income and of evaders in society for alternative distributions of \( F \) and varying levels of the capital income tax (moral costs linear in the share of evaded income). Uniform distribution on \([0,1]\); log-normal distribution (with parameters LN(0,1)); Weibull distribution (with parameters WBL(1,1.5)).
Table 2: Predicted steady state shares of evaded income and of evaders in society for alternative distributions of $F$ and varying levels of the capital income tax (moral costs linear in the level of evaded income). Uniform distribution on $[0,1]$; log-normal distribution (with parameters LN(0,1)); Weibull distribution (with parameters WBL(1,1.5)).

<table>
<thead>
<tr>
<th>$F(.)$</th>
<th>$\tau$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform</td>
<td>$n_*$</td>
<td>0.0387</td>
<td>0.0653</td>
<td>0.0997</td>
</tr>
<tr>
<td></td>
<td>$\tilde{e}_*$</td>
<td>0.0340</td>
<td>0.0481</td>
<td>0.0495</td>
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<tr>
<td>log-normal</td>
<td>$n_*$</td>
<td>0.0005</td>
<td>0.0032</td>
<td>0.0105</td>
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<tr>
<td></td>
<td>$\tilde{e}_*$</td>
<td>0.0005</td>
<td>0.0017</td>
<td>0.0036</td>
</tr>
<tr>
<td>Weibull</td>
<td>$n_*$</td>
<td>0.0076</td>
<td>0.0166</td>
<td>0.0309</td>
</tr>
<tr>
<td></td>
<td>$\tilde{e}_*$</td>
<td>0.0058</td>
<td>0.0106</td>
<td>0.0135</td>
</tr>
</tbody>
</table>

Figure 1: Underlying distribution: Uniform (on [0,1]). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$. Stars indicate steady state levels.
Figure 2: Underlying distribution: Log-normal (LN(0,1)). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$. Stars indicate steady state levels.

Figure 3: Underlying distribution: Weibull (WBL(0,1.5)). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$. Stars indicate steady state levels.
Figure 4: Underlying distribution: Uniform (on [0,1]). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$ (moral costs linear in the level of evaded income). Stars indicate steady state levels.

Figure 5: Underlying distribution: Log-normal (LN(0,1)). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$ (moral costs linear in the level of evaded income). Stars indicate steady state levels.
Figure 6: Underlying distribution: Weibull (WBL(0,1.5)). Dynamics of the share of evaded income (left) and the share of evaders in society (right) for different levels of $\tau$ and initial values $k_0 = 1$ and $n_0 = 0.3$ (moral costs linear in the level of evaded income). Stars indicate steady state levels.