A Macroeconomic Theory of Banking Oligopoly

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Abstract

We study the behavior and economic impact of oligopolistic banks in a tractable macro environment with solid micro-foundations for money and banking. Our model has three key features: (i) banks as oligopolists; (ii) liquidity constraint for banks that arises from mismatched timing of payments; and (iii) search frictions in credit, labor and goods markets. Our main findings are: First, it is optimal to have a small, yet greater than one, number of banks. That is, it is welfare-maximizing to have the banking sector as oligopolistic. When the number of banks is low and banks are not liquidity constrained, bank competition improves welfare. But each bank receives a smaller share of the aggregate deposit as the number of banks rises. When the number of banks is so high that banks become liquidity constrained, having more banks leads to lower welfare. This is because bank lending is now limited by the amount of deposits a bank can gather. As banks start to charge a higher loan rate to improve their financial conditions, what follow are reduced wages, discouraged firm entry, lower aggregate output yet higher unemployment, all of which leads to dampened welfare. Second, the interest rate spread also reacts non-monotonically to the number of banks. It first decreases, then increases, and finally decreases again as the number of banks climbs. Third, with entry of banks, there may exist at most three equilibria of the following types: one is stable and Pareto dominates, another is unstable and ranks second in welfare, and the third is stable yet Pareto inferior. Finally, inflation can change the nature of the equilibrium. Low inflation promotes a unique good equilibrium, high inflation cultivates a unique bad equilibrium, but medium inflation can induce all three equilibria of the aforementioned types.

Key Words: Banking; Oligopoly; Interest Rate Spread; Liquidity; Market Frictions.

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1 Introduction

We study the behavior and economic impact of oligopolistic banks in a tractable macro environment with solid micro-foundations for money and banking. Nowadays in many countries, the banking sector is clearly an oligopoly, in the sense that it consists of a few large banks who control a significant proportion of the banking business across the country. Figure 1 provides some cross-country evidence of the oligopolistic structure of the banking sector. The theoretical literature on banking, however, often ignores this fact and treats the banking sector as being composed of either one monopoly bank or a continuum of competitive banks without any market power. Nevertheless, as is well known from the theory of Cournot competition, oligopolistic firms may behave very differently from monopolistic or competitive firms, which then leads to very different economic outcomes. To overcome this gap between theory and evidence, we construct a macroeconomic model of banking oligopoly.

Figure 1 Concentration of the Banking Sector

Data source:

In our dynamic, general equilibrium setting, banks serve as financial intermediaries as they take in demand deposits from households and then lend money to firms to help them start up. Our model has three key features. First, banks are oligopolists, and the market structure of the banking sector is endogenously determined. In particular, individual banks are large in the sense that each bank has a large number of loan officers working
to provide loans. Each loan officer may or may not get to make a loan to a firm within a period, although there is no uncertainty regarding how many new loans are made by an individual bank. Banks compete for shares in the credit market by choosing the measure of loan officers at work, which is essentially choosing the volume of new loans to make every period. Free entry determines the number of banks in equilibrium.

Second, we incorporate a liquidity constraint into banks' decision problems. It is simply that every period only an exogenous part of the loan repayments arrive at the bank before the time when the bank must make payments to new and existing loans that require funding. Demand deposits, as a source of funds other than loan repayments, can be particularly helpful for banks in dealing with such timing mismatch. Nevertheless, the liquidity constraint faced by a bank may bind to the extent that a bank's lending activity is constrained by how much demand deposit it manages to raise from households.

Third, our model has search frictions in credit, labor and goods markets. Credit market frictions naturally give rise to well-defined intensive margin (i.e., the size of a loan) and extensive margin (i.e., the volume of loans issued by a bank) of credit. Goods market frictions, as in Lagos and Wright (2005), provide a rigorous micro-foundation for a medium of exchange, i.e., money. In addition, such a structure is very tractable as it renders the equilibrium money distribution degenerate. This is particularly convenient for us in that we aim to examine the macroeconomic consequences of banking.

Moreover, labor and goods market frictions together help us build a solid microfoundation for banking. Because of labor market frictions, it may take several periods before a firm gets to recruit a worker and start generating revenue. Therefore, lending to firms directly means that households may not get to receive repayments right away. This puts households in a disadvantage because they might not have enough money to spend when they receive a chance to buy in the frictional goods market. In contrast, banks can intermediate between a large set of households and firms. As a result, banks can manage inflows and outflows of funds without uncertainty even though households and firms face idiosyncratic matching risks. Thus, banks can meet withdrawal demands from households who need money for consumption.

With these features, banks in our model interact strategically where they compete against each other in terms of the volume of loans to make. Potentially, each bank has three sources of funds for making loans: repayments from previous loans, demand deposit and the its own savings. In addition to the first, and very natural source, banks generally prefer using demand deposit to their own savings. Maintaining savings is costly due to inflation. Moreover, banks only pay interests on the end-of-period household deposit balances. This means that banks essentially get to use part of the household funds for free because every period a fraction of the households will need to withdraw money (to make payments in the frictional goods market) before the end of the period, and thus before interests apply. For this reason, banks find it more economical to rely heavily on demand deposits to solve the funding-liquidity problem, rather than maintaining their own savings.
We summarize our results into two categories: First, we take the number of banks as exogenous and characterize the equilibrium outcomes. Then, we allow the number of banks to be endogenously determined and obtain policy results.

When the number of banks is exogenous, we find that: Firstly, welfare responds to the number of banks non-monotonically, loosely speaking, with a hump shape. The highest welfare corresponds to a small, yet greater than one, number of banks. In other words, it is optimal to have an oligopolistic banking sector. To see this, note that there are three types of banking equilibria, depending on the number of banks.

(i) The first type of equilibrium corresponds to the case where the number of banks is low and banks are not liquidity constrained. When the number of banks is low, the liquidity constraint for banks does not bind. Banks rely on loan repayments and demand deposits to make new loans. Within this domain, as the number of banks increases, competition among banks intensifies, which leads to more firms getting funded, more workers getting employed, and more output being produced. Thus, welfare increases with the number of banks in the first type of equilibrium.

(ii) The second type of equilibrium corresponds to the case where the number of banks is intermediate and that banks are liquidity constrained but do not yet resort to their own savings as a source of funds. Banks become liquidity constrained when the number of banks is sufficiently high. This is because with more banks in competition, each bank receives a smaller share of the aggregate demand deposit. It is true that increased demand for deposits from banks will stimulate the size of the aggregate deposits supplied by all households. But banks will be more likely to face a binding liquidity constraint with a higher of banks in competition, as long as the drop in deposit size received by individual banks due to a further split of market dominates the growth in the aggregate deposit size.

When the liquidity constraint starts to bind, the amount of credit issued by individual banks is constrained by the amount of demand deposits it attracts from households. In order to overcome the shortage of funds, banks negotiate more aggressively with firms and demand higher loan repayments when issuing new loans. This has a sequence of negative impact on welfare: First, firms cut wages in order to meet higher loan requirements. Second, firm entry is discouraged due to more costly credit. Third, less firm entry leads to fewer firms producing, lower aggregate output and higher unemployment. Therefore, once the liquidity constraint turns binding, welfare strictly decreases with the number of banks within this second class of equilibrium. What is at the core of this result is the following: on one hand, each bank’s incentive to issue loans become stronger as more banks competing against each other. On the other hand, the amount of deposits available per bank shrinks with more banks at play. The two forces together create tension in banks’ liquidity needs. This is a unique result out of studying banking oligopoly. Unlike productive firms in a Cournot model, oligopolistic banks must deal with the problem that how much liquidity it can “generate” for (the producer side of) the economy is limited by how much liquidity it can gather from (the consumer side of) the same economy.

(iii) The third type of equilibrium occurs when the number of banks is so high that
banks start to use their own savings as a source of funds to lend. The economy enters this particular type of equilibrium when there is no more room for banks to raise the loan requirements on firms. In this situation, banks start to save on their own in order to maintain another source of funds, in addition to repayments and deposits. This worsens banks’ financial conditions because of the inflation tax. Thus welfare drops further as the economy turns from the second to the third class of equilibrium. Moreover, within the third class of equilibrium, welfare increases with the number of banks as competition stimulates lending. Nevertheless, the increase in welfare is negligibly small because banks are all stuck in a bad financial situation.

Secondly, the interest rate spread, \( i.e., \) the difference between the loan rate and the deposit rate, also reacts non-monotonically to the number of banks. When the number of banks is low and the economy is in the first class of equilibrium (call it the "good" equilibrium as it is Pareto dominant), the interest rate spread decreases with the number of banks. On one hand, banks lower their loan requirements \( i.e., \) the loan rate on firms due to competition. On the other hand, the deposit rate remains unchanged with the number of banks because ample supply of liquidity (that is, deposits from households). Thus the spread shrinks as the number of banks rises. When the number of banks is sufficiently high and the economy enters the second class of equilibrium (call it the "unstable" equilibrium), binding liquidity constraint prompts banks to raise loan rates aggressively in order to improve their financial conditions. At the same time, banks are also willing to accept a higher deposit rate in order to maintain their share of the deposit market. Nevertheless, the rise in the loan rate dominates that in the deposit rate, and thus causes the spread to widen. Finally, when the number of banks is high enough to put the economy in the third class of equilibrium (call it the "bad" equilibrium), banks start to maintain savings of their own. Similar to the case with the good equilibrium, intensified competition among banks will induce banks to demand a lower loan rate while the deposit rate remaining the same, which again makes the spread shrink. But overall, the spread remains higher than that in a good equilibrium because of the worsening financial conditions of banks.

When the number of banks is endogenous, we also have two main results:

First, there may exist a unique "good" equilibrium, a unique "bad" equilibrium, or all three types of equilibria, corresponding to the same set of parameters. Both the "good" and "bad" equilibria are stable because in such an equilibrium the individual bank’s profit curve crosses the zero line at a negative slope. But the second class of equilibria are unstable.

Second, inflation worsens banks’ liquidity conditions and thus decreases welfare. Moreover, inflation can change the nature of the equilibrium. The economy tends to be put in a unique good equilibrium by low inflation, a unique bad equilibrium by high inflation, and in all three possible equilibria by intermediate levels of inflation.

Our theoretical framework is partly built upon Wasmer and Weil (2004) (henceforth WW) and Berentsen, Menzio and Wright (2011) (henceforth BMW). In particular, WW has search frictions in the credit and labor markets, while BMW has search frictions in the
labor and goods markets (and do not touch on the credit market). Although WW focuses on how credit market conditions affect labor market outcomes, there is an important piece missing in the model in that the source of the amount to lend by creditors is unaccounted for. In contrast, we specifically fill in this missing piece of the puzzle by modeling a full-fledged banking sector who gather funds from households before lending to firms. Moreover, this banking sector is the main focus of our paper as we aim to study how decisions of oligopolistic banks influence the entire economy.

Our way of modeling the banking sector is novel, in the sense that (i) we model banks as oligopolists in a tractable macro environment with solid micro-foundation for money and banking. With these features, we bring our unique contribution to the vast literature on banking (see Gorton and Winton [2003] and Freixas and Rochet [2008] for surveys). The theoretical banking literature mainly consists of microeconomic models of banking, but macroeconomic structures of banking have also started to catch up in recent years especially after the wake of the 2008 Financial Crisis. To name a few, Gertler and Kiyotaki (2010, 2015) and Gertler, Kiyotaki and Prestipino (2016), etc. These are macro models of banking with a particular focus on potential crises in the spirit of Diamond and Dybvig (1983). In contrast, our current paper focuses on the long-run effect of having a banking sector with oligopolists.

Our paper is closest in relation to the theories of money, credit and banking, by which we refer to theories that have a rigorous micro-foundations for money and banking, and focus particularly on bank lending activities. In this regard, the literature narrows down to just a few papers, e.g., Berentsen, Camera and Waller (2007) (henceforth BCW), and Sun (2007, 2011). In all of these papers, only Sun (2007) considers banks as oligopolists. In Sun (2007), banks serve as delegated monitors and there arises a micro-foundation for money due to lack of double coincidence of wants. In the model, the number of banks is exogenous, the amount of bank lending is fixed, but the repayment of a loan is endogenous. Bank money can help reveal information of individual banks and thus improve welfare relative to the use of government-issued fiat money alone. Moreover, competition among banks strictly improves welfare by stimulating aggregate lending. In contrast, our model has frictional credit markets and allows for an endogenous market structure of banking oligopoly. Moreover, we examine the consequence of liquidity problem among the oligopolist banks. With these unique features, our paper provides a rich set of welfare implications of banking relative to the previous literature. Namely, oligopolistic competition among banks does not always improve welfare; and inflation can change the nature of the equilibrium.

There is also a theoretical literature on network models of interbank lending (e.g., Gai and Kapadia, 2010; Gai, Haldane and Kapadia, 2011; etc.). Typically in these models,
the number of banks is finite as in our model. However, our model draws sharp contrast to
this particular literature in that the latter are not micro-founded models of banking and
do not have strategic interaction of banks. Furthermore, this class of models focuses on
stability of a financial network and analyzes propagation of shocks across the network and
thus contagion among banks. In this sense, the topic of our model is at a distance with
this literature because we do not model aggregate uncertainty, nor idiosyncratic risks at
the bank level. In addition, we examine stability of an equilibrium, rather than fragility
of a financial system.

2 Model Environment

Time is discrete and has infinite horizons. Each time period $t$ consists of three subperiods.
A labor market is active in the first subperiod. A decentralized goods market is active
in the second subperiod. A centralized market and a credit market are simultaneously
active in the third subperiod. The economy is populated by a measure one of households.
A household’s preference is given by

$$U = u(q) + X,$$

where $q$ is consumption of decentralized goods and $X$ centralized goods. Moreover, the
utility function has the usual properties that $u' > 0$, $u'' < 0$. Each household has a
worker who can work for firms and earn wages. Firms are owned by households. All goods
are perishable across periods. The government issues fiat money, which is intrinsically
worthless and durable. Firms require funding to recruit workers. There is a banking sector
composed of $N$ banks, where $N$ is finite. Banks take in demand deposits of fiat money
from households and lend money to firms. Throughout the paper, we use centralized
goods as the numeraire.

In subperiod 1 of period $t$, firms (with funding from banks) and workers are randomly
matched in the labor market. The meetings are bilateral. Once met, the two parties
bargain over the real wage $w$. Matching is governed by function $\mathcal{N}(u, v)$, where $u$
is the measure of unemployed workers and $v$ is the measure of vacancy. The matching
probabilities for a firm and a worker are respectively denoted by

$$\lambda^f = \frac{\mathcal{N}(u, v)}{v}; \quad \lambda^h = \frac{\mathcal{N}(u, v)}{u}.$$

In subperiod 2 of $t$, each employed worker produces $y$ units of goods at no cost. Then
households and firms are randomly matched in a decentralized goods market. Again,
matching is bilateral and is governed by $\mathcal{M}(1, S)$, where $S$ is the measure of firms selling
in the market. Here all households are buyers and thus the measure of buyers is simply
one. Thus in the decentralized goods market, the matching probabilities for a firm and a
household are respectively given by

\[ \alpha^f = \frac{M(1,S)}{S}; \quad \alpha^h = M(1,S). \]

Once matched, the parties engage in Kalai bargaining over the terms of trade, \((q,d)\), where a real balance \(d\) is used to purchase an amount \(q\) of decentralized goods. It costs \(c(q)\) to produce \(q\) units of decentralized goods, where \(c(0) = 0\), \(c' > 0\), and \(c'' \geq 0\). The rest of the goods, \(y - c(q)\), becomes inventory and will be sold in the centralized market.

In subperiod 3 of \(t\), firms and households trade centralized goods in a competitive market. The credit market is active simultaneously with the centralized goods market. Banks take in demand deposits from households and provide long-term loans to fund firm recruiting. For deposits in the bank, households earn a net interest rate \(i_d\) every period, which is paid at the beginning of subperiod 3 of \(t + 1\). Each bank has a large number of loan officers who assist particularly in lending. At the beginning of subperiod 3, each bank selects from a pool of idle loan officers. The selected loan officers take instructions from their own bank on how to bargain with matched firms, and then they are sent to the credit market to search for firms seeking funds.

New firms have free entry to the credit market at a cost \(k > 0\). Loan officers and firms are randomly matched in a bilateral fashion. Matching is governed by the function \(L(B,E)\), where \(B = \sum_{n=1}^{N} B_n\) is the total measure of loan officers from all banks, \(B_n\) is the measure of loan officers from bank \(n\), and \(E\) is the measure of firms in the credit market. Accordingly, matching probabilities for a firm and a loan officer are respectively denoted by

\[ \phi^f = \frac{L(B,E)}{E}; \quad \phi^b = \frac{L(B,E)}{B}. \]

Once a loan officer and a firm are matched, they bargain over the contract terms. The contract states that the loan officer is to lend the firm a real amount \(\gamma > 0\) (in terms of centralized goods) every period in subperiod 3 until the firm finds a worker. In return, the firm will repay a real amount \(a\) every period starting the period it finds a worker to produce, until either the loan officer and the firm are separated with an exogenous probability \(s_c\), or the firm and the worker are exogenously separated with probability \(s\). The employment separation shock \((s)\) occurs at the onset of each subperiod 1, while the contract separation shock \((s_c)\) occurs at the end of each subperiod 1.

All bank deposits and loans are made with fiat money. Purchase of decentralized goods can be paid with either fiat money or bank money. The use of bank money can be thought of using a bank’s IOU. In reality, this corresponds to the use of checks or debit cards. The money stock \(M_t\) grows at a net rate \(\pi\) every period. The government collects a lump-sum tax \(T\) and pays a benefit \(b\) to each unemployed household. Government transfers are made at the beginning of each subperiod 3. Figure 1 provides a graph of timing in this environment.
3 Values and Decisions

3.1 Firms

There are three types of firms: firms not associated with any loan officer or worker (type 0), firms with a loan officer but not a worker (type 1) and firms who have both a loan officer and a worker (type 2). Let us first consider the value of a type-0 firm at the beginning of subperiod 3 of $t$, represented by $I_{0}^{t}$. In the rest of the paper, we suppress the time index $t$, and use a “hat” to denote period $t+1$ values. A type-0 firm can enter the credit market at cost $k > 0$ to search for a loan officer for funding. The firm will be matched with a loan officer with probability $\phi$. Thus, the value of a type-0 firm at the beginning of subperiod 3 is given by

$$I_{0}^{t} = \max\{0, \phi \beta U_{1}^{t} (a) + (1 - \phi)\beta I_{0}^{t} - k\},$$

where $U_{1}^{t} (a)$ represents the value of a type-1 firm with a contract of repayment $A$ (for short, contract $A$) in $t+1$. If not matched, the firm remains a type-0 firm.

Now consider a type-1 firm with contract $A$ at the beginning of subperiod 1 of $t$. This firm is funded by the bank to recruit a worker. With probability $\lambda$, the firm finds a worker and (after successfully negotiates wages) moves into subperiod 2 as a type-2 firm of value $V_{2}^{t} (a)$. Otherwise, it remains a type-1 firm. Thus,

$$U_{1}^{t} (a) = \lambda V_{2}^{t} (a) + (1 - \lambda)\beta U_{1}^{t} (a).$$

Next, consider a type-2 firm with contract $A$ at the beginning of subperiod 2 with value $V_{2}^{t} (a)$. First, a contract separation shock randomly hits such a firm. With probability $s$, the firm separates from the loan officer and will no longer repays the loan from that point on. In this case, we simply use $a = 0$ to denote the status of a firm with a worker. Wage,
however, is not affected by this shock. With probability $1 - s_c$, the firm remains in the loan contract. Next, the firm’s worker produces $y$ units of goods. Then the decentralized market starts, where the firm meets a household with probability $\alpha_f$. The two engage in bargaining over $(q, d)$. After the trade, the firm moves into the next subperiod with $y - c(q)$ units of real goods and a money balance of $\rho d$, where $\rho = 1/(1 + \pi)$. If the firm is not matched with a household, the firm moves into subperiod 3 with $y$ units of real goods and zero money balance. Thus,

$$V_2^f(a) = s_c \left( \alpha_f W_2^f[y - c(q), \rho d, 0] + (1 - \alpha_f) W_2^f(y, 0, 0) \right) + (1 - s_c) \left( \alpha_f W_2^f[y - c(q), \rho d, a] + (1 - \alpha_f) W_2^f(y, 0, a) \right),$$

where $W_2^f(\cdot, \cdot, \cdot)$ is the value of a type-2 firm at the beginning of subperiod 3 of $t$. It is given by

$$W_2^f(x, z, a) = x + z - w - a + s \beta \hat{I}_0 + (1 - s) \beta \hat{V}_2^f(a),$$

where $x$ is the amount of inventory and $z$ is the real money balance. In subperiod 3, this firm pays wage $w$ and make loan repayment $a$. Any remaining profit, $x + z - w - a$, will be rebated in a lump sum to households as dividend. The employment relationship is subject to a separation shock at the end of subperiod 3. With probability $s$, the relationship ends. Then the firm becomes type-0 again and will participate in the credit market of period $t + 1$. Otherwise, the firm moves on still as a type-2.

### 3.2 Households

There are two types of households, employed and unemployed. First consider an employed worker at the beginning of subperiod 3 of $t$ with value $W^h_e(z)$, where $z$ is the amount of demand deposit (made in the previous period) due for principal and interests from the bank in the current AD. Recall that demand deposit pays an interest and households can use bank money to pay for goods. Thus, households will deposit all holdings of fiat money in the bank as long as $i_d > 0$. The household’s value is given by

$$W^h_e(z) = \max_{X, \hat{z}} \left\{ X + \beta \hat{U}^h_e(\hat{z}) \right\}$$

$$s.t. \ X + \hat{z} = w + \Pi - T + (1 + i_d) z,$$

where $\hat{z}$ is the new demand deposit to be made in the current subperiod, $\Pi$ is dividend from firms, $T$ is the lump-sum tax. Moreover, $U^h_e$ is the value of the household at the beginning of subperiod 1 of $t$, and is given by

$$U^h_e(z) = s V^h_w(z) + (1 - s) V^h_e(z).$$
The employment relationship gets hit by the separation shock with probability $s$. If so, the worker becomes unemployed and continues with value $V_{u}^{h}$. Otherwise, she continues as an employed worker with value $V_{e}^{h}$. Then in the decentralized market, the household purchases from a firm with probability $h$, where she pays a real balance $d$ for $q$ units of goods. That is,

$$V_{e}^{h}(z) = \alpha^{h}[u(q) + W_{e}^{h}(\rho z - \rho d)] + (1 - \alpha^{h})W_{e}^{h}(\rho z). \quad (7)$$

Similarly, the values of an unemployed worker are given by the following:

$$W_{u}^{h}(z) = \max_{X, \hat{z}} \{X + \beta \hat{U}_{u}^{h}(\hat{z})\} \quad (8)$$

$$s.t. \quad X + \hat{z} = b + \Pi - T + (1 + i_{d})z.$$

$$U_{u}^{h}(z) = \lambda^{h}V_{u}^{h}(z) + (1 - \lambda^{h})V_{u}^{h}(z) \quad (9)$$

$$V_{u}^{h}(z) = \alpha^{h}[u(q) + W_{u}^{h}(\rho z - \rho d)] + (1 - \alpha^{h})W_{u}^{h}(\rho z). \quad (10)$$

In the above, the unemployed worker finds a firm to work for with probability $\lambda^{h}$ in subperiod 1. Moreover, $b$ is the unemployment insurance.

### 3.3 Banks

Banks $n = 1, 2, \cdots, N$ issue demand deposits to households in each subperiod $3$. The demand deposit is a contract between a bank and a household, defined as follows.

**Definition 1** *The demand deposit contract is written in subperiod 3 of period $t$. It states that: (i) the household is to deposit $z$ in the bank by the end of $t$ and is free to withdraw any amount less than or equal to $z$ before subperiod 3 of $t + 1$. (ii) The bank is to pay a net interest rate of $i_{d}$ on the household’s bank balance at the beginning of subperiod 3 of $t + 1*."

Consider any bank $n$ in subperiod 3. The bank sends a measure $B_{n}$ of type-0 loan officers to the credit market. Sending a type-0 loan officer incurs a real cost $\kappa > 0$. Bargaining will be analyzed later. Let $\mathcal{A} = \{a : a \in [0, \infty)\}$ be the set of all possible contracts and $a \in \mathcal{A}$ represents a contract with a particular periodic repayment level $a$. Moreover, $A$ represents the contract term negotiated by loan officers and firms in the current credit market. Now define $g_{i}(a)$ as the PDF of the distribution of all type-$i$ contracts of the bank at the beginning of subperiod 3 of $t$, with $i = 1, 2$. That is, $g_{ni}(a)$ represents the measure of type-$i$ contracts with repayment requirement $a$ for bank $n$. Let $K_{n}$ be the bank’s savings at the beginning of subperiod 3 of $t$. Let $\hat{Z}_{n}$ be the amount of demand deposits to accept in the current credit market. Moreover, $D_{n} \in [0, \hat{Z}_{n}]$ is the
expected amount of withdrawal in the decentralized goods market immediately coming 
up. Finally, let $\bar{B}_{-n}$ be the aggregate decisions of all banks other than bank $n$. That is, 
\[ \bar{B}_{-n} = \sum_{n \neq k=1}^{N} B_k. \]

The bank’s periodic profit is given by 
\[ \Pi_{bn} = \rho K_n + \int_{a \in A} a g_{n2} (a) + (1 - \kappa_d) \hat{Z}_n - \kappa B_n - (1 + i_d) \rho Z_{nD} \]
\[ - \gamma \left[ \int_{a \in A} g_{n1} (a) + B_n \phi^b (B_n, \bar{B}_{-n}, E) \right] - D_n - \hat{K}_n - \kappa_f, \]  
(11)
where $\kappa_d \in (0, 1)$ is the variable cost of handling demand deposits. To cover costs over 
the current subperiod 3 to the next one, the bank has its own savings $\rho K_n$, together 
with incoming repayments $\int_{a \in A} a g_{n2} (a)$ and new demand deposits being issued $\hat{Z}_n$. In 
terms of banking costs, there are the cost of handling demand deposits $\kappa_d \hat{Z}_n$, cost of 
sending new loan officers to the credit market $\kappa B_n$, cost of funding new and existing loans 
$\gamma \left[ \int_{a \in A} g_{n1} (a) + B_n \phi^b (B_n, \bar{B}_{-n}, E) \right]$, cost of paying principal and 
interests on previous demand deposits $(1 + i_d) \rho Z_{nD}$, cost of meeting demand of 
withdrawals $D_n$, a fixed cost $\kappa_f$. In addition, the bank will also choose a new amount of 
its own savings $\hat{K}_n$ to put aside. Note that $\int_{a \in A} g_{ni} (a)$ is the stock of type-$i$ contracts 
created by the bank. Moreover, $B_n \phi^b (B_n, \bar{B}_{-n}, E)$ is the measure of new type-1 contracts 
created in the current subperiod 3.

Taking $(i_d, D_n, E, A, \bar{B}_{-n})$ as given, the bank chooses $\left( B_n, \hat{Z}_n, \hat{K}_n \right)$ to maximize the 
present value of current and future profits:
\[ V_n (g_{n1}, g_{n2}, Z_{nD}, K_n) = \max_{B_n, \hat{Z}_n, \hat{K}_n} \left\{ \Pi_{bn} + \beta V_n \left( \hat{g}_{n1}, \hat{g}_{n2}, \hat{Z}_{nD}, \hat{K}_n \right) \right\} \]  
(12)

s.t. 
\[ \rho K_n + \hat{Z}_n \geq \gamma \left[ \int_{a \in A} g_{n1} (a) + B_n \phi^b (B_n, \bar{B}_{-n}, E) \right] \]
\[ + \kappa B_n + (1 + i_d) \rho Z_{nD} + \kappa_f \]
\[ \hat{K}_n \geq 0 \]
\[ \hat{Z}_{nD} = \hat{Z}_n - D_n \]
\[ \hat{g}_{n1} (a) = \begin{cases} 
(1 - \lambda^f) \left[ g_{n1} (a) + B_n \phi^b (B_n, \bar{B}_{-n}, E) \right], & \text{if } a = A \\
(1 - \lambda^f) g_{n1} (a), & \text{if } a \neq A
\end{cases} \]  
(16)
\[ \hat{g}_{n2} (a) = \begin{cases} 
(1 - s_c) (1 - s) g_{n2} (a) \\
+ \lambda^f \left[ g_{n1} (a) + B_n \phi^b (B_n, \bar{B}_{-n}, E) \right], & \text{if } a = A \\
(1 - s_c) (1 - s) g_{n2} (a) + \lambda^f g_{n1} (a), & \text{if } a \neq A
\end{cases} \]  
(17)

Note that only a fraction $\epsilon \in [0, 1]$ of the repayments are received by the bank in AD of $t$. 

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Condition (13) is the liquidity constraint faced by the bank by the end of \( t \). There are costs that must be covered by this time. They are cost of lending associated with \( \kappa \) and \( \gamma \), payment of principal and interests on previous demand deposits, and the fixed cost. What causes the liquidity constraint is that only a fraction \( \epsilon \in [0, 1] \) of the loan repayments are received by the bank by the end of \( t \). Because of the delay in some of the repayments, issuing new demand deposits and bank savings can both be helpful for relieving the liquidity condition of the bank.

Condition (14) states that the bank’s savings must be nonnegative. Condition (15) is the law of motion for the amount of demand deposit due at the beginning of subperiod 3. The rest of the conditions in the bank’s decision problem are the laws of motion for the distributions of contracts. At the beginning of subperiod 3 of \( t + 1 \), the type-1 contracts with particular requirement \( A \) consist of all of those previous and the newly-added type-1 contracts of \( A \) that are not matched with a worker in the immediately previous labor market. Similarly, the evolution of type-2 contracts with \( A \) must include those existing type-2 contracts that are not hit by a separation shock, and those newly added to the pool of type-2. Finally, measures of the contracts with requirements other than the currently offered term \( A \) evolve in their due courses given labor market shocks.

3.4 Bargaining solutions

3.4.1 The credit market

In the credit market, loan officers and firms engage in Nash bargaining. Let \( \eta_b \) be the loan officer’s bargaining power against the firm and \( \mathcal{V}_{n1} (A) \) be the marginal value of a type-1 contract to bank \( n \). If the two parties agree on a deal, then the loan officer brings the value \( \mathcal{V}_{n1} (A) \) back to the bank, and the firm receives \( \beta U_1^f (A) \). If no deal, then the bank receives nothing and the firm remains the value \( I_{10}^f \). Therefore, bargaining solution splits the surplus between the two parties according to

\[
\frac{\mathcal{V}_{n1} (A)}{\beta \left[ U_1^f (A) - I_{10}^f \right]} = \frac{\eta_b}{1 - \eta_b}.
\]

3.4.2 The labor market

Firms and workers engage in Nash bargaining in the labor market. Effectively, the wage rate \( w \) solves

\[
\frac{V_2^f (A) - \beta U_1^f (A)}{V_e^h (z) - V_u^h (z)} = \frac{\eta_f}{1 - \eta_f},
\]

where \( \eta_f \) is the firm’s bargaining power against the worker. If both parties agree, then the firm receives \( V_2^f (A) \) and the worker \( V_e^h (z) \). Otherwise, the firm gets \( U_1^f (A) \) and the worker \( V_u^h (z) \).
3.4.3 The decentralized goods market

In the decentralized goods market, matched firm and household bargain over the terms of trade \((q, d)\) according to the Kalai protocol. In particular, bargaining is to maximize the surplus of the household subject to the condition that total trade surplus is split according to the respective bargaining powers. Let \(\mu\) be the bargaining power of a household. The bargaining problem between a firm and a type \(i = e, u\) household is given by

\[
\max_{(q, d; z)} \left\{ u(q) + W^h_i (\rho z - \rho d) - W^h_i (\rho z) \right\} \tag{20}
\]

\[
s.t. \quad u(q) + W^h_i (\rho z - \rho d) - W^h_i (\rho z) = \frac{\mu}{1 - \mu} \left\{ W^f_{2} [y - c(q), \rho d, A] - W^f_{2} (y, 0, A) \right\}.
\]

4 Equilibrium

Let the distributions of type-\(i\) contracts aggregated across all \(N\) banks be denoted by

\[
g_i (a) = \sum_{n=1}^{N} g_{ni} (a), \quad i = 1, 2.
\]

Moreover, define \(g_2 (0)\) as the measure of type-2 firms free of loan repayments. We have the following definitions of equilibrium:

**Definition 2** A symmetric banking equilibrium \((\bar{\mathcal{Z}}, \bar{B}_n > 0)\) consists of values

\[
\left\{ \{\mathcal{V}_n\}_{n=1}^{N}, \left\{ U^h_i, V^h_i, W^h_i \right\}_{i=e,u}, \left\{ I^f_0, U^f_1, V^f_2, W^f_2 \right\} \right\},
\]

decision rules \(\{X, z_i\}_{i=e,u}, \left\{ B_n, \bar{\mathcal{Z}}_n, \bar{K}_n \right\}_{n=1}^{N}\), measures \(\{E, u, v, \{D_n\}_{n=1}^{N}\}\), prices, terms of trade and contract \((w, q, d, i_d, A)\), and distributions \(\{g_i (a)\}_{i=1,2}\) such that given policy \((b, T, \pi)\), the following are satisfied:

1. All decisions are optimal;
2. All bargaining results are optimal in (18), (19) and (20);
3. Free entry of firms and banks is such that
   \[
   I^f_0 = 0, \quad \Pi_{bn} = 0, \quad \forall \ n; \tag{21}
   \]
4. Matching in the labor markets satisfies:
   \[
   \mathcal{N}(u, v) = (1 - u)s, \tag{23}
   \]
   \[
   s_e g_2 (A) = s g_2 (0); \tag{24}
   \]
5. All competitive markets clear. In particular, the clearing of money and demand deposit market requires:

\[(1 - u) \dot{z}_c + u \dot{z}_u + \sum_{n=1}^{N} K_n = \frac{M}{P} \quad (25)\]
\[(1 - u) \dot{z}_c + u \dot{z}_u = \sum_{n=1}^{N} \dot{Z}_n; \quad (26)\]

6. Consistency: the distributions satisfy:

\[\hat{g}_i (a) = \sum_{n=1}^{N} \hat{g}_{ni} (a), \quad i = 1, 2 \text{ and } \forall a\]

where \(\hat{g}_{ni} (a)\) obeys laws of motion (16)-(17);

7. Symmetry: \((B_n, \tilde{Z}_n, \tilde{K}_n)\) are respectively the same for all \(n\).

8. Government balances budget:

\[bu = T - \pi \frac{M}{P}. \quad (27)\]

**Definition 3** A stationary equilibrium is one in which all real values and distributions are time-invariant. In particular,

\[g_1 (A) = \left( \frac{1}{\lambda} - 1 \right) \mathcal{L} (B, E) \quad (28)\]
\[g_2 (A) = \frac{\mathcal{L} (B, E)}{1 - (1 - s_c) (1 - s)} \quad (29)\]
\[g_i (a) = 0 \quad \forall \quad a \neq A \in \mathcal{A}. \]

Equations (28)-(29) are derived from (16)-(17) given \(\hat{g}_{ni} (A) = g_{ni} (A)\) for all \(n\) and \(i\) in the steady state.

**Lemma 1** In the steady state, the firm values are given by

\[U_1^f (A) = \frac{k}{\beta \rho} = \frac{\lambda^f}{1 - \beta (1 - \lambda)} V_2^f (A) \quad (30)\]
\[V_2^f (A) = \frac{y - w - (1 - s_c) A + \alpha^f [\rho d - c(q)] + s_c (1 - s) \beta V_2^f (0)}{1 - \beta (1 - s_c) (1 - s)} \quad (31)\]
\[V_2^f (0) = \frac{y - w + \alpha^f [\rho d - c(q)]}{1 - \beta (1 - s)}. \quad (32)\]

**Proof.** These results are straightforward to derive given (21).
Lemma 2 The household choice of deposit balance $z$ is independent of the current balance $z$. The functions $W^h_i(z)$ are linear with $dW^h_i(z)/dz = 1 + i_d$ for all $z \in [0, \infty)$ and $i = e, u$.

Proof. This is the classic Lagos-Wright result. It is straightforward from (5) and (8) that

$$W^h_e(z) = w + \Pi - T + (1 + i_d)z + \max_{\hat{z}} \left\{ \beta \hat{U}^h_e(\hat{z}) - \hat{z} \right\}$$  \hspace{1cm} (33)

$$W^h_u(z) = b + \Pi - T + (1 + i_d)z + \max_{\hat{z}} \left\{ \beta \hat{U}^h_u(\hat{z}) - \hat{z} \right\}.$$  \hspace{1cm} (34)

Hence the lemma. \blacksquare

Proposition 1 The optimal bargaining solution in the decentralized goods market is the same regardless of the type of the household. That is, for any $i = e, u$,

$$d_i(z) = \begin{cases} z^*, & \text{if } z \geq z^* \\ z, & \text{if } z < z^* \end{cases}$$

$$q_i(z) = \begin{cases} q^*, & \text{if } z \geq z^* \\ \Psi^{-1}(\rho z), & \text{if } z < z^*. \end{cases}$$

Proof. Given (5), (8) and Lemma 2, we have for $i = e, u$,

$$W^h_i(\rho z - \rho d) - W^h_i(\rho z) = -(1 + i_d)\rho d.$$  

Then given (4), we have

$$W^f_2(y - c(q), \rho d) - W^f_2(y, 0) = \rho d - c(q).$$

Together, the bargaining problem in (20) becomes the same for $i = e, u$ and boils down to

$$\max_{(q,d) \leq z} \left[ u(q) - (1 + i_d)\rho d \right]$$  \hspace{1cm} (35)

s.t. $u(q) - (1 + i_d)\rho d = \frac{\mu}{1 - \mu} \left[ \rho d - c(q) \right].$

If the money constraint does not bind, i.e., $d < z$, then we can rearrange the bargaining constraint to get

$$\rho d = \frac{(1 - \mu)u(q) + \mu c(q)}{(1 - \mu)(1 + i_d) + \mu} = \Psi(q).$$

Then use the above to eliminate $\rho d$ in the objective of (35). The bargaining problem is reduced to

$$\max_q \{ u(q) - (1 + i_d)c(q) \}$$
Therefore, the optimal choice of \( q \) solves
\[
\frac{u'(q^*)}{c'(q^*)} = 1 + i_d. \tag{36}
\]
Accordingly, the optimal choice of payment \( z^* \) is given by
\[
\rho z^* = \Psi(q^*). \tag{37}
\]
If the money constraint binds, i.e., \( d = z \), then trade quantity can be directly solved from the bargaining constraint. That is, \( q(z) = \Psi^{-1}(\rho z) \).

**Proposition 2** In the steady state, household decisions are such that:

(i) \( \hat{z}_e = \hat{z}_u \);

(ii) If \( \beta \rho (1 + i_d) < 1 \), then \( d_i(z) = z \) for \( i = e, u \). Moreover, \( \hat{z}_e = \hat{z}_u = \hat{z} \) in the steady state, where \( \hat{z} \) satisfies
\[
\alpha^h \frac{u'(\Psi^{-1}(\rho \hat{z}))}{\Psi'(\rho \hat{z})} + \left(1 - \alpha^h\right) (1 + i_d) = \frac{1}{\beta \rho}. \tag{38}
\]

(iii) If \( \beta \rho (1 + i_d) = 1 \), then \( d_i(z) < z \) for \( i = e, u \). Moreover, \( \hat{z}_e = \hat{z}_u > z^* \).

**Proof.** Let \( q_i(\cdot) \) and \( d_i(\cdot) \) be the bargaining solutions in the decentralized goods market. Recall from (7) and (10) that
\[
V^h_i(z) = \alpha^h[u(q_i(z)) + W^h_i(\rho z - \rho d_i(z))] + (1 - \alpha^h)W^h_i(\rho z), \quad i = e, u.
\]
Furthermore,
\[
\frac{dU^h_i(z)}{dz} = s \frac{dV^h_i(z)}{dz} + (1 - s) \frac{dV^h_e(z)}{dz} = s \left[ \alpha^h[u'(q_e(z))q_e'(z) + \rho (1 + i_d) (1 - d_u'(z))] + (1 - \alpha^h) \rho (1 + i_d) \right] + (1 - s) \left[ \alpha^h[u'(q_e(z))q_e'(z) + \rho (1 + i_d) (1 - d_u'(z))] + (1 - \alpha^h) \rho (1 + i_d) \right] = \alpha^h \frac{\alpha^h[u'(q_u(z))q_u'(z) - \rho (1 + i_d) d_u'(z)]}{\Psi'(\rho \hat{z})} + \rho (1 + i_d)
\]
and
\[
\frac{dU^h_i(z)}{dz} = \lambda^h \frac{dV^h_i(z)}{dz} + (1 - \lambda^h) \frac{dV^h_u(z)}{dz} = \lambda^h \left[ \alpha^h[u'(q_e(z))q_e'(z) + \rho (1 + i_d) (1 - d_u'(z))] + (1 - \alpha^h) \rho (1 + i_d) \right] + \left(1 - \lambda^h\right) \left[ \alpha^h[u'(q_e(z))q_e'(z) + \rho (1 + i_d) (1 - d_u'(z))] + (1 - \alpha^h) \rho (1 + i_d) \right] = \alpha^h \frac{\lambda^h[u'(q_u(z))q_u'(z) - \rho (1 + i_d) d_u'(z)]}{\Psi'(\rho \hat{z})} + \rho (1 + i_d)
Finally, an interior optimal choice of $\hat{z}_i$ from (5) and (8) requires
\[ \beta \frac{dU_i^h(\hat{z}_i)}{dz} = 1, \quad i = e, u. \]  
(39)

Therefore, households’ steady-state money holdings, $(\hat{z}_e, \hat{z}_u)$ are jointly solved from
\[ \frac{1}{\beta} = \alpha^h \left\{ \begin{align*}
& s[u'(q_u) q_u' (z_u) - \rho (1 + i_d) d''_u (z_u) ] \\
& + (1 - s) [u'(q_e) q'_e (z_e) - \rho (1 + i_d) d''_e (z_e) ] \end{align*} \right\} + \rho (1 + i_d) \]  
(40)
\[ \frac{1}{\beta} = \alpha^h \left\{ \begin{align*}
& \lambda^h [u'(q_e) q'_e (z_e) - \rho (1 + i_d) d''_e (z_e) ] \\
& + (1 - \lambda^h) [u'(q_u) q_u' (z_u) - \rho (1 + i_d) d''_u (z_u) ] \end{align*} \right\} + \rho (1 + i_d). \]  
(41)

The above implies
\[ u'(q_u) q_u' (z_u) - \rho (1 + i_d) d''_u (z_u) = u'(q_e) q'_e (z_e) - \rho (1 + i_d) d''_e (z_e). \]  
(42)

Immediately it follows that
\[ \hat{z}_e = \hat{z}_u. \]

If the money constraint does not bind for the households, then we have $q'_i = d''_i = 0$. Therefore,
\[ \beta \frac{dU_i^h(\hat{z}_i)}{dz} = \beta \rho (1 + i_d). \]

But then the first-order condition (39) requires
\[ \beta \rho (1 + i_d) = 1. \]

Therefore, the money constraint must be binding for both types of households if $\beta \rho (1 + i_d) < 1$. If this is the case, then (38) follows from (39). Obviously, if the money constraint does not bind, then $\hat{z}_e = \hat{z}_u > z^*$. □

In the steady state, per-capita divided from firms is given by
\[ \Pi = \left[ \alpha^f (\rho d + y - c(q)) + (1 - \alpha^f) y - w - A \right] g_2 (A) + \left[ \alpha^f (\rho d + y - c(q)) + (1 - \alpha^f) y - w \right] g_2 (0). \]  
(43)

The two components in $\Pi$ are respectively dividends from firms still making loan repayments and those without repayments. The measure of firms participating the labor market is
\[ v = g_1 (A) + \mathcal{L} (B, E) = \frac{\mathcal{L} (B, E)}{\lambda^f}, \]  
(44)
given (28) and $B = NB_n$. Moreover, in equilibrium total withdrawal faced by a bank is given by
\[ D_n = \frac{\alpha^h d(\hat{z})}{N}. \]  
(45)
Finally, we have the following results for the steady state:

\[
L(B,E) = \mathcal{N}(u,v) \\
\lambda^f = \frac{\mathcal{N}(u,v)}{v} = \frac{L(B,E)}{v} \\
\alpha^f = \frac{\mathcal{M}(1,g_2(A) + g_2(0))}{g_2(A) + g_2(0)} \\
\alpha^h = \mathcal{M}(1,g_2(A) + g_2(0)).
\]

The first equation in the above is because in equilibrium, the inflow of type-1 firms (from successful matching in the credit market) must be equal to the outflow of them (due to successful matching in the labor market). Finally, let \( \xi \) be the Lagrangian multiplier for the bank’s liquidity constraint (13).

### 4.0.4 Banking equilibrium

**Theorem 4** If exists, the stationary banking equilibrium must have the money constraint always binding for households in the decentralized goods market. In particular, the equilibrium deposit rate \( i_d \) satisfies

\[
\beta \rho (1 + i_d) = \frac{1 + \xi - \kappa_d}{1 + \xi}.
\]

There may exist multiple equilibria of the following three types:

(i) \( \xi = \hat{K}_n = 0 \). That is, the liquidity constraint does not bind for individual banks, and that banks do not save. In this case, the steady state can be determined by solving \((B,E,u,N)\) from the following System I:

\[
\beta \phi^f \lambda^f V_2^f (A) - k[1 - \beta(1 - \lambda^f)] = 0 \\
\mathcal{L}(B,E) - (1 - u) s = 0 \\
[\phi^b(B,E) + \frac{B}{N} \frac{\partial \phi^b}{\partial B_n}] \left[ (1 - \lambda^f) \beta V_1 + \lambda^f \beta V_2 - \gamma \right] = \kappa \\
[(1 - \rho (1 + i_d))(1 - \alpha^h) - \kappa_d] \tilde{z} + Ag_2(A) - \gamma[g_1(A) + B\phi^b(B,E)] - \kappa B - N \kappa_f = 0.
\]

(ii) \( \xi > 0, \hat{K}_n = 0 \). In this case, the steady state can be determined by solving \((B,E,u,\xi,N)\) from System II:

\[
\beta \phi^f \lambda^f V_2^f (A) - k[1 - \beta(1 - \lambda^f)] = 0 \\
\mathcal{L}(B,E) - (1 - u) s = 0 \\
[\phi^b(B,E) + \frac{B}{N} \frac{\partial \phi^b}{\partial B_n}] \left[ (1 - \lambda^f) \beta V_1 + \lambda^f \beta V_2 - \gamma (1 + \xi) \right] - \kappa (1 + \xi) = 0 \\
[1 - \rho (1 + i_d)(1 - \alpha^h)] \tilde{z} + \epsilon Ag_2(A) - \gamma[g_1(A) + B\phi^b(B,E)] - \kappa B - N \kappa_f = 0 \\
(1 - \epsilon) Ag_2(A) - (\alpha^h + \kappa_d) \tilde{z} = 0.
\]
(iii) $\xi > 0, \dot{K}_n > 0$. In this case, the steady state can be determined by solving $(B, E, u, \xi, K_n, N)$ from System III:

$$
\beta \phi^f \lambda^f V_2^f (A) - k [1 - \beta (1 - \lambda^f)] = 0
$$

$$
\mathcal{L} (B, E) - (1 - u) s = 0
$$

$$
[\phi^b (B, E) + \frac{B}{N} \frac{\partial \phi^b}{\partial B_n}] \left[ (1 - \lambda^f) \beta V_1 + \lambda^f \beta V_2 - \gamma (1 + \xi) \right] - \kappa (1 + \xi) = 0
$$

$$
[1 - \rho (1 + i_d) (1 - \alpha^b)] \tilde{z} + \rho N K_n + \epsilon A g_2 (A) - \gamma [g_1 (A) + B \phi^b (B, E)] - \kappa B - N \kappa_f = 0
$$

$$
\beta \rho (1 + \xi) - 1 = 0
$$

$$
(1 - \epsilon) A g_2 (A) - (\alpha^b + \kappa_d) \tilde{z} - N K_n = 0.
$$

In all of the above three cases, the contract term $A$ solves

$$
\frac{V_1}{\beta [U_1^f (A) - I_0^f]} = \eta_b \left( 1 - \eta_b \right), \quad (47)
$$

where

$$
V_2 = \frac{(1 + \epsilon \xi) A}{1 - \beta (1 - s_c) (1 - s)}, \quad (48)
$$

$$
V_1 = \frac{\beta \lambda^f V_2 - \gamma (1 + \xi)}{1 - \beta (1 - \lambda^f)}. \quad (49)
$$

**Proof.** Consider the decisions of individual bank $n$. The first-order condition for $\dot{Z}_n > 0$ is given by

$$
1 - \kappa_d + \xi + \beta \dot{V}_3 = 0. \quad (50)
$$

The first-order condition for $B_n > 0$ is

$$
\left[ \phi^b (B_n, \bar{B}_{-n}, E) + B_n \frac{\partial \phi^b}{\partial B_n} \right] \left[ (1 - \lambda^f) \beta \dot{V}_1 + \lambda^f \beta \dot{V}_2 - \gamma (1 + \xi) \right] = \kappa (1 + \xi). \quad (51)
$$

The first-order condition for $\dot{K}_n$ is

$$
\beta \dot{V}_4 - 1 \leq 0, \quad \dot{K}_n \geq 0. \quad (52)
$$

Moreover, the Envelope Theorem yields

$$
V_1 = -\gamma (1 + \xi) + \beta \left[ (1 - \lambda^f) \dot{V}_1 + \lambda^f \dot{V}_2 \right], \quad (53)
$$

$$
V_2 = A + \epsilon \Lambda \xi + \beta (1 - s_c) (1 - s) \dot{V}_2, \quad (54)
$$

$$
V_3 = -\rho (1 + i_d) (1 + \xi), \quad (55)
$$

$$
V_4 = \rho (1 + \xi). \quad (56)
$$
Equations (50) and (55) imply
\[ 1 - \kappa_d + \xi - \beta \rho (1 + i_d) (1 + \xi) = 0. \]
Thus,
\[ \beta \rho (1 + i_d) = \frac{1 + \xi - \kappa_d}{1 + \xi} < 1 \] (57)
given \( \kappa_d > 0 \). Furthermore, (52) and (56) imply
\[ \beta \rho (1 + \xi) - 1 \leq 0, \quad \hat{K}_n \geq 0. \] (58)

Consider \( \xi = 0 \). Then the above yields
\[ \beta \rho (1 + \xi) - 1 = \beta \rho - 1 < 0 \]
given \( i_d > 0 \) and \( \beta \rho (1 + i_d) < 1 \). Thus, \( \xi = 0 \) implies \( \hat{K}_n = 0 \). That is, the bank does not save as long as the liquidity constraint (13) does not bind.

Now consider \( \hat{K}_n > 0 \). Then (58) yields
\[ \beta \rho (1 + \xi) = 1. \]
Combined with (57), we have
\[ \frac{1 - \kappa_d + \xi}{1 + i_d} = 1. \]
Thus,
\[ \xi = i_d + \kappa_d > 0. \]
That is, the liquidity constraint must be binding as long as the bank decide to save. Then we have the following possible cases in equilibrium:

(i) \( \xi = 0, \hat{K}_n = 0 \). In this case,
\[ \beta \rho (1 + i_d) = 1 - \kappa_d. \]

We can solve for \( (B, E, u, N) \) from the following equations: free-entry of firms (21), labor market matching condition (23), bank’s decision on \( B_n \), (51), and bank’s zero-profit condition.\(^2\)

(ii) \( \xi > 0, \hat{K}_n = 0 \). In this case, we can solve for \( (B, E, u, \xi, N) \) from the following equations: (21), (23), (51), bank’s zero-profit condition, and (13) holding with equality.

(iii) \( \xi > 0, \hat{K}_n > 0 \). In this case, we can solve for unknowns \( (B, E, u, \xi, K_n, N) \) from the following equations: (21), (23), (51), bank’s zero-profit condition, and (13) and (58) both holding with equality.

In all of the above cases, the loan repayment \( A \) is determined by (47). \( \blacksquare \)

\(^2\)Obviously the number of banks should be an integer. However, for analytical convenience, we solve for \( N \) from the bank’s zero profit condition, which may not be an integer.
4.0.5 Welfare

Welfare is defined as the weighted average of household values at the beginning of subperiod 3. That is,

\[ \mathcal{W}_1 = (1 - u) \left[ \alpha^h W^h_e (\bar{z} - d) + \left(1 - \alpha^h\right) W^h_e (\bar{z}) \right] + u \left[ \alpha^h W^h_u (\bar{z} - d) + \left(1 - \alpha^h\right) W^h_u (\bar{z}) \right] \]

\[ = (1 - u) \left[ w + \beta U^h_e (\bar{z}) \right] + u \beta U^h_u (\bar{z}) + \Pi + i_d \bar{z} - \alpha^h (1 + i_d) d - \pi \left(\bar{z} + N \hat{K}_n\right), \quad (59) \]

where the second equality is according to condition (27), Lemma 2 and Proposition 2. In the above, \( U^h_e (\bar{z}) \) and \( U^h_u (\bar{z}) \) are given by

\[ U^h_u (\bar{z}) = \frac{1}{1 - \beta} \left\{ \lambda^h \left[ w - b + \beta \frac{(1 - \lambda^h - \sigma)(w - b)}{1 - \beta (1 - \lambda^h - \sigma)} \right] + \alpha^h u (\bar{q}) - \bar{z} \right\} \]

\[ + b (1 - u) + \Pi - \frac{\pi M}{\beta^u} + (1 + i_d) \rho(\bar{z} - \alpha^h d) \]

\[ U^h_e (\bar{z}) = \frac{(1 - s - \lambda^h) (w - b)}{1 - \beta (1 - s - \lambda^h)} + U^h_u (\bar{z}). \]

In the above definition, households do not receive banking dividends. To analyze results for the case where the number of banks \( N \) is taken as given, we also consider the definition of welfare that households own banks and thus receive (nonnegative) banking dividends. That is,

\[ \mathcal{W}_2 = \mathcal{W}_1 + \max [N \Pi_{bn}, 0]. \quad (60) \]

In the case of free-entry of banks, it does not matter which definition we use because \( \Pi_{bn} = 0. \)

5 Numerical Results

We conduct numerical exercises to obtain further results. We adopt the following functional forms:

\[ u(q) = \frac{q^{1 - \sigma_u}}{1 - \sigma_u} \]

\[ c(q) = \frac{\gamma q}{\sigma_u} \]

\[ \mathcal{L}(B, E) = \omega_L B^{1 - \chi_L} E^{\chi_L} \]

\[ \mathcal{N}(u, v) = \omega_N u^{1 - \chi_N} v^{\chi_N} \]

\[ \mathcal{M}(1, S) = \omega_G S^{\chi_G}. \]

Moreover, the parameter values are listed in Table 1. For baseline computation, we set policy variables as \( \pi = 0.02 \) and \( b = 0.5w \), where \( w \) is the equilibrium wage. Moreover, to highlight the effects of liquidity constraint, we set \( \epsilon = 0. \) We summarize our numerical results into two categories: one set of results are obtained taking the number of banks \( N \)
as given. That is, we solve the three systems listed in Theorem 4 without the competitive banking condition $\Pi_{bn} = 0$. For the second set of results, we endogenize $N$ through the competitive banking condition. That is, we solve the exact three systems as listed in Theorem 4.

5.1 Equilibrium with $N$ fixed

In this section, we assume that the number of banks is exogenously given and analyze the equilibrium without entry or exit of banks. This is helpful for better understanding the equilibrium with entry, whose results will be presented in the next section. Figure 2 depicts how an individual bank’s profit varies with the number of banks in the economy, i.e., $\Pi_{bn}(N)$. Panel A provides an overview of the curve for $N \geq 1$ and Panel B is a partial view of the function given $N \geq 6$. The partial view is meant to provide a better view of the segment where the equilibrium switches types as $N$ changes. The red color represents type I equilibria mentioned in Theorem 4, i.e., equilibria with $\xi = \hat{K}_n = 0$. The yellow color represents type II equilibria with $\xi > 0$ and $\hat{K}_n = 0$. The green color represents type III equilibria with $\xi, \hat{K}_n > 0$. As is shown in Figure 2, bank profit first decreases, then increases and finally decreases again as $N$ rises. Moreover, the banking equilibrium is of type I for lower $N$, type II for intermediate levels of $N$, and type III for higher $N$.

Correspondingly, Figure 4 shows how welfare changes with the number of banks in the economy. Panel A illustrates welfare without banking dividends, whereas Panel B is for welfare with banking dividends. Consider Panel A. Welfare is strictly increasing for type I equilibria (red), strictly decreasing for type II equilibria (yellow), and rather flat for type III equilibria (green). In Panel B, types II and III equilibria remain qualitatively the same. However, welfare in type I equilibria becomes hump-shaped. Figure 5 illustrates how other equilibrium variables change with the number of banks.

The intuition for the above results is the following: As in a standard Cournot model, oligopolistic competition stimulates quantities supplied. Similarly in our model, bank entry induces more intense competition, which leads to more loans being issued in aggregate. As a result, more firms get funding, more workers find jobs, and more output are produced. Moreover, as banks’ incentives to lend become stronger, firms get to negotiate a lower repayment $A$, which allows them to offer a higher wage $w$. (The responses of $A$ and $w$ can be respectively seen in Figure 5, as will the other variables mentioned in the explanation below.) All of these elements contribute to a positive effect on welfare as illustrated in Panel A of Figure 4 for the red-colored type I equilibrium. Nevertheless, an individual bank’s profit strictly decreases with $N$ as is the case with Cournot competition. Therefore, the hump-shaped segment in Panel B of Figure 4 is due to declining banking dividend. This is essentially a macroeconomic effect of Cournot competition.

Notice that type I equilibrium ($\xi = \hat{K}_n = 0$) only occurs for lower $N$. When the number of banks is small, all else equal there is sufficient demand deposits available for
each individual bank to the extent that the liquidity constraint (13) does not bind. In this case, banks have no incentive to save in that saving is costly due to inflation. But the liquidity constraint will bind eventually when the number of banks becomes sufficiently high. That is because there are two opposing effects associated with an increasing number of banks. On one hand, intensified competition prompts individual banks to gather more funds in order to make more loans. On the other hand, each individual bank can only receive a smaller share of the aggregate demand deposits given higher \( N \). As a result, the liquidity constraints eventually starts to bind. But when the binding constraint is not too tight, banks can still meet their lending goals by solely relying on demand deposits. Hence type II equilibria (yellow; \( \xi > 0 \) and \( \hat{K}_n = 0 \)) follows type I (red) as \( N \) increases.

Within type II equilibria, deposit rate \( i_d \) increases with bank’s liquidity shadow price \( \xi \) (see equation 57 for the relationship between \( i_d \) and \( \xi \)). That is, as banks become more desperate for funds, they are more willing to pay a higher price for them. In the meanwhile, banks compensate for this higher cost of demand deposit by negotiating for a higher repayment \( A \). Thus \( A \) is strictly increasing in \( N \) for type II equilibria. Moreover, banks negotiate so hard that their profits end up increasing with \( N \) in this type of equilibria. Nevertheless, welfare strictly decreases with \( N \) for type II equilibria because of the following: First, as banks demand higher repayments, firms have to cut wages. Second, with more banks in the economy, liquidity constraint is tightened such that each individual bank sends fewer loan officers to the credit market. This effect is so strong that the aggregate volume of loans \( B \) falls even with more banks present. Lower \( B \) implies fewer type-0 firms get funded to recruit, which in turn implies a higher unemployment rate and lower output. All of these factors contribute to a negative effect on welfare as \( N \) goes up.

However, it is intuitive that type II equilibria cannot be sustained for very long. Indeed, soon type II equilibria turn into type III (green; \( \xi, \hat{K}_n > 0 \)) as \( N \) goes further up. This is because repayment \( A \) is bounded above by output level \( y \). When there is no room to further raise \( A \), banks have to start saving for themselves even though the latter is subject to inflation tax. Over this segment, welfare is generally increasing but remains rather flat. This is because at this point \( N \) is rather high that the effect of another bank’s entry diminishes. This can also be obviously seen from a rather small yet positive response of aggregate loan volume \( B \).

5.2 Equilibrium with \( N \) variable (bank entry)

With bank entry, equilibrium can be either unique or multiple. As is obvious in Panel B of Figure 2, the function \( \Pi_{bn} (N) \) crosses the zero-profit line three times. This means that for the baseline example (\( \pi = 0.02 \)), there exist multiple equilibria of all three types. Figure 3 presents two other possible cases: Keeping all other parameters fixed, given \( \pi = 0 \) there exists a unique equilibrium of type I. Given \( \pi = 0.04 \), however, there exists of a unique equilibrium of type III. Given the general curvature of the function \( \Pi_{bn} (N) \), it is
also possible to have multiple equilibria of two types, either with types I and II coexisting, or with types II and III coexisting. These two possible cases can be indirectly seen from Figure 6 on the welfare effects of inflation. There are two turning points of the curve, one at which colors red and yellow joining each other (at $\pi = 0.019$) and the other at which colors yellow and green joining each other (at $\pi = 0.026$). The inflation rates respectively corresponding to these turning points give rise to multiple equilibria of two types.

It is also worth noting that types I and III equilibria are stable but type II is not. That is, if the economy is slightly shocked to the left of type II equilibrium, bank profit turns negative and thus there will be banks exiting and thus the economy will move to the type I equilibrium. In contrast, if the economy is slightly shock to the right of type II equilibrium, then bank profit becomes positive which induces further entry of banks. Hence the economy will move toward type III equilibrium. With similar arguments, both types I and III equilibria are stable.

It is clearly shown in Figure 4 that type I equilibria Pareto dominate, type II ranks second and type III is Pareto inferior.

5.3 Welfare effects of inflation

Figure 6 depicts the equilibrium effects of inflation. First, inflation generally decreases welfare regardless of the type of equilibrium. First of all, it is well known that inflation has a negative effect on welfare because it depreciates the real value of money. Moreover, in our model, *ceteris paribus* inflation worsens a bank’s liquidity condition. It is obvious from Figure 6 that inflation strictly increases the deposit rate. The intuition is that, as the real value of money declines, banks compete more intensely for funds by raising the deposit rate they pay. This naturally tightens banks’ liquidity constraint. In fact, it is straightforward to see from (46) that $i_d$ and $\xi$ are positively related. As a result, the higher the inflation rate, the more likely that banks become liquidity constrained, and thus the lower the welfare.

Second, inflation can change the nature of the equilibrium. In particular, low inflation ($\pi < 0.019$ in our example) only promotes type I equilibrium (red; stable and good), whereas high inflation $\pi > 0.026$ only cultivates type III (green; stable yet bad). The intermediate levels of inflation ($0.019 \leq \pi \leq 0.026$), however, can give rise to multiple equilibria of all three types. Again, the reason is that inflation tightens up banks’ liquidity conditions. Figure 7 illustrates how bank profit and repayment level $A$ vary with the number of banks for given levels of inflation. As is shown in Figure 7, higher inflation makes bank liquidity constraint start binding at a lower given number of banks. That is, the red section of the bank profit curve is the shortest at $\pi = 0.03$. Moreover, both the yellow and the green sections of the bank profit curve shifts up given higher inflation. Due to the curvature of the bank profit function, the unique good equilibrium is more likely to occur at lower inflation rates, the unique bad equilibrium tends to take place at higher inflation rates, whereas all three types can occur at medium levels of inflation.
6 Conclusion

We have constructed a tractable macroeconomic model of banking to study the behavior and economic impact of oligopolistic banks. Our model has three key features: (i) banks as oligopolists; (ii) liquidity constraint for banks that arises simply from timing mismatch of cashflows; and (iii) frictions in credit, labor and goods markets. We found that: First, it is optimal to have a small, yet greater than one, number of banks. That is, it is welfare-maximizing to have the banking sector as oligopolistic. When the number of banks is low and banks are not liquidity constrained, bank competition improves welfare. But each bank receives a smaller share of the aggregate deposit as the number of banks rises. When the number of banks is so high that banks become liquidity constrained, having more banks leads to lower welfare. This is because bank lending is now limited by the amount of deposits a bank can gather. As banks start to charge a higher loan rate to improve their financial conditions, what follow are reduced wages, discouraged firm entry, lower aggregate output yet higher unemployment, all of which leads to dampened welfare. Second, the interest rate spread also reacts non-monotonically to the number of banks. It first decreases, then increases, and finally decreases again as the number of banks climbs. Third, with entry of banks, there may exist at most three equilibria of the following types: one is stable and Pareto dominates, another is unstable and ranks second in welfare, and the third is stable yet Pareto inferior. Finally, inflation can change the nature of the equilibrium. Low inflation promotes a unique good equilibrium, high inflation cultivates a unique bad equilibrium, but medium inflation can induce all three equilibria of the aforementioned types.

References


Table 1. Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.992</td>
<td>Discount factor</td>
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<tr>
<td>$s$</td>
<td>0.041</td>
<td>Employment separation rate</td>
</tr>
<tr>
<td>$s_c$</td>
<td>0.000</td>
<td>Loan separation rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.200</td>
<td>Recruitment cost</td>
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<tr>
<td>$b$</td>
<td>0.500</td>
<td>UI as a ratio of real wage</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.000</td>
<td>Banks’ fraction of repayment received in AD</td>
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<tr>
<td>$\rho$</td>
<td>0.980</td>
<td>$\rho = 1/(1 + \pi)$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.000</td>
<td>Reserve requirement ratio</td>
</tr>
<tr>
<td>$y$</td>
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<td>Productivity</td>
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<td>$\varsigma_u$</td>
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<td>$\sigma_u$</td>
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<td>Risk aversion of the CRRA utility function</td>
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<tr>
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<tr>
<td>$\theta$</td>
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<td>Power of the cost function</td>
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<td>Labor market matching efficiency</td>
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<td>$\chi_N$</td>
<td>0.680</td>
<td>Labor market matching elasticity of $v$</td>
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<td>Credit market matching efficiency</td>
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<td>LW market matching efficiency</td>
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<td>$\eta_f$</td>
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<td>Firm bargaining power in the labour market</td>
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<td>$\eta_b$</td>
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<td>loan officer bargaining power in the credit market</td>
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<td>$\mu$</td>
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<td>HH bargaining power in the LW market</td>
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<tr>
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<td>Separation rate between a firm and a loan officer</td>
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<td>$\kappa_d$</td>
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<td>Variable cost of handling demand deposits</td>
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<td>Firms’ entry cost</td>
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<tr>
<td>$\kappa$</td>
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<td>Cost of sending loan officers to CM</td>
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<tr>
<td>$\kappa_f$</td>
<td>0.005</td>
<td>Bank’s fixed cost</td>
</tr>
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</table>
Figure 2. Bank Profit ($\pi = 0.02$)
Figure 3. Bank Profit
Figure 4. Welfare

A. Welfare without Banking Dividends, $W_1(N)$

B. Welfare with Banking Dividends, $W_2(N)$
Figure 5. Other Effects of the Number of Banks
Figure 6. Effects of Inflation
Figure 7. Inflation, Bank Profit and the Number of Banks