Estimating and Accounting for the Output Gap with Large Bayesian Vector Autoregressions

James Morley¹ and Benjamin Wong²

¹University of New South Wales
²Reserve Bank of New Zealand

June 13, 2017

Abstract

We demonstrate how Bayesian shrinkage can address problems with utilizing large information sets to calculate trend and cycle via a multivariate Beveridge-Nelson (BN) decomposition. We illustrate our approach by estimating the U.S. output gap with large Bayesian vector autoregressions that include up to 138 variables. Because the BN trend and cycle are linear functions of historical forecast errors, we are also able to account for the estimated output gap in terms of different sources of information as well as underlying structural shocks given identification restrictions. Our empirical analysis suggests that, in addition to output growth, the unemployment rate, CPI inflation, and, to a lesser extent, housing starts, consumption, stock prices, real M1, and the federal funds rate are important conditioning variables for estimating the U.S. output gap, with estimates largely robust to incorporating additional variables. Using standard identification restrictions, we find that the role of monetary policy shocks in driving the output gap is small, while oil price shocks explain about 10% of the variance over different horizons.

JEL Classification: C18, E17, E32

Keywords: Beveridge-Nelson decomposition, output gap, Bayesian estimation, multivariate information

---

¹Email: Morley: james.morley@unsw.edu.au Wong: benjamin.wong@rbnz.govt.nz

²We thank participants at a seminar at the University of Melbourne, as well as at the 25th Symposium of the Society for Nonlinear Dynamics and Econometrics, for helpful comments and suggestions. The views expressed in this paper do not necessarily represent the views of the Reserve Bank of New Zealand. Any errors are our responsibility.
1 Introduction

Interpretation of macroeconomic data often involves decomposing time series into trend and cycle, especially as related concepts such as the neutral rate of interest, the output gap, and trend inflation are crucial inputs into macroeconomic policy decision making. The macroeconomic literature is replete with statistical methods to conduct such decompositions (e.g. Hodrick and Prescott [1997], Christiano and Fitzgerald [2003]). These methods are often univariate and so only rely on a single time series (i.e. the series being detrended) for implementation. One challenge with univariate detrending is that the interpretation of the estimated trend and cycle from a statistical filter often needs to be corroborated “off-model” with other sources of information. While one could allow an explicit role for multivariate information to help conduct and interpret trend-cycle decomposition (e.g. Kozicki [1999], Garratt, Robertson, and Wright [2006], Sinclair [2009], Garrison, Lee, and Shields [2016], Chan and Grant [2017]), practical challenges still remain in terms of which variables should be included in the information set or even with how large the information set can be to keep estimation tractable.

We address these issues associated with specifying and interpreting multivariate information within the context of a particular approach to calculating trend and cycle, namely the Beveridge and Nelson (BN) (1981) decomposition. The BN decomposition provides estimates of the trend and cycle for a time series by looking at its long-horizon conditional forecast. Vector autoregressions (VARs) are widespread and well developed models for forecasting and provide a natural starting point for incorporating multivariate information to calculate trend and cycle via the BN decomposition (see Evans and Reichlin [1994]). However, their use can give rise to practical concerns such as overfitting or a mechanical decrease in the signal-to-noise ratio with more information, the latter a theoretical property of the multivariate BN decomposition as proven by Evans and Reichlin. We show that Bayesian shrinkage can circumvent these practical challenges and that estimation of trend and cycle utilizing information sets containing well over a hundred variables is both feasible and can avoid overfitting.

We also show how to infer which sources of information are the most important for estimating trend and cycle, providing a guide for variable selection and setting the appropriate size of the information set in practice. In particular, we demonstrate that, because the estimated trend and cycle from a multivariate BN decomposition are linear functions of the historical forecast errors, the contribution of various sources of information to the estimated trend and cycle can be easily determined. Given the forecast errors and identification restrictions, an extension to structural VAR (SVAR) analysis is also straightforward when the objective is to infer which economic shocks are important for driving trend and cycle.

A key empirical finding from our analysis is that the U.S. output gap estimated using a
large Bayesian VAR (BVAR) with 138 variables is similar to estimates for smaller BVARs with 23 or even just 8 variables, if appropriately selected. The reason for the robust results, despite information sets of very different sizes, is because of the Bayesian shrinkage and our ability to determine which variables are the most important sources of information. In particular, the most important variables for estimating the U.S. output gap in addition to output growth are the unemployment rate, CPI inflation, and, to a lesser extent, housing starts, consumption, stock prices, real M1, and the federal funds rate, with estimates largely robust to incorporating additional variables. Identifying monetary policy shocks and oil price shocks using standard restrictions in our 23 variable benchmark system, we find they respectively account for approximately 4% and 10% of the unconditional variance of the U.S. output gap. We also find that neither type of shock explains very much of the variance of movements in trend, consistent with consistent with monetary neutrality and traditional theories of growth that assume technology shocks are the main determinant of the long-run level of output.

Our approach and analysis are heavily influenced by [Evans and Reichlin (1994)], who consider a multivariate generalization of the original univariate BN decomposition in [ Beveridge and Nelson (1981)]. Figure 1 presents an example to illustrate two key insights from [Evans and Reichlin (1994)] that we build on. First, adding relevant multivariate information into the forecasting model significantly alters the profile of the estimated output gap. This can be seen by comparing the output gap obtained from a BN decomposition based on a univariate AR(4) model of U.S. output growth in the top panel of the figure with various multivariate BN decompositions based on VARs containing larger information sets, ranging from 2 to 7 variables, in the bottom panel. The univariate BN estimate of the output gap lacks both persistence and amplitude and also moves counter-cyclically with the NBER chronology of expansions and recessions, while the multivariate ones are more persistent, have larger amplitude, and move more pro-cyclically relative to the NBER chronology. Second, the larger the information set, the lower the signal-to-noise ratio. This can be seen as when the number of variables increases, the estimated output gap in the bottom panel clearly becomes larger in amplitude, with the 7-variable VAR producing the largest amplitude estimate of the output gap. [Evans and Reichlin]’s insights are powerful because they suggest that multivariate information should serve some role in making inference about the business cycle, but increasing the amount of information needs to be balanced with concerns about overfitting and specifically what is the appropriate signal-to-noise ratio. Our goal in this paper is to provide a practical solution to balance these tradeoffs and address other challenges with large multivariate models. In

---

1The data are for the U.S. economy and are described in full detail in Section 3. The 2-variable VAR includes output growth and the unemployment rate. The 3-variable VAR includes output growth, CPI inflation, and the federal funds rate. The 7-variable VAR includes all of the variables in the 2 and 3 variable system, as well as capacity utilization, the growth of industrial production, and the growth of real personal consumption expenditure. All of the VARs were estimated with 4 lags using least squares.
particular, we suggest how to specify and estimate a multivariate forecasting model to handle various information sets as well as how to illustrate the types of inferences one can draw about the estimated trend and cycle.

The remainder of this paper proceeds as follows. Section 2 describes the BN decomposition, its multivariate generalization, and how to infer the role of different sources of information and underlying structural shocks given identification restrictions. Section 3 describes how to use Bayesian shrinkage to utilize large information sets and describes the data to be included in the large BVARs. Section 4 presents resulting estimates of the U.S. output gap and considers the role of multivariate information in determining these estimates. Section 5 considers SVAR analysis to consider possible causal determinants of the U.S. output gap as well as trend growth. We conclude by summarizing our key findings and also suggesting possible extensions.

2 The BN Decomposition

Beveridge and Nelson (1981) define the trend of a time series as its long-horizon conditional expectation minus any deterministic future movements in the time series. In particular, let \( \{y_t\} \) be a time series with a trend component that follows a random walk with a constant drift, \( \mu \), then the BN trend, \( \tau_t \), at time \( t \) is

\[
\tau_t = \lim_{j \to \infty} E_t[y_{t+j} - j \cdot \mu].
\]

The intuition behind the BN decomposition is that the long-horizon the expectation of the cycle is zero, meaning that the long-horizon conditional expectation of the time series will just reflect the trend component. Therefore, one only needs to specify a forecasting model for the first difference of the time series, \( \{\Delta y_t\} \), to generate these forecasts. For univariate time series, ARMA models are a natural choice for the forecasting model, as considered in Beveridge and Nelson (1981). In the multivariate context, linear VARs are a natural choice, as considered in Evans and Reichlin (1994).

Let \( \{\Delta x_t\} \) represent a vector of stationary variables that includes \( \{\Delta y_t\} \). We can write a finite-order VAR in companion form as

\[
(\Delta x_t - \mu) = F(\Delta x_{t-1} - \mu) + H\nu_t
\]

where \( F \) is the companion matrix, \( \mu \) a vector of unconditional means, \( H \) maps the forecast errors to the companion form, and \( \nu_t \) is a vector of serially uncorrelated forecast errors.

---

2 By framing the stationary variables as being in differences, we can apply the BN decomposition to the integrated levels of the variables \( \{x_t\} \), including \( \{y_t\} \), although variables that are naturally stationary in their levels could be included in \( \{\Delta x_t\} \) and the BN decomposition would implicitly be applied to the accumulation of their levels.
with covariance matrix $\Sigma$. Given stationarity, $(I - F)^{-1}$ exists and, from equation (2), the cumulative sum at time $t$ of expected future deviations of the vector process from its unconditional mean can be written as,

$$E_t \sum_{j=1}^{\infty} (\Delta x_{t+j} - \mu) = F(I - F)^{-1}(\Delta x_t - \mu). \quad (3)$$

Then, denoting $\tau_t$ and $c_t$ respectively as the vector of BN trends and BN cycles of $x_t$ at time $t$, we can solve for these following Morley (2002):

$$\tau_t = x_t + F(I - F)^{-1}(\Delta x_t - \mu) \quad (4)$$
$$c_t = -F(I - F)^{-1}(\Delta x_t - \mu). \quad (5)$$

Note that the BN cycles can be written as a function of the history of forecast errors through recursive substitution. Letting $\Gamma_i \equiv F^i(I - F)^{-1}$ for notational convenience and repeatedly lagging and substituting equation (2) into equation (5), we get the following representation of the BN cycles as a function of the forecast errors:

$$c_t = -\Gamma_1(\Delta x_t - \mu) = -\Gamma_1 \{F(\Delta x_{t-1} - \mu) + H\nu_t \} = -\Gamma_1 H\nu_t - \{F\Gamma_1(\Delta x_{t-1} - \mu) \} = -\Gamma_1 H\nu_t - \{\Gamma_2(\Delta x_{t-1} - \mu) \} = -\left\{ \sum_{i=0}^{t-1} \Gamma_{i+1}H\nu_{t-i} \right\} - \Gamma_{t+1}(\Delta x_0 - \mu) \approx -\left\{ \sum_{i=0}^{t-1} \Gamma_{i+1}H\nu_{t-i} \right\}. \quad (6)$$

In practice, the initial condition (i.e. $\Delta x_0 - \mu$) is either treated as fixed when conducting conditional maximum likelihood estimation or set to zero according to its unconditional expectation. Note that, even if the initial condition were treated as fixed, $\Gamma_{t+1}$ exponentially decays to zero as $t$ increases given a stationary vector process. So the approximation in the last line should be highly accurate for almost all of the sample under conditional maximum likelihood estimation. Meanwhile, in our application, we use least squares estimation with backcast observations at the sample average. Thus, $\Delta x_0 - \mu$ is zero and there is no approximation, although we note it would be straightforward to include fixed values instead when accounting for the role of different sources of information. Then, in terms of representing the changes in BN trends as a function of the forecast errors, we
can work off equation $\eqref{4}$ to get the following:

$$
\Delta \tau_t = \{x_t + \Gamma_1(\Delta x_t - \mu)\} - \{x_{t-1} + \Gamma_1(\Delta x_{t-1} - \mu)\} = \mu + \Gamma_0 H \nu_t.
$$

Both equations $\eqref{6}$ and $\eqref{7}$ express the BN cycles and changes in BN trends as functions of all current and past forecast errors. To isolate a given BN trend or cycle as a function of the forecast errors, we can define a selection vector, $e_i$, as a vector of zeros and one as its $i^{th}$ element. Then, to account for the contribution of the forecast error for the $j^{th}$ variable to the BN trend or cycle of the $s^{th}$ ordered variable, we pre-multiply $c_t$ or $\Delta \tau_t$ in equations $\eqref{6}$ or $\eqref{7}$ respectively with $e_s$ and post-multiply with $e_j$.

### 2.1 Interpretation

Equations $\eqref{6}$ and $\eqref{7}$ provide a natural starting point for two different, but related, ways of looking at the BN trends and cycles. We briefly discuss each of these in turn.

**Sources of Information** Because equations $\eqref{6}$ and $\eqref{7}$ express the BN trends and cycles as functions of all historical forecast errors within the multivariate forecasting model, it is possible to relate these trends and cycles to different sources of information by relating them to forecast errors for each variable in the VAR. Thus, it can be easily determined what multivariate information is important for inferences about trend and cycle for a given variable. If the trend and cycle of a given variable does not depend on the forecast error for another variable, it turns out to have very little effect whether to include the other variable in the VAR when conducting a multivariate BN decomposition for the variable of interest. Importantly, though, this is not the basis for a variance decomposition because the forecast errors can be correlated across equations. However, as shown in our application, it is extremely useful for understanding how many and which variables should be included when estimating trend and cycle for a given variable.

**The Role of Structural Shocks** As noted, to the extent that the forecast errors in $\nu_t$ are correlated, variance decompositions of trends and cycles based on the forecast errors are not possible. However, given the forecast errors and identification restrictions, causal inference such as a variance decompositions based on structural shocks are possible. Recall that $\Sigma_\nu$ is the covariance matrix associated with the forecasting model presented in equation $\eqref{2}$. Let $\epsilon_t$, where $A\epsilon_t = \nu_t$, represent the vector of structural shocks with covariance matrix $\mathbf{I}$. The matrix $A$ maps the the structural shocks to the reduced form forecast errors to by a series of identification restrictions that satisfy $\Sigma_\nu = AA'$ (see, for example, Kilian, 2013; Stock and Watson, 2016, for textbook treatments and details). In practice, $A$ could be a lower triangular Cholesky decomposition of $\Sigma_\nu$, although numerous
other schemes for identifying the structural shocks are also widely used and available. Then, once an identification scheme is specified, it is straightforward to plug $A\epsilon_t$ in place of $\nu_t$ in equations (6) and (7) such that the BN trend and cycle become functions of the structural shocks. Because the structural shocks are orthogonal, this allow direct inference using variance decompositions or related measures about how much, say, monetary policy shocks, oil price shocks, or other structural shocks contribute to the trend and cycle of a given variable.

With any SVAR analysis, causal interpretations of the drivers of the trend and cycle directly rely on the plausibility of the identification of the structural shocks. Therefore, it is crucial to note that, while inferences about sources of information are invariant to identification restrictions, the causal analysis will clearly be highly dependent on a particular identification scheme. At the same time, it should be noted that, again given orthogonality, it is possible to examine the causal effects of a given structural shock without necessarily identifying all other structural shocks.

3 Bayesian Shrinkage and the Data

In this section, we first specify a BVAR as the forecasting model in order to use Bayesian shrinkage to address problems with utilizing large information sets to calculate trend and cycle via a multivariate BN decomposition. Then we discuss the data we will use in a large BVAR to estimate the U.S. output gap.

Bayesian shrinkage methods for VARs are well developed (e.g. [Litterman, 1986] [Robertson and Tallman, 1999]), including recently in a large BVAR context (see [Banbura, Giannone, and Reichlin, 2010]). Letting $Y_t = (y_{1,t} \ldots y_{n,t})'$ be a vector of $n$ random variables, we model $Y_t$ as a VAR of lag order $p$:

$$
Y_t = \beta_1 Y_{t-1} + \ldots + \beta_p Y_{t-p} + u_t
$$

$$
Y_t = \begin{bmatrix} \beta_1 & \beta_2 & \ldots & \beta_p \\ Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-p} \end{bmatrix} + u_t = \begin{bmatrix} \beta_1 & \beta_1 & \ldots & \beta_1 \\ \beta_2 & \beta_2 & \ldots & \beta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_p & \beta_p & \ldots & \beta_p \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ \vdots \\ u_{n,t} \end{bmatrix}
$$

(8)

where $\mathbb{E}(u_t'u_t) = \Sigma_u$ and $\mathbb{E}(u_t'u_{t-i}) = 0 \ \forall i > 0$. Assuming the variables in $Y_t$ are stationary and have been demeaned prior to estimation, the VAR in equation (8) can
be cast into the companion form presented in equation (2) in a straightforward manner (see, for example, Hamilton, 1994) and the BN trends and cycles can be calculated and expressed as functions of historical forecast errors, as discussed previously.

As motivated above, while the Evans and Reichlin (1994) show that additional relevant multivariate information lowers the signal-to-noise ratio, their result leaves unanswered the question of how large this information set should be or how one can practically estimate a very large system in case many variables appear to be relevant. Meanwhile, it is a virtual certainty that parameter proliferation within a VAR framework will lead to overfitting that generates a spurious decrease in the signal-to-noise ratio as more variables with any estimated degree of Granger causality are added into the VAR system. The mechanical result in terms of a decrease in the signal-to-noise ratio was illustrated earlier in Figure 1 with the larger VAR system using least squares estimation producing the estimated U.S. output gap with the largest amplitude.

Here, we consider how to build on the insights of the BVAR literature in order to best utilize Bayesian shrinkage (e.g., see Litterman, 1986; Robertson and Tallman, 1999) to make estimation of BN trends and cycles with larger information sets tractable and not subject to a mechanical and possibly spurious decrease in the signal-to-noise ratio as more variables are included in the system. A basic insight from the large BVAR literature (see Banbura, Giannone, and Reichlin, 2010) is that one should apply more shrinkage as the system gets larger. We follow this approach, although we highlight that our proposed approach to shrinkage differs from Banbura, Giannone, and Reichlin’s in three key ways. First, while Banbura, Giannone, and Reichlin (2010) shrink variables towards random walks or noise depending on their persistence, we shrink the equation for the target variable (i.e. output growth when estimating the output gap) towards a pre-specified signal-to-noise ratio, as discussed below, and the other variables to noise (if not differenced) or implicitly random walks in their levels (if differenced in the VAR). Second, we optimize our shrinkage parameter relative to an out-of-sample one-step-ahead forecast, rather than targeting the in-sample fit of a smaller system. Three, as mentioned above, we demean all our data prior to estimation, and thus we do not estimate a vector of constants as part of the model. Implicitly, this means we use the sample average to estimate the unconditional mean of a given time series, which is equivalent to placing a flat/improper prior on the constants, if they were included.

Our choice to shrink the target-variable equation towards a particular signal-to-noise ratio merits some discussion, as it is the most nonstandard feature of our procedure relative to the wider BVAR literature. Typical BVAR methods would shrink a variable like output towards to a random walk. The underlying idea is that because a random walk provides a competitive forecast for many macroeconomic variables, shrinking towards a random walk balances overfitting, which worsens the forecasting performance of the model, with a more parsimonious and accurate forecasting model. If forecasting is the sole objective,
shrinkage towards a random walk might be a good choice. However, forecasting is not our sole objective. In particular, while we aim to specify a competitive forecasting model, hence our choice to optimize based on out-of-sample forecasting performance, the ultimate aim of our analysis is to estimate a variable such as the output gap with a multivariate BN decomposition. By definition, if the forecasting model for the level of the target variable $y_t$ were a pure random walk with drift, the BN decomposition would imply no cycle (i.e. the time series would be equivalent to the trend, as its long horizon forecast minus drift is equal to the current level of the time series). In other words, the tighter we shrink towards the random walk, the smaller would be the BN cycle. This is not a desirable property as it could conflict with our aim of more shrinkage as the number of variables increase. Put differently, applying shrinkage in the usual fashion in the BVAR literature would mechanically shrink the size of the BN cycle as the number of variables increases, with a no cycle in the limit as the number of variables goes to infinity. Our solution, then, is to shrink the target variable equation towards a pre-specified signal-to-noise ratio, which we label as $\delta$. To interpret this signal-to-noise ratio, $\delta = x$ implies $x\%$ of the variance in the forecast error for the $\{\Delta y_t\}$ is due to permanent shocks to $\{y_t\}$. 

Kamber, Morley, and Wong (2017) demonstrate that one can perform a univariate BN decomposition with a pre-specified $\delta$ because there is a direct mapping from $\delta$ to the AR coefficients in an AR($p$) model. Let $\rho$ be the sum of AR coefficients in an AR($p$) regression of output growth. The mapping between the two is $\rho = 1 - 1/\sqrt{\delta}$. In Kamber, Morley, and Wong (2017), the estimation of the output gap from a univariate AR($p$) model of output growth treats $\rho$ as being fixed and so can be viewed as a dogmatic prior on the signal-to-noise ratio. Here, in the multivariate environment, we place a prior on $\delta$, but it is not dogmatic because the multivariate information can move the posterior away from the prior depending on how well the multivariate information helps in forecasting $\{\Delta y_t\}$. A prior on $\delta$ amounts to placing a prior on the sum of the autoregressive coefficients in the target variable equation, which we label $\rho(\delta)$.

Let $\beta_{ij}^t$ denote the VAR coefficient of the $t^{th}$ lag of variable $j$ in the $i^{th}$ equation of the VAR, which are elements of matrix of VAR slope coefficients $\beta$ which we introduced in equation (8). Let the target variable, $\{\Delta y_t\}$, be the $s^{th}$ variable in our BVAR, and $\rho(\delta)$ be the sum of the autoregressive coefficients in the target variable equation consistent with a pre-specified $\delta$. We set the prior means and variances as follows:
Equations (9) and (10) imply all of the differenced variables in the VAR will shrink towards random walks in their levels or white noise for stationary level variables in the VAR. The variances \( \sigma_i^2 \) and \( \sigma_j^2 \) are set by taking the variance of the residuals on an AR(4) estimated using least squares on each of the individual time series as per the usual practice (e.g. Banbura, Giannone, and Reichlin 2010; Koop 2013). The factor \( 1/l^2 \) shrinks coefficients at longer lags closer to zero, embedding the Minnesota prior structure that shorter lags are more important than longer lags in modeling the dynamics of a time series. Equations (11) and (12) implement the prior on the signal-to-noise ratio. As discussed, we shrink the sum of the AR coefficients in the target variable equation to \( \rho(\delta) \). In our application, we set a prior on \( \delta = 0.25 \), consistent with Kamber, Morley, and Wong (2017). In order to shrink towards \( \delta \), rather than a random walk in the limit, we apply more shrinkage to the sum of the autoregressive coefficients than for the individual coefficients – i.e. we require \( \chi << \lambda \). We thus set \( \chi = \lambda/10 \).

Our main estimation strategy centers on the shrinkage hyper-parameter \( \lambda \). Intuitively, \( \lambda \) serves as an information processing parameter that controls the degree of multivariate information entering into the estimation of the BN trend and cycle. As previously discussed, our approach is to shrink \( \lambda \) closer to zero as the number of time series increase so as to first prevent overfitting, but also to control the degree of the multivariate information entering into the estimate of the BN trend and cycle and so prevent a mechanical decrease of the signal-to-noise ratio. Therefore, we adopt a similar principle to the approach in Banbura, Giannone, and Reichlin (2010), who increase the degree of shrinkage as the number of variables increases in order to deal with possible overfitting. While Banbura, Giannone, and Reichlin’s procedure of applying shrinkage is designed to target the in-sample fit of a smaller 3-variable system with that of the larger system, we adopt a similar principle to applying shrinkage, but deviate slightly in how we choose \( \lambda \). In particular, we set \( \lambda \) to minimize the one-step-ahead out-of-sample root mean squared forecast.
error (RMSFE) of the target variable \{\Delta y_t\}. This approach is based on the principle that overfitting compromises out-of-sample forecasting performance. Intuitively, this approach means that larger systems with a greater potential to overfit will be subject to a larger degree of shrinkage in order to achieve similar or better out-of-sample forecasting performance.\(^2\)

To conduct Bayesian estimation of the model, we first cast the VAR in equation (8) into a system of multivariate regressions:

\[
Y = X\beta + u \quad (13)
\]

where \(Y = [Y_1, \ldots, Y_T]'\), \(X = [X_1, \ldots, X_T]'\) with \(X_t = [Y_{t-1}', \ldots, Y_{t-p}']'\), and \(u = [u_1, \ldots, u_T]'\). We employ a Normal-Inverse Wishart prior, which has the form

\[
vec(\beta) | \Sigma \sim N(vec(\beta_0), \Sigma \otimes \Omega_0) \quad \text{and} \quad \Sigma \sim IW(S_0, \alpha_0) \quad (14)
\]

where the prior parameters \(\beta_0\), \(\Omega_0\), \(S_0\), and \(\alpha_0\) are set to be consistent with equations (9), (10), (11), and (12) and the expectation of \(\Sigma\) being \(\text{diag}(\sigma_1^2, \ldots, \sigma_n^2)\). The prior from (14) can then be implemented by choosing the following dummy observations in order to match the moments of the prior (see, e.g. Del Negro and Schorfheide, 2011; Woźniak, 2016):

\[
Y_d = \begin{pmatrix} 0_{k,n} \\ diag(\sigma_1 \ldots \sigma_n) \\ 0_{1,s-1} \end{pmatrix}, \quad X_d = \begin{pmatrix} 0_{n,k} \\ J_p \otimes \text{diag}(\sigma_1 \ldots \sigma_N) / \lambda \\ 0_{1,n-s} \\ 0_{1,n-s} \end{pmatrix} \quad (15)
\]

where \(Y_d\) and \(X_d\) are dummy observations, \(J_p = \text{diag}(1, \ldots, p)\), \(S_0 = (Y_d - X_d B_0)'(Y_d - X_d B_0)\), \(B_0 = (X_d'X_d)^{-1}X_d'Y_d\), \(\Omega_0 = (X_d'X_d)^{-1}\), and \(\alpha_0 = T_d - k\), where \(T_d\) is the number of rows for both \(Y_d\) and \(X_d\) and \(k = n \times p\) where \(n\) is again the number of variables in the VAR and \(p\) is the lag order.\(^5\) The first block of dummy observations places the prior on all of the individual VAR slope coefficients, the second block imposes the priors on the covariance matrix, and the third block implements the prior on the signal-to-noise ratio.

Augmenting the regression in equation (13) with the dummy observations gives the following:

\[
Y^* = X^*\beta + u^* \quad (16)
\]

where \(Y^* = [Y', Y_d']'\), \(X^* = [X', X_d']'\) and \(u^* = [u', u_d']'\). Estimating the BVAR then

---

\(^2\)We set up our procedure as using the first 20 years of data to train the coefficients and thereafter conduct out-of-sample forecasting by sequentially expanding the window one observation at a time.

\(^5\)Note that because we demean all the variables prior to estimation, we do not include a constant in our BVAR. Thus the number of parameters in each equation in \(n \times p\).
simply amounts to conducting least squares regression of $Y^*$ on $X^*$. Therefore, the posterior distribution has the form

$$vec(\beta)|\Sigma, Y \sim N(vec(\tilde{\beta}, \Sigma \otimes (X^{**}X^*)^{-1})$$

(17)

$$\Sigma|Y \sim IW(\tilde{\Sigma}, T_d + T - k + 2)$$

(18)

where $\tilde{\beta} = (X^{**}X^*)^{-1}X^{**}Y^*$ and $\tilde{\Sigma} = (Y^* - X^*\tilde{\beta})' (Y^* - X^*\tilde{\beta})$.

**Data** For our empirical application, we consider data from 1959Q2 to 2016Q4. We use a 23-variable BVAR for our benchmark specification. The raw data are the oil price, real GDP, the Consumer Price Index (CPI), the unemployment rate, hourly earnings, the fed funds rate, stock prices, the slope of the yield curve, the GDP deflator, employment, income, real personal consumption expenditure (PCE), industrial production, capacity utilization, housing starts, producer price index for all commodities, the PCE deflator, hours worked, nonfarm real output per hour, total reserves, non-borrowed reserves, real M1, and real M2. Much of the 23-variable system is informed by [Banbura, Giannone, and Reichlin]’s 20-variable system, which is in turn informed by an influential monetary VAR study by [Christiano, Eichenbaum, and Evans] (1999). In particular, [Banbura, Giannone, and Reichlin] (2010) suggest their medium-sized 20-variable BVAR system contains a sufficiently broad information set for macroeconomic forecasting purposes and so we believe it serves as a reasonable starting point for a model which should contain most, if not all, of the relevant information for estimating the output gap via the BN decomposition. However, we also consider a 138-variable BVAR, many of the additional variables of which are just subcomponents of the 23-variable system, to make it comparable to the large BVAR (see [Banbura, Giannone, and Reichlin] 2010) or FAVAR (e.g. Bernanke, Boivin, and Eliasz 2005) studies employed within the wider empirical literature. Finally, we consider an 8-variable system that contains a subset of variables of the baseline model, as discussed in the next section. All of the raw data are sourced from FRED and IFS. We leave definitions and details of the data to the appendix.

We take natural logarithms of series when appropriate and then take first differences of any series if either a unit root test cannot reject stationarity at a 5% level of significance or a simple t-test can reject an equal sample mean for the first half and latter half of the sample at 10% level of significance. We transform the data in this way because we have assumed stationarity of the variables in our VAR in order to construct BN trends and cycles, as noted in Section 2. All series, once rendered stationary, are backcast with their

---

6The simple sample splitting approach for testing for a break in mean is for convenience given the large number of series to process. However, it would certainly be possible to consider more formal tests for structural breaks in mean at unknown breakpoints.

7Preliminary analysis showed that, despite the shrinkage, incorporating very persistent series in the
sample average so as to keep the initial observations as part of the estimation sample. We conduct all of our estimation with 4 lags, as is standard for quarterly data.

4 Output Gap Estimates

Figure 2 presents the estimated BN output gap based on our benchmark 23-variable BVAR. The estimated output gap moves with the NBER chronology of business cycle peaks and troughs reasonably well. In comparison to Figure 1, it appears the use of Bayesian shrinkage has been successful in processing multivariate information without necessarily producing an output gap that has a mechanically outsized amplitude.

As discussed previously, equation (6) shows that the BN cycle can be represented as a function of the forecast errors for the various \( n \) variables, where the share of the \( i^{th} \) variable, \( S_i \), as a source of information for the BN cycle for the \( s^{th} \) variable at time \( t \) can be expressed as,

\[
S_i = e_s c_t e_i
\]  

(19)

where output growth is the \( s^{th} \) variable in the system. We then calculate the standard deviation of the various \( n \) components to understand which sources of multivariate information are most important for the estimation of the output gap. These shares are presented in Figure 3. It can be seen from the figure that the two most important sources of multivariate information for the output gap in our 23-variable system are the unemployment rate and CPI inflation. We also observe that five other variables, personal consumption expenditure, housing starts, the federal funds rate, real M1, and stock prices are also somewhat important.

To confirm the importance of these particular variables as sources of information, we consider a more parsimonious 8-variable BVAR, which includes the seven most important variables from our 23-variable benchmark BVAR, along with output growth in order to estimate the output gap. We also consider a large 138-variable BVAR to see if other macroeconomic variables might provide useful information.

The top panel of Figure 4 presents the output gap estimates from the three different specifications containing 8, 23 and 138 variables, respectively. The output gap estimates for these three specifications are largely similar. The reason for the similarity is that Bayesian shrinkage has suppressed all but the most important sources of multivariate information for output gap estimation. This is even true for the 8-variable BVAR that produces a smaller amplitude output gap than the estimate based on a 7-variable VAR estimated by least squares in Figure 1. Meanwhile, the bottom panel of Figure 4 compares
the output gap estimate from our benchmark BVAR with the estimate for a 22-variable BVAR that omits the unemployment rate, which was the most important source of information according to our analysis in Figure 3. The output gap estimate now differs substantially from the benchmark estimate, supporting an approach that determines the main sources of information for a given BN cycle, rather than, say, using a factor model approach to process the information in a large dataset. In particular, it is the inclusion of key variables that matters, not necessarily the general size of the information set.

Figure (5) plots the out-of-sample root mean square error of output growth as a function of our shrinkage parameter, λ, for the three specifications in terms of the number of variables in the BVAR. As expected, our procedure produces more shrinkage as the system gets larger. The shrinkage parameter, λ, minimizes the one-step-ahead RMSFE at 0.49, 0.11, and 0.04, for our 8, 23 and 138 variable specifications, respectively. Therefore, while our procedure of optimizing the degree of shrinkage differs from Banbura, Giannone, and Reichlin (2010), targeting out-of-sample fit achieves the same goal of tightening the degree of shrinkage as we increase the size of the BVAR, suggesting this is a viable alternative option for choosing shrinkage in large BVARs.

We also note that the out-of-sample fit improves as we move away from λ = 0. Recall that the prior implies output growth is shrunk towards an AR(4) with a pre-specified sum of coefficients that coincides with a prior on the signal-to-noise ratio, δ. As shown in Kamber, Morley, and Wong (2017), maximum likelihood estimates of δ are far in excess of 1, and so it is not surprising that the fit when δ = 0.25, as implied by the prior, can be improved upon. However, we also note that, due to likely overfitting, the out-of-sample fit deteriorates much quicker for the larger models as we loosen the degree of shrinkage. Therefore, our objective of targeting the out-of-sample forecasting performance balances out the improvement of fit from being less restrictive on the signal-to-noise ratio and adding relevant multivariate information against possible overfitting.

Interestingly, the minimum achievable out-of-sample RMSE does fall marginally as we increase the size of the information set. This suggest that more data does contain more information to improve out-of-sample forecasts, which one could exploit with judiciously choices on the degree of shrinkage. For comparison, we also plot a horizontal line to indicate the out-of-sample forecasting performance of an AR(1) model. As can be seen, our BVARs produce very competitive forecasts relative to an AR(1) and even beat the AR(1) model when we increase the information set to 138 variables. Our benchmark does only marginally worse than an AR(1) model in terms of its out-of-sample forecasting performance. This addresses a key critique by Nelson (2008), who argues that the BN decomposition based on an AR(1) model is relevant because it is an appropriate forecasting model that matches the autocovariance structure of the output growth data. We show that with shrinkage, it is possible to specify models that do as well, or may even beat an AR(1) model in an out-of-sample forecasting exercise, supporting our approach as possible
way to conduct BN decompositions without necessarily generating spurious cycles.

We draw three general conclusions from the preceding analysis. First, Evans and Reichlin (1994) show that multivariate information will help the BN decomposition produce larger cycles relative to univariate BN decompositions. However, given least squares estimation of a VAR, the addition of multivariate information mechanically lowers of the signal-to-noise ratio, implying a larger amplitude cycle. That the cycle is a mechanical function of the size of the information set is a relevant obstacle to using such multivariate BN decompositions for practical analysis. However, we show that shrinkage is helpful in filtering information, so the general issues that Evans and Reichlin (1994) discuss are no longer an impediment to using multivariate BN decomposition for practical analysis. As long as a researcher specifies a set of relevant multivariate information, the output gap estimate should be robust to varying the size of the information set. Second, we show that it is crucial to determine what is important information. We do this by examining the shares of different forecast errors in driving the BN cycle. Not surprisingly, discarding or omitting important information has a noticeable impact on the output gap estimate. Third, we show shrinkage is an important tool to be able to consider very large datasets. That we have successfully estimated an output gap using 138 variables suggest that we may have a feasible solution to a scenarios not dissimilar to one we observe in a policy environment. Policy institutions such as central banks typically monitor and observe large volumes of data, which they use to inform their view of the degree of economic slack. We show that we may possess a viable solution to directly incorporating all the information into formal trend-cycle decomposition aimed at estimating the output gap.

In sum, we have demonstrated that it is possible to utilize Evans and Reichlin’s insights about using multivariate information to estimate the output gaps, but without the disadvantage of a mechanical lowering of the signal-to-noise ratio or overfitting. We have also showed that it is possible to produce an output gap with as few as 8 important variables, with the unemployment rate and CPI inflation being particularly important sources of multivariate information.

**Why was the estimated output gap less deep during the Great Recession than in the early 1980s?**

The estimates in Figures 2 and 4 suggest that the output gap was more negative in the early 1980s than during the Great Recession. Given that output fell by more in the Great Recession, this directly suggests that the decline in trend output during the Great Recession must have been substantial. This is consistent with the assessment by the President of the Federal Reserve Bank of St Louis, James Bullard, who suggests the Great Recession resulted in large permanent decreases in output that cannot be expected to be reversed (see Bullard, 2012). A lower trend output also means that our output gap
estimate was around zero by 2014Q1 and was as positive as 1% by early 2016. Because we model multivariate information, we can study which source of multivariate information help explain trend growth. Our analysis suggest that the information from consumption during and post Great Recession played the main role in lowering the estimate of trend output.

The top panel of Figure 6 presents the change in trend output (cumulated because of the stochastic trend) that is accounted for by the forecast errors of consumption, comparing two periods, 1980Q1-1983Q1 and 2008Q1-2013Q4. The former period encompassed the twin recessions in the early 1980s, and serves as a contrast as we try to understand what happened during the Great Recession. Recall that we estimate a large negative output gap in the 1981-1983 recession, but less so in the Great Recession. Evidently, the forecast errors of consumption only lowered the estimate of trend growth during the twin recessions in the early 1980s by less than -0.5%. By contrast, the forecast errors of consumption lowered the estimate of trend output by about 2% during the period between 2008Q1 to 2013Q4. Therefore, because the estimate of trend output was substantially lowered during and after the Great Recession, this corresponds to less deep output gap as our BVAR attributes much of the decline in real GDP with the Great Recession to have had permanent effects.

Looking at the profile of consumption growth during the two time periods in the bottom panel, one striking observation is consumption stayed sluggish throughout the Great Recession. Compare this with the early 1980s, where there was a sharp fall in the consumption growth rate in 1980Q2, but which was immediately reversed in the following quarter. Meanwhile, during the 1982-83 recession, consumption growth did not collapse, but was comparatively strong throughout most of the recession. For the Great Recession, consumption growth stayed largely negative during the whole of the NBER dated recession, and consumption growth remained weak afterwards. In fact, consumption growth in the period 2011-2013 was weaker than the period from 1982-1983, when NBER still considered the U.S. to be in a recession. We can conjecture why consumption stayed sluggish in the aftermath of the Great Recession. To the extent the permanent income hypothesis holds and agents perceived slower growth the permanent income, consumption growth should be sluggish. Our estimates of much of the fall of output being permanent in the Great Recession due to the behaviour of consumption is consistent with this conjecture. Meanwhile, we do not wish to give the impression that the Great Recession only produced a small negative output gap. Looking across the whole sample period, our estimated output gap was still relatively deep, but this is despite our estimates of a large fall in trend output.
What does multivariate information add?

While it is apparent from the previous analysis that the addition of multivariate information substantially impacts our estimates, we now explore exactly what the multivariate information adds beyond a univariate model in the vein of Kamber, Morley, and Wong (2017). A natural way to this is to plot the univariate estimate against our multivariate estimate. The top panel compares the benchmark estimate of the output gap against the “BN Filter” by Kamber, Morley, and Wong (2017). We also compare our multivariate estimate against the prior mean, which we capture by estimating the output gap with a very tight prior with $\lambda$ very close to zero. Recall that this recovers the prior on the signal-to-noise ratio of $\delta = 0.25$. The only difference between this approach and the Kamber, Morley, and Wong (2017) approach is that the estimated AR dynamics differ, even though the signal-to-noise ratio is identical. We plot these comparisons in the top panels of Figure 7. Two observations are immediately striking. First, the estimated output gap incorporating multivariate information tends to be of larger amplitude relative to the univariate “BN Filter” approach or the prior. Second, it appears that the output gap incorporating multivariate information appears to be capturing booms in the sense that there is a large positive output gap near the peak of NBER expansions. The BN Filter or the output gap based on the prior mean only appear to be capturing troughs, but the estimated output gaps appear to fall before the start of the NBER recessions. A suggestive interpretation is that perhaps multivariate information is more helpful in capturing booms, but perhaps less so for capturing troughs.

To investigate this possibility in greater detail, we track the shrinkage parameter in pseudo-real-time. Recall the shrinkage parameter, $\lambda$, is used to allow multivariate information into the detrending problem and is chosen based on minimizing the one-step-ahead out-of-sample RMSFE. We can interpret a larger $\lambda$ as implying a greater role for multivariate information. We thus track $\lambda$ for our 23-variable benchmark and 138-variable specifications by increasing the sample, one observation at a time. The bottom panels of Figure 7 presents the pseudo-real-time values of $\lambda$. Looking at large movements in $\lambda$, there is some evidence that multivariate information appears to be overfitting in recessions, but appears to be more useful in expansions. For example, we can observe that the shrinkage parameter collapses at the trough of the 2008/09 recession, but appears to be rising steadily in the current expansion.

We know from observing the raw data that the dynamics of real output growth are

---

8 We conduct the Kamber, Morley, and Wong (2017) procedure of maximizing the amplitude-to-noise ratio with an AR(4) to make directly comparable to our BVAR and find $\delta = 0.25$, consistent with Kamber, Morley, and Wong (2017).
9 We set $\lambda \approx e^{-15}$ to do this calculation. It produces slightly different results, although a similar shape, to the BN filter. This is because of a different prior on lagged differences.
10 Note that whatever the size of the pseudo-real-time sample, we still keep an initial window size of 20 years, or 80 observations to train the coefficients—i.e. the out of sample root mean square error is only calculated from the 81st observation onwards.

17
such that recessions tend to be such that we get large negative observations in recessions, but we do not observe large positive growth rates in expansions. Instead, expansions tend to be dominated by output growth of close to mean growth rates. One possible interpretation is because we get a large negative growth rate in recession, one may not require multivariate information in recessions to understand the permanent and transitory effects of recessions. By contrast, because in expansions, output growth rates tend to often be at average levels, multivariate information can help in providing identification about permanent and transitory movements in output.

We also note that the asymmetry in output growth rates has given rise to the idea that output gaps and business cycles can be asymmetric and modelling nonlinearities (e.g., see Hamilton 1989, Morley and Piger 2012). Despite the multivariate information being informative for estimating a sizeable positive output gap, our output gap still appears to be somewhat asymmetric. Because we know that our estimated BN cycle is a function of the historical forecast errors, this asymmetry must, almost by construction, reflect the asymmetry of the estimated forecast errors, which we argue should be quite prevalent given our observation about output growth dynamics.

Our conclusions may be related to Harding and Pagan (2010), who document linear univariate models struggle to identify business cycle peaks. We show that by incorporating multivariate information, the model is better able to estimate significantly positive output gaps, and thus may provide some evidence that multivariate information may be more useful in expansions than in recessions.

5 SVAR Analysis

So far, our analysis has abstracted from causality given that all we have been doing is associate movements in the estimated output gap with information embedded in various forecast errors. We will now turn to conducting more structural analysis in order to examine possible causal determinants of those movements. We will SVAR analysis to identify two widely-considered structural shocks: a monetary policy shock and an oil price shock. We identify a monetary policy shock by ordering the federal funds rate after “slow moving” variables, but before “fast moving” ones in a Cholesky decomposition. Our identification strategy is similar in spirit to work by, inter alia, Christiano, Eichenbaum, and Evans (1999) and Bernanke, Boivin, and Eliasz (2005), where the idea is that financial market variables are in the fast moving block because they can respond contemporaneously to monetary policy shocks, while slow moving variables take at least a quarter to respond. The “fast moving” variables in our benchmark 23 variable specification are real M1 and M2, stock prices, non-borrowed reserves, total reserves, and the slope of the yield curve. We also identify an oil price shock by drawing on Kilian and Vega (2011), who show that oil prices do not appear responsive to macroeconomic news and thus can be taken to be
pre-determined. This in essence orders the oil price first in a Cholesky decomposition
and also has precedence in the wider SVAR literature studying oil price shocks (e.g. see
Edelstein and Kilian [2009] Wong [2015]). Our system is partially identified, in the sense
that we only identify 2 out of 23 potential structural shocks in our benchmark system and
we do not attempt to disentangle any of the remaining 21 unidentified shocks.

We first study how much a given structural shock has driven the historical BN trend
and cycle by performing a variance decomposition. To setup a variance decomposition
of the BN cycle, we first observe that \( E \nu_t = 0 \). Working off Equation (6), it can be
verified that the difference between the actual \( h \)-step-ahead BN cycle and the conditional
expectation of the BN cycle at time \( t - 1 \) is,

\[
\begin{align}
  c_{t+h} - E_{t-1}c_{t+h} &= \sum_{i=0}^{h} \Gamma_{i+1}H\nu_{t+i-h} \\
  &= \sum_{i=0}^{h} \Gamma_{i+1}HAe_{t+i-h},
\end{align}
\]

where the second equality follows from the identification associated with the structural
shocks from the SVAR (cast into companion form). Since \( E(\nu_t'\nu_{t-i}) = 0, i > 0 \), the total
variance can therefore be written as

\[
Var(c_{t+h} - E_{t-1}c_{t+h}) = \sum_{i=0}^{h} \Gamma_{i+1}H\Sigma_{\nu}H'H_{i+1}'.
\]

It follows that a variance decomposition of the \( h \)-step-ahead variation in the BN cycle can
be calculated using equations (21) and (22):

\[
FEVD_{j,h}^c = \left[ \sum_{i=0}^{h} e_s\Gamma_{i+1}HAe_j \right]^2 e_s \left[ \sum_{i=0}^{h} \Gamma_{i+1}H\Sigma_{\nu}H'H_{i+1}' \right] e_s'.
\]

where \( FEVD_{j,h}^c \) is the \( h \)-step-ahead share of the variance of the cycle or output gap due
to the \( j^{th} \) structural shocks, where output growth is once again the \( s^{th} \) variable in the
system. Similarly, to perform a variance decomposition of the BN trend, from equation
(7), it is straightforward to verify that the variance of the change in trend can be written as

\[
Var(\Delta \tau_t - E_{t-1}\Delta \tau_t) = \Gamma_0H\Sigma_{\nu}H'\Gamma_0'.
\]
and the share of the variance can be similarly decomposed as

\[
FEVD^\tau_j = \frac{\left[\sum_{h=0}^{\infty} e_s \Gamma_0 H A e_j^\prime \right]^2}{e_s \left[\Gamma_0 H \Sigma \nu H \Gamma_0^\prime \right] e_s^\prime}.
\] (25)

Note that due to the random walk trend, the variance of the BN trend becomes infinitely large as the time horizon goes to infinity. Consequently, a decomposition of the contemporaneous variance of the change in the trend is sufficient to provide insight into how much of the variation of the BN trend is due to the various identified structural shocks given the random walk trend implies a one-off permanent shift in the random walk trend.

Figure 8 presents a variance decomposition of the output gap and trend growth. For the output gap, we present the share of monetary policy shocks and oil price shocks at \( h = 0, h = 4, \) and \( h = \infty \). Both oil price shocks and monetary policy shocks do not explain more than 10% of the variance of the output gap. While monetary policy shocks can explain about 7% of the variance of the output gap contemporaneously, its share quickly dissipates and it only explains about 4% of the unconditional variance. Therefore, it appears the role of monetary policy shocks in driving the output gap can also be relatively short lived and its impact relatively front-loaded. Our findings are consistent with the wider SVAR literature, which often reports that monetary policy shocks explain only a small part of the business cycle. Oil price shocks explain a larger share and can explain about 10% of the variance of the output gap at all horizons. Both shocks each explain a negligible share of trend growth, about 5% for oil price shocks and less than 4% for monetary policy shocks. The latter result is consistent with the idea of money neutrality, as we do not expect a monetary policy shock to have any permanent effects on the level of output.

Although variance decompositions are useful to gain an overall perspective of the importance of shocks over history, we can also calculate historical contribution of shocks to the output gap to understand specific episodes. This analysis is presented in Figure 9. For monetary policy shocks, we can observe that they explain a large share of the positive output gap before the 1980 recession, consistent with anecdotal evidence that the Fed may have been overly heating the economy. Although we observe monetary policy shocks contributed to some of the negative output gap in 1981/82, consistent with the Volcker disinflation, the overall output gap in 1981/82 was estimated to be large and negative, and monetary policy shocks only contributed to part of the negative gap, and were not a dominant cause. A recent interpretation of the events leading to the Great Recession suggest that Chairman Greenspan was perhaps running the economy too hot before 2008 (e.g., see Taylor 2012). Our historical decomposition does not appear to support this story. We find that, while monetary policy shocks did contribute modestly to a rising positive output gap in the early 2000s, this contribution largely turned negative by 2005,
while the estimate output gap continued to increase up until the advent of the Great Recession.

We find that with oil price shocks, they tend to contribute positively to the output gap when oil prices are low and contribute negatively when oil prices are high. This can be seen from the negative contribution of oil price shocks throughout the 2000s, and the positive contribution in the late 1990s. We also observe a positive contribution turning negative around 1990, consistent with the timing when the First Gulf War caused oil prices to rise from a low starting level. We also observe oil prices contribute negatively to the output gap around 1979/80, consistent with the timing of the Iranian hostage crisis and the start of the Iraq-Iran War.

Overall, we find that the contribution of the identified monetary policy and oil price shocks line up well with a number of historical episodes. Our analysis thus provide support the tools developed in this paper may provide an additional way to understand and interpreting estimated trends and cycles.

6 Conclusion

In this paper, we show how to apply the Beveridge-Nelson decomposition to obtain estimates of trend and cycle using very large multivariate models with Bayesian shrinkage. We also show how to account for and interpret the various sources of multivariate information. In our empirical application, we present estimates of the U.S. output gap with information sets ranging from eight to over one hundred variables. We find the U.S. unemployment rate and CPI inflation, together with, to a lesser extent, housing starts, real M1, real consumption, the federal funds rate, and stock prices to be important sources of information in estimating the U.S. output gap. We also show how to conduct structural analysis using the tools developed in this paper.

Future Extensions  We view two advantages of the tools we have developed in our paper which can motivate future extensions and applications. The first advantage of our approach is casting the detrending problem within a linear regression framework, using appropriate methods of shrinkage allow us to utilize potentially large datasets for inferring trend and cycle. Many time series problems can naturally be cast into the form that we introduce in this paper. For example, policy institutions often construct an output gap by monitoring very broad set of data of differing frequency. One could, with some extra work, cast the problem into a VAR with mixed frequency, and thus allow information from monthly data to directly enter the problem of nowcasting the output gap, where real GDP is a variable which is often only available at quarterly frequency. Another potential extension is joint detrending. While we only target a single variable to estimate the output gap in our analysis, one could cast the problem in the form where one could
jointly detrend variables to obtain estimates about the natural rate of unemployment, trend inflation, the natural rate of interest and potential output within a unified and consistent framework. A second advantage of our approach is the ability to interpret trend and cycle by appealing to tools from a well-developed SVAR literature. This allows us to be able to meaningfully discuss shocks driving the trend and cycle, and be able to attribute causality. The standard frameworks of trend-cycle decomposition using time series methods like unobserved components models can struggle to attribute causality, as well as being often more difficult to estimate than the models we propose, especially given large information sets. For example, Kamber and Wong (2017) adapt methods introduced in this paper to estimate the role of foreign shocks in driving trend inflation and the inflation gap for a number of open economies. One could similarly adapt the tools we introduce in this paper to answer current relevant policy questions such as what drives low neutral interest rates and what drives financial cycles. But we leave these extensions to future research.

**Data Appendix**

IFS in the mnemonic field refers to the time series being sourced from the International Financial Statistics. Otherwise, the time series is sourced from the Federal Reserve Economic Data (FRED), and the mnemonic field refers to the FRED mnemonic.
<table>
<thead>
<tr>
<th>Series</th>
<th>Mnemonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.: Commodity Price: W Texas Interm Spot Price (US$/Barrel)</td>
<td>IFS</td>
</tr>
<tr>
<td>Real Gross Domestic Product, 3 Decimal</td>
<td>GDPC96</td>
</tr>
<tr>
<td>Real Personal Consumption Expenditures</td>
<td>PCECC96</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Durable Goods</td>
<td>PCDGx</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Services</td>
<td>PCESVx</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Nondurable Goods</td>
<td>PCNDx</td>
</tr>
<tr>
<td>Real Gross Private Domestic Investment, 3 decimal</td>
<td>GPDIC96</td>
</tr>
<tr>
<td>Fixed Private Investment</td>
<td>FPIx</td>
</tr>
<tr>
<td>Gross Private Domestic Investment: Fixed Investment: Nonresidential: Equipment</td>
<td>Y033RC1Q027SBEAx</td>
</tr>
<tr>
<td>Private Nonresidential Fixed Investment</td>
<td>PNFIx</td>
</tr>
<tr>
<td>Private Residential Fixed Investment</td>
<td>PRFIx</td>
</tr>
<tr>
<td>Shares of gross domestic product: Gross private domestic investment: Change in private inventories</td>
<td>A014RE1Q156NBEA</td>
</tr>
<tr>
<td>Real Government Consumption Expenditures and Gross Investment</td>
<td>GCEC96</td>
</tr>
<tr>
<td>Real Government Consumption Expenditures and Gross Investment: Federal</td>
<td>A823RL1Q225SBEA</td>
</tr>
<tr>
<td>Federal Government Current Receipts</td>
<td>FGRECPTx</td>
</tr>
<tr>
<td>State and Local Consumption Expenditures &amp; Gross Investment</td>
<td>SLCEx</td>
</tr>
<tr>
<td>Real Exports of Goods and Services, 3 Decimal</td>
<td>EXPGSC96</td>
</tr>
<tr>
<td>Real Imports of Goods and Services, 3 Decimal</td>
<td>IMPGSC96</td>
</tr>
<tr>
<td>Real Disposable Personal Income</td>
<td>DPIC96</td>
</tr>
<tr>
<td>Nonfarm Business Sector: Real Output</td>
<td>OUTNFB</td>
</tr>
<tr>
<td>Business Sector: Real Output</td>
<td>OUTBS</td>
</tr>
<tr>
<td>Industrial Production Index</td>
<td>INDPRO</td>
</tr>
<tr>
<td>Industrial Production: Final Products (Market Group)</td>
<td>IPFINAL</td>
</tr>
<tr>
<td>Industrial Production: Consumer Goods</td>
<td>IPCONGD</td>
</tr>
<tr>
<td>Industrial Production: Materials</td>
<td>IPMAT</td>
</tr>
<tr>
<td>Industrial Production: Durable Materials</td>
<td>IPDMAT</td>
</tr>
<tr>
<td>Industrial Production: Nondurable Materials</td>
<td>IPNMAT</td>
</tr>
<tr>
<td>Industrial Production: Durable Consumer Goods</td>
<td>IPDCONGD</td>
</tr>
</tbody>
</table>
Industrial Production: Durable Goods: Automotive products
Industrial Production: Nondurable Consumer Goods
Industrial Production: Business Equipment
Industrial Production: Consumer energy products
Capacity Utilization: Manufacturing (SIC)
All Employees: Total Nonfarm Payrolls
All Employees: Total Private Industries
Civilian Employment Level
Civilian Labor Force Participation Rate
Civilian Unemployment Rate
Unemployment Rate: 16 to 19 years
Unemployment Rate: 20 years and over, Men
Unemployment Rate: 20 years and over, Women
Number of Civilians Unemployed for Less Than 5 Weeks
Number of Civilians Unemployed for 5 to 14 Weeks
Number of Civilians Unemployed for 15 to 26 Weeks
Number of Civilians Unemployed for 27 Weeks and Over
Employment Level: Part-Time for Economic Reasons, All Industries
Business Sector: Hours of All Persons
Nonfarm Business Sector: Hours of All Persons
Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing
Housing Starts: Total: New Privately Owned Housing Units Started
Privately Owned Housing Starts: 5-Unit Structures or More
Housing Starts in Midwest Census Region
Housing Starts in Northeast Census Region
Housing Starts in South Census Region
Housing Starts in West Census Region
Personal Consumption Expenditures: Chain-type Price Index
Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index)
Effective Federal Funds Rate
3-Month Treasury Bill: Secondary Market Rate
6-Month Treasury Bill: Secondary Market Rate
1-Year Treasury Constant Maturity Rate
10-Year Treasury Constant Maturity Rate
Moody’s Seasoned Aaa Corporate Bond Yield
Moody’s Seasoned Baa Corporate Bond Yield
Moody’s Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity
6-Month Treasury Bill Minus Federal Funds Rate
10-Year Treasury Constant Maturity Minus Federal Funds Rate
Real St. Louis Adjusted Monetary Base
Real M1 Money Stock
Real M2 Money Stock
Real MZM Money Stock
Commercial and Industrial Loans, All Commercial Banks
Consumer Loans at All Commercial Banks
Total Nonrevolving Credit Owned and Securitized, Outstanding
Real Estate Loans, All Commercial Banks
Total Consumer Credit Owned and Securitized, Outstanding
Households and Nonprofit Organizations; Total Assets, Level
Households and Nonprofit Organizations; Total Liabilities, Level
Households and Nonprofit Organizations; Credit Market Instruments; Liability, Level
Households and Nonprofit Organizations; Net Worth, Level
Households and Nonprofit Organizations; Total Financial Assets, Level
Households and nonprofit organizations; real estate at market value, Level
Households and Nonprofit Organizations; Total Financial Assets, Level
Shares of gross domestic product: Exports of goods and services
Shares of gross domestic product: Imports of goods and services
Industrial Production: Manufacturing (SIC)
Industrial Production: Residential utilities

FEDFUNDS
TB3MS
TB6MS
GS1
GS10
AAA
BAA
BAA10YM
TB6SMFFM
T10YFFM
AMBSLREALx
M1REALx
M2REALx
MZMREALx
BUSLOANSx
CONSUMERx
NONREVSx
REALLx
TOTALSLx
TABSHNOx
TLBSHNOx
CMDEBT
TNWBSHNOx
TFAABSHNO
HNOREMQ027x
TFAABSHNOx
B020RE1Q156NBEA
B021RE1Q156NBEA
IPMANSICS
IPB51222S
<table>
<thead>
<tr>
<th>Metric</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Production: Fuels</td>
<td>IPFUELS</td>
</tr>
<tr>
<td>Average (Mean) Duration of Unemployment</td>
<td>UEMPMEAN</td>
</tr>
<tr>
<td>Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing</td>
<td>CES0600000007</td>
</tr>
<tr>
<td>Total Reserves of Depository Institutions</td>
<td>TOTRESNS</td>
</tr>
<tr>
<td>Reserves of Depository Institutions, Nonborrowed</td>
<td>NONBORRES</td>
</tr>
<tr>
<td>5-Year Treasury Constant Maturity Rate</td>
<td>GS5</td>
</tr>
<tr>
<td>3-Month Treasury Bill Minus Federal Funds Rate</td>
<td>TB3MFFFM</td>
</tr>
<tr>
<td>5-Year Treasury Constant Maturity Minus Federal Funds Rate</td>
<td>T5YFFM</td>
</tr>
<tr>
<td>Moody’s Seasoned Aaa Corporate Bond Minus Federal Funds Rate</td>
<td>AAAFFM</td>
</tr>
<tr>
<td>Total Consumer Loans and Leases Owned and Securitized by Finance Companies, Outstanding</td>
<td>DTCTHFNMM</td>
</tr>
<tr>
<td>Securities in Bank Credit at All Commercial Banks</td>
<td>INVESTM</td>
</tr>
<tr>
<td>Nikkei Stock Average, Nikkei 225</td>
<td>NIKKEI225</td>
</tr>
<tr>
<td>Nonfinancial Corporate Business; Total Liabilities, Level</td>
<td>TLBSNCCBx</td>
</tr>
<tr>
<td>Nonfinancial Corporate Business; Nonfinancial Assets, Level</td>
<td>TTAABSNCCBx</td>
</tr>
<tr>
<td>Nonfinancial Corporate Business; Net Worth, Level</td>
<td>TNWMVBSNCCBx</td>
</tr>
<tr>
<td>Nonfinancial noncorporate business; total liabilities, Level</td>
<td>NNBTLQ027Sx</td>
</tr>
<tr>
<td>Nonfinancial noncorporate business; total assets, Level</td>
<td>NNBTAQ027Sx</td>
</tr>
<tr>
<td>Nonfinancial Noncorporate Business; Proprietors’ Equity in Noncorporate Business (Net Worth), Level</td>
<td>TNWBSNNBx</td>
</tr>
<tr>
<td>Corporate Net Cash Flow with IVA</td>
<td>CNCFx</td>
</tr>
<tr>
<td>U.S.: Industrial Share Prices (2010=100)</td>
<td>IFS</td>
</tr>
</tbody>
</table>
References


Figure 1: Estimated Output Gap from BN decompositions

Percent deviation from trend. Shaded bars refer to NBER recessions. See footnote [1] for descriptions of the 2 variable, 5 variable and 7 variable VAR systems.
Figure 2: Estimated Output Gap from 23 Variable Benchmark

Percent deviation from trend. Shaded bars refer to NBER recessions.
Figure 3: Standard Deviation of Shares
Figure 4: Estimated Output Gap of Various Specifications

Percent deviation from trend. Shaded bars refer to NBER recessions.
Figure 5: One Step Ahead Out of Sample Root Mean Square Error

The horizontal axis represents the tightness of the prior on the hyperparameter $\lambda$. The vertical axis represents the one step ahead, out of sample root mean square error. 8 variables, 23 variables and 138 variables refer to the number of variables in the various BVARs. The horizontal line is the one step ahead out of sample root mean square error forecasting output growth using an AR(1) estimated using least squares.
Figure 6: Role of Consumption

Cumulative change in trend is in terms of percentage change. Shaded bars refer to NBER recessions. Real personal consumption expenditure in quarterly percent change.
Figure 7: Role of Multivariate Information

Output Gap in top panels in terms of percent deviation from trend. Shaded bars refer to NBER recessions.
Figure 8: Variance Decomposition

Notes: Percentage of total variation
Figure 9: Historical Decomposition of the Output Gap

Percent deviation from trend. Shaded bars refer to NBER recessions.