

# The redistributive effects of bank capital regulation

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March 2018

## Abstract

We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to hold equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it widens the spread between the returns to equity and to deposits, so that the bulk of the cost of raising additional capital accrues to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

**Keywords:** limited market participation, bank capital structure, capital regulation

**JEL classifications:** G18, G2, G21

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We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to hold equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it widens the spread between the returns to equity and to deposits, so that the bulk of the cost of raising additional capital accrues to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

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# 1 Introduction

The regulation of financial institutions, and banks in particular, has been at the forefront of the policy debate for a number of years. Much of the concern over financial institutions relates to the perceived negative consequences associated with a bank's failure, and with how losses may be distributed across various stakeholders, such as borrowers (either corporate or individual) and creditors, including depositors and the government, with the ultimate bearers of the losses being households and shareholders.

A primary tool for bank regulation is the imposition of minimum capital standards, which amount to requirements that banks limit their leverage and issue at least a minimal amount of equity. Capital regulation has two primary roles. First, by creating a junior security held by bank shareholders, capital (i.e., equity) stands as a first line of defense against losses. Second, by forcing shareholders to have "skin in the game", capital helps control moral hazard problems that may arise as a result of investment decisions by levered banks. In fact, recent calls among regulators, policy makers, and academics (see, e.g., Admati et al., 2013) have been for banks to dramatically increase the amount of capital they issue as way of reducing risk and ultimately increasing social welfare.

What is less understood in the discussion related to bank capital regulation is who bears the costs, to the extent that there are any, associated with requiring banks to increase their capitalization. If banks', or bankers', primary goal is to maximize profits, and capital structure is chosen taking this objective into account, then the imposition of a leverage constraint, or any other restriction on banking activities, should lead to a reduction in bank profitability and consequently in the return available to bank claimants. Although this may be more than compensated by increases in social welfare through other channels, such as a reduced social burden or the internalization of externalities connected with a bank failure, at the bank level, the reduction in bank profitability falls on the shoulders of its various claimants, such as depositors and shareholders. Do shareholders primarily bear the costs associated with regulation, or are depositors the parties more affected by tighter regulatory requirements?

This is an important consideration for understanding the incidence of regulation, particularly if part of the aim of (capital) regulation is to protect specific agents. So far, there has been little study of which party bears the brunt of any regulatory burden.

To tackle these questions, we present a general equilibrium model of banks' optimal capital structures where all parties are risk neutral, but investors are reluctant to participate in financial markets. This key friction, which is well documented in the literature on household finance (see, e.g., Guiso and Sodini, 2013) and much studied in the literature on asset pricing in contexts of limited market participation (see, e.g., Allen and Gale, 1994), implies that investors must be induced through additional compensation to hold any financial assets. In particular, investors/households will be unwilling to become equity holders, and perhaps even depositors, unless they receive compensation sufficient to overcome their reluctance.

In our model, banks exist to channel funds from investors, who have limited options for storing their savings, into productive but risky investments. Investors are disinclined to invest in anything other than in storage, but can be induced to hold equity and deposits if their returns are sufficiently high relative to that of storage. As investors are otherwise risk neutral, however, there is no premium for holding risk and thus effects related purely to leverage changes (as in Modigliani and Miller, 1958) are shut down.

Banks can finance themselves with either debt or equity, but using too much debt exposes them to default and, hence, to bankruptcy risk. Given that bankruptcy is costly, and that banks ultimately are owned by shareholders and thus try to maximize shareholder value, they endogenously limit the amount of deposit financing to reduce expected bankruptcy costs. However, raising equity capital is difficult because investors face costs in becoming equity-holders and thus need to be compensated, with greater compensation demanded the larger amount of equity capital the banks wants to raise. In other words, investors' participation in financial markets, as holders of bank capital (i.e., equity) or deposits, is endogenous, and will depend on the difference in the returns of deposits versus equity. However, the equilibrium return is itself endogenous, reflecting the general equilibrium nature of the problem. The

trade-off between these two forces leads to an optimal capital structure, with banks always finding it optimal to raise some amount of deposit financing.

As a first important step, we characterize the equilibrium return to equity holders as well as to depositors. While the marginal holder of capital is indifferent between being an equity holder or a depositor, inframarginal equity holders earn a strictly positive rent as a result of investing in a bank's equity. The bank therefore creates value for investors by channeling funds from storage into real investment projects, allowing investors to earn a return that more than compensates them for the disutility they associate with being an equity market investor. When the expected return to investment projects is sufficiently high, all available funds are put to productive use, with no funds going into storage. In this scenario, even depositors earn a premium in that their expected return is strictly higher than what they would earn in storage.

We show that the market solution yields no distortions that can be directly improved with capital regulation since the individually optimal capital structure decisions for banks in our baseline model coincide with what a central planner would choose. This occurs because banks are competitive and all profits ultimately accrue to the banks' shareholders. The model thus exhibits a (second best) efficient benchmark solution that provides us with an ideal laboratory in which to study how the distortion (on bank profits and, thus, social output) affects the claimants of the bank, namely depositors and shareholders who are, ultimately, represented by the investors who make portfolio decisions on how to allocate their savings.<sup>1</sup>

We identify two main sources of inefficiency associated with binding minimum capital requirements. One is that, when project returns are relatively low and not all funds are being invested in productive projects requiring banks to hold greater amounts of capital reduces further the number of projects that are funded. This, in turn, reduces aggregate

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<sup>1</sup>As described below, we later introduce various frictions that can be at least partly resolved through capital requirements, so that capital regulation plays a role in solving a social problem. We show that even in that context, where capital regulation can increase social welfare, the incidence of the regulation falls differentially on different classes of investors. In particular, it is not the households that are most inclined to be equity investors that bear the brunt of the regulatory burden.

surplus since investment projects yield a higher surplus than investing in storage. While bank default risk goes down, depositors don't benefit from the tighter regulation since their expected utility is pinned down by the outside option given by the return to the storage technology. Moreover, once we endogenize the return to the storage technology, as we do in Section 4.1, depositors stand to lose from the introduction of a binding capital requirement.

The second inefficiency arises at the other extreme, once all investment funds are being allocated to productive projects. Here, an increased requirement to hold capital raises the overall cost borne by investors. The reason is that satisfying the capital requirement necessitates that a larger number of investors is induced to become equityholders. These investors will choose to do so only if their compensation, relative to what they can otherwise earn, increases. But the only way for that to happen is for the return to depositors to go down. Thus, much of the increased cost is ultimately borne by those investors who remain as depositors. In other words, much of the incidence of the increased costs associated with satisfying the capital requirement falls on those investors who are least willing to be financial market participants, rather than on investors who, by holding equity, are the residual claimants of the banks.

We then extend our baseline model to consider various market failures that may justify a need to introduce minimum capital requirements. Specifically, we consider four settings that are commonly discussed as giving rise to a need for regulation: externalities arising from "fire sales" of bank assets that occur when many banks fail at once; distortions introduced by deposit insurance, which pushes banks to rely excessively on deposits; social losses to other stakeholders that result from bank failures; and risk shifting problems for banks that lead them to put in too little effort in improving project returns (see, e.g., Holmström and Tirole, 1997). For all of these, we first show that capital regulation can help increase efficiency, so that there is a role for regulation. More importantly for our purposes, we then show that nevertheless the incidence of regulation follows a similar pattern as above, with depositors bearing much of the burden, or benefiting little when there is scope for the bank to benefit

from capital regulation. In other words, while the effect of capital regulation depends very much on the precise market failure that regulation should address, the incidence tends to always fall in the same way.

Our work is related to various strands of literature. A sizable literature<sup>2</sup> has considered the role of bank capital as a buffer or as an incentive mechanism to control moral hazard in partial equilibrium frameworks where the return to equity is exogenously given. Most of these studies support the view that banks, if left unregulated, hold inefficiently low levels of capital because of market failures or the presence of externalities, which in turn renders them excessively prone to failure.<sup>3</sup> Thus, imposing minimum capital standards on banks should increase welfare, and much of the literature has focused on discussing the mechanisms through which capital standards act, or on estimating their welfare effects (Van den Heuvel, 2008). Far less explored, however, is the question of where the incidence of such regulatory intervention falls, for which deriving securities' returns as equilibrium variables is critical.

Our findings that higher capital standards may be to the detriment of depositors is related to the work by Besanko and Thakor (1992) and Repullo (2004). Both contributions discuss the consequences of tighter capital standards in a spatial model of imperfect bank competition where returns to bank equity are exogenously fixed. They find that deposit rates fall because capital regulation forces banks to substitute some deposits for equity, which in turn prompts lenders to compete less aggressively for depositors. In contrast, in our model all markets are perfectly competitive, and equity returns, deposit returns, and the degree of equity market participation are all obtained endogenously in general equilibrium. This latter aspect, in particular, enables us to study the incidence of regulation on all bank claimants. From this perspective, our model is closest to Allen, Carletti, and Marquez (2015), where limited participation in the equity market is the key friction, although it is exogenously fixed. By

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<sup>2</sup>See e.g., Holmström and Tirole (1997); Hellmann, Murdock, and Stiglitz (2000); Morrison and White (2005); Dell'Ariccia and Marquez (2006); Allen, Carletti, and Marquez (2011); Mehran and Thakor (2011); see Thakor (2014) for a survey.

<sup>3</sup>Berger and Bouwman (2013) document empirically that well-capitalized banks are more likely to withstand a financial crisis.

contrast, in our model, the degree of participation is endogenous and is a function of the wedge in the expected return between equity and deposit markets.

Our finding that depositors bear most of the burden of capital requirements is reminiscent of studies that have investigated the incidence of taxation. For example, Huizinga, Voget, and Wagner (2014) have documented empirically that international corporate income taxation of banks is reflected in higher pre-tax interest rate margins, suggesting that the incidence of taxation falls primarily on bank customers and depositors rather than on shareholders.

By focusing on the endogenous degree of participation in the bank equity market, our paper is also related to the literature analyzing how investors' limited participation in financial markets has implications for the equilibrium pricing of assets. For instance, Allen and Gale (1994) study how limited market participation can lead to amplified volatility of asset prices. Vissing-Jørgensen (2002) uses limited market participation to help explain part of the equity premium puzzle.<sup>4</sup> Several studies in the household finance literature also feature an endogenous degree of household participation in equity markets which arises due to heterogeneous household characteristics, just like in our model. For instance, Lusardi, Michaud, and Mitchell (2017) show in a lifecycle model that the heterogeneous cost of acquiring financial knowledge limits stock market participation of less wealthy households and can account for a significant portion of wealth inequality. Our paper contributes to this literature by showing that investors' reluctance to hold risky assets not only affects household wealth, asset allocations, or stock volatility, but it also has implications for the capital structures of financial firms.

The paper proceeds as follows. The next section lays out the model. Section 3 contains the main analysis of the model, and the characterization of equilibrium. Section 4 looks at social welfare and studies the effects of a binding capital requirement. Section 5 extends the baseline model to include various settings where there is a social inefficiency which capital

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<sup>4</sup>Many other authors assume households face a (fixed) cost of participating in financial markets, and study various implications for asset pricing and portfolio allocation. For a recent example, see, for instance, Favilukis (2013). For an application to macroeconomics and the value of money, see Chatterjee and Corbae (1992).

regulation can help address. Section 6 contrasts our results with those from a setting where existing banks are forced to change their capital structures as part of a recapitalization. Finally, Section 7 concludes. All proofs are relegated to Appendix A.

## 2 A frictionless benchmark

We develop a simple one period ( $T = 0, 1$ ) benchmark model of financial intermediation with banks and investors that can provide funds in the form of equity capital or deposits. There exist two investment options: one is a *storage* technology which yields in  $t = 1$  a return of one on every unit of funds invested at  $t = 0$ ; the other is a *risky* investment which, for every unit of funds invested at  $t = 0$ , yields in  $t = 1$  a risky return of  $r \sim U[0, R]$ .

There is a continuum of mass  $M$  of risk-neutral investors, endowed with one unit of wealth each. Investors may either invest directly in the storage technology, or they can place their wealth in a bank, either as depositors or as equity holders. Investors are reluctant to hold any financial security so that they have to be induced to participate in financial markets. Specifically, each investor incurs a non-pecuniary *participation* cost  $\underline{c}$  to hold financial securities, including bank deposits.<sup>5</sup> To invest in more sophisticated financial assets such as equity, each investor  $i$  incurs an additional non-pecuniary *sophistication* cost  $c_i \sim U[0, C]$ , which is i.i.d. across investors.

The cost  $c_i$  incurred by investors may stand for the cost of acquiring the necessary skills to trade in equity markets or for heterogeneous taste for safer assets in the investor population, and a consequent disutility associated with holding junior, leveraged claims. The assumption that investors bear a cost of participating in financial markets is a standard feature in much of the asset pricing literature on limited market participation, such as Allen and Gale (1994) or, more recently, Favilukis (2013), who studies a dynamic model of investor behavior. One

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<sup>5</sup>While we allow for the possibility that investors may be skeptical of any financial asset, including deposits (i.e.,  $\underline{c} > 0$ ), our results do not rely on this assumption and continue to hold even if  $\underline{c} \leq 0$ , so that investors view a bank deposit as equivalent to storage or, indeed, have a preference for bank deposits because they view deposits as providing a liquidity service which other assets do not.

difference in our framework is that we allow this cost to be heterogeneous, rather than assuming the same cost for all investors. Doing so incorporates the realistic feature that if higher returns are anticipated, more investors will choose to hold equity, as documented recently by Lin (2017) for banking institutions.<sup>6</sup>

Banks are primarily vehicles that provide investors with access to the risky technology. Each bank finances itself with an amount of capital  $k$  and an amount of (uninsured) deposits  $1 - k$  and invests in the risky technology.<sup>7</sup> This implies that, by becoming shareholders in a bank, investors de facto take a position in the risky technology. We denote the promised per unit deposit rate as  $r_D$ , and the equilibrium expected return to bank deposits and to bank capital as  $u$  and  $\rho$ , respectively. To ensure that the risky investment is actually viable and not strictly worse than investing in storage, we assume that  $E[r] \geq 1 + \underline{c}$ , which is equivalent to assuming that  $R > 2(1 + \underline{c})$ .

Banks are subject to bankruptcy if they are unable to repay their debt obligations. This occurs when  $r < (1 - k)r_D$ , that is, when the realized return from the risky technology is lower than the total promised repayment to depositors. Bankruptcy is costly, and for simplicity we make the extreme assumption that in the event of bankruptcy, all the project's return is dissipated.<sup>8</sup> Finally, we assume that the banking sector is perfectly competitive. Free entry reduces excess returns to zero, and banks behave as price takers with respect to the equilibrium expected return on bank capital  $\rho$  and deposits  $u$ .

### 3 Optimal capital structure

The equilibrium of the model is pinned down by the following conditions:

1. Investors optimally decide whether to invest in deposits, equity, or storage so as to

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<sup>6</sup>A similar assumption on investors' heterogeneity related to sophistication and financial market participation costs can be found in recent papers such as Kacperczyk et al. (2014) and Black et al. (2018).

<sup>7</sup>Given there are constant returns to scale, normalizing the size of every bank to 1 is without loss of generality.

<sup>8</sup>We relax this assumption later in Section 5.1.

- maximize their expected utility;
2. Banks choose capital  $k$  and the deposit rate  $r_D$  to maximize expected profits;
  3. Free entry reduces bank excess returns to zero in equilibrium;
  4. The markets for bank equity and deposits clear.

We start by analyzing investors' optimal investment strategy for given returns  $\rho$  and  $u$ . At  $t = 0$ , each investor decides whether to hold financial securities depending on whether they are compensated for the respective cost they need to incur. An investor choosing to hold equity obtains a return  $\rho$  and incurs total costs  $\underline{c} + c_i$ . An investor holding deposits obtains a return  $u$  against the participation cost  $\underline{c}$ . Investing in storage yields 1 and does not entail any cost. Thus, an investor is willing to hold deposits over storage if  $u \geq 1 + \underline{c}$ . Similarly, an investor is willing to hold equity over deposits and storage if  $\rho - (\underline{c} + c_i) \geq u - \underline{c} \geq 1$ , that is if  $c_i \leq \rho - u$  with  $u \geq 1 + \underline{c}$ . It follows that whenever  $\rho - u < C$  there exists a marginal investor with sophistication cost  $\hat{c} = \rho - u$  who is indifferent between equity and deposits.

We can now calculate the total investor demand for bank equity,  $K$ , as follows. For a given spread  $\rho - u$  with  $u \geq 1 + \underline{c}$ , the mass  $K$  of investors that are willing to become sophisticated and hold bank equity is

$$K = \begin{cases} 0 & \text{if } \rho - u \leq 0 \\ \frac{M}{C} \int_0^{\rho - u} c \, dc = \frac{M}{C}(\rho - u) & \text{if } 0 \leq \rho - u < C \\ M & \text{if } \rho - u \geq C. \end{cases} \quad (1)$$

Similarly, the mass  $D$  of investors willing to hold deposits is equal to

$$D = \begin{cases} 0 & \text{if } u < 1 + \underline{c} \\ \in [0, \frac{M}{C}(C - (\rho - u))] & \text{if } u = 1 + \underline{c} \\ \frac{M}{C}(C - (\rho - u)) & \text{if } u > 1 + \underline{c}. \end{cases} \quad (2)$$

The remaining investors,  $S = M - K - D \geq 0$ , represent the demand for storage.

Turning to the banks' problem, we can derive the supply of equity, each bank chooses its capital  $k$  and its promised deposit rate  $r_D$  to solve:

$$\max_{k, r_D} E[\Pi_B] = \frac{1}{R} \int_{r_D(1-k)}^R (r - (r_D(1-k))) dr - \rho k \quad (3)$$

subject to

$$E[U_D] = \frac{1}{R} \int_{r_D(1-k)}^R r_D dr \geq u \quad (4)$$

$$E[\Pi_B] \geq 0 \quad (5)$$

$$0 \leq k \leq 1. \quad (6)$$

Expression (3) represents the excess return of the bank. The first term captures the bank's expected return from investing in the risky technology, net of the payment  $r_D(1-k)$  to depositors. Such a return is positive only when the bank does not go bankrupt, that is for  $r \geq r_D(1-k)$ . The second term  $\rho k$  reflects the expected return to shareholders for providing capital (i.e., equity) to the bank. Constraint (4) captures depositors' participation constraint. It requires that the payoff depositors receive when the bank remains solvent is at least equal to their opportunity cost  $u$ . Constraints (5) and (6) ensure that the bank is active and that the chosen capital structure lies within the feasible range.

In equilibrium both the equity market and the market for deposits have to clear. Aggregating the individual choices of capital and deposits of all banks gives the aggregate supply of equity and deposits. Denoting as  $N$  the number of banks, market clearing then requires

$$Nk \leq K,$$

so that the supply of bank capital,  $Nk$ , does not exceed the demand from (1), and

$$N(1-k) \leq D,$$

so that the supply of deposits does not exceed the demand as given in (2).

We can now characterize the equilibrium.

**Proposition 1.** *The model has a unique equilibrium, which is characterized as follows. There is a value  $\bar{R} > 1 + \underline{c}$  such that:*

i) For  $R \leq \bar{R}$ ,  $k = \frac{4(1+\underline{c})}{R} - 1 > 0$ ,  $r_D = \frac{R}{2}$ ,  $\rho = \frac{2(1+\underline{c})^2}{4(1+\underline{c})-R} > E[r]$ ,  $u = 1 + \underline{c}$ , and

$$N = \frac{M}{C} \frac{R(R-2(1+\underline{c}))(1+\underline{c})}{(R-4(1+\underline{c}))^2} \leq M.$$

ii) For  $R > \bar{R}$ ,  $k = \sqrt{\frac{R}{R+8C}}$ ,  $r_D = \frac{R}{2}$ ,  $\rho = \frac{R}{4} \left(1 + \sqrt{\frac{(R+4C)^2}{(R+4C)^2 - 16C^2}}\right) > E[r]$ ,  $u = \frac{R}{4} \left(1 + \sqrt{\frac{R}{R+8C}}\right) > 1 + \underline{c}$ , and  $N = M$ .

The threshold  $\bar{R}$  is defined in the Appendix.

The proposition shows that banks always find it optimal to raise a combination of equity and deposits, with the exact amounts depending on the profitability of the risky technology as measured by the highest possible cash flow,  $R$ . The choice between equity and deposits entails a trade-off: on the one hand, equity requires a greater return to compensate investors for incurring the costs of becoming sophisticated; on the other hand, despite requiring a lower return, deposits expose the bank to the risk of default. Under the optimal capital structure the bank balances these two forces, and exposes itself to default risk by using leverage since doing so allows a greater premium to be paid to investors willing to become equity holders.

When project expected returns are low (i.e., for  $R \leq \bar{R}$ ), the amount of funds allocated to the banking sector, which is simply  $N$ , is less than the total funds available  $M$  so that some investors choose to keep their funds in storage. In this region, which we will refer to as *partial inclusion*, the equilibrium return to depositors is pinned down by the return to storage, 1, and the participation cost,  $\underline{c}$ .

For higher levels of the project's expected return (i.e., for  $R \geq \bar{R}$ ), the investment projects are sufficiently profitable that the equilibrium allocation has all funds flowing to the banking sector,  $N = M$ , and investors either hold deposits or become equity holders. In this region,

which we will refer to as *full inclusion*, the equilibrium return to depositors is no longer driven by the return to storage, but rather increases as project returns increase.

Finally, the proposition also shows that, irrespective of the specific capital structure banks choose, the equilibrium return for equity holders is greater than the expected project return, while that for depositors is lower, that is  $\rho > E[r] > u$ . In either region, the return to equity  $\rho$  satisfies  $\rho - u = \hat{c}$ . This implies that all investors with  $c_i < \hat{c}$  invest in equity and earn an excess return  $\rho - c_i$  from their investments. This excess return is larger the smaller is  $c_i$  so that net returns are not equalized across equity holders, even if gross returns are. By contrast, investors with  $c_i > \hat{c}$  invest in either deposits or storage, and receive an excess return  $u - \underline{c} > 1$  from investing in deposits only when  $R > \bar{R}$  and all funds are employed in the financial system. In equilibrium, the difference  $\rho - u$  varies with the total amount of capital  $Nk$  used in the banking sector. In particular,  $\rho - u$ , and thus  $\hat{c}$ , increases with  $Nk$  as investors with higher sophistication costs have to be induced to hold equity. This in turn increases the excess return obtained by investors with  $c_i < \hat{c}$ .

We next analyze how the equilibrium varies with the profitability of the bank's project. In what follows, for simplicity we will refer interchangeably to the case  $N < M$  or  $R \leq \bar{R}$  and to the case  $N = M$  or  $R > \bar{R}$ .

**Corollary 1.1.** *An increase in the project's profitability, as measured by an increase in  $R$ , leads to:*

- i) a decrease in  $k$  for  $R \leq \bar{R}$  and an increase thereafter;*
- ii) an increase in  $\rho$  for any  $R$  and an increase in  $u$  for  $R > \bar{R}$ ;*
- iii) an increase in the probability of bankruptcy for  $R \leq \bar{R}$  and a decrease thereafter;*
- iv) an increase in  $N$  for  $R \leq \bar{R}$ ;*
- v) an increase in total capital as measured by  $K = Nk$  for any  $R$ .*

The corollary shows that capital structure at the individual bank level exhibits a non-monotonic dependence on  $R$ . To understand this, suppose that returns to deposits were unaffected by an increase in  $R$ . Increased profitability of projects would then always lead

banks to choose a riskier capital structure (i.e., higher leverage) in exchange for greater returns conditional on success of the project. This is exactly what happens for  $R < \bar{R}$ , where deposit returns are fixed at  $u = 1 + \underline{c}$  by depositors' outside option. In contrast, under full inclusion, the equilibrium return to deposits increases as project returns increase. This general equilibrium feedback to deposit returns reduces the benefit of higher leverage, and at the same time increases the expected bankruptcy costs since the deposit rate must rise. As a consequence, banks reduce their leverage once the point of full inclusion has been reached.

The shift in individual bank capital holdings and the expected returns to equity and deposits is illustrated in Figure 1.

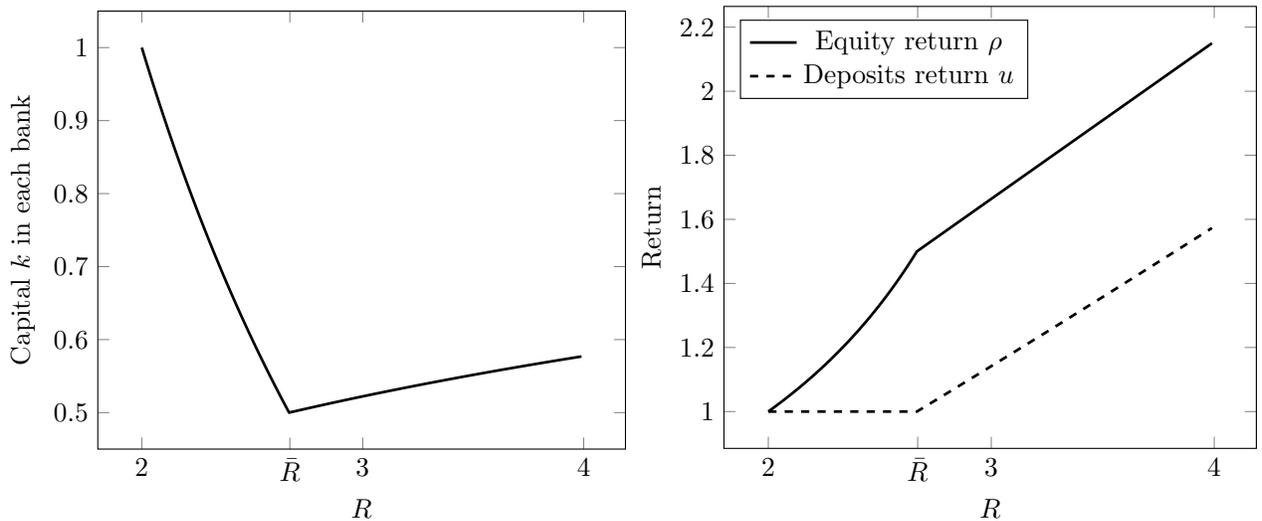


Figure 1: Equilibrium level of bank capital  $k$  and returns  $u$ ,  $\rho$  as a function of  $R$ . Parametrization:  $C = 1, \underline{c} = 0$ .

We now turn to study how the equilibrium changes with investors' characteristics, that is  $\underline{c}$  and  $C$ .

**Corollary 1.2.** *i) An increase in the participation cost  $\underline{c}$  increases  $k$  and  $u$ , and decreases  $\rho$ ,  $N$  and  $K$  for  $R \leq \bar{R}$ ; while it leaves everything constant thereafter; ii) An increase in the maximum sophistication cost  $C$  leaves  $k$  and  $\rho$  unchanged and decreases  $N$  and thus  $K$  for  $R \leq \bar{R}$ ; while it reduces  $k$ ,  $u$ , and  $K$  and increases  $\rho$  for  $R > \bar{R}$ ; iii) An increase in either  $\underline{c}$  or  $C$  increases the threshold  $\bar{R}$ .*

The participation cost affects the equilibrium only in the case of partial inclusion, when some investors make use of storage. By increasing the gross return to depositors, a higher  $\underline{c}$  increases the bank's default risk. As a consequence, banks find it optimal to increase their individual capital holdings. However, as projects become less profitable net of the participation cost, it becomes optimal to operate fewer projects so as to employ more equity capital at the individual bank level while remunerating investors less on aggregate to become equity holders. By contrast, increases in  $C$  do not affect banks' optimal capital structures directly, but reduce the number of banks since overall participation costs increase.

Finally, a change in investors' characteristics affects the degree of financial inclusion through changes in the threshold  $\bar{R}$ . In particular, any increase in investors' participation costs, either through increases in  $\underline{c}$  or in  $C$ , reduces the amount of funds used in the financial sector, thus reducing  $N$  and raising the point at which full inclusion (i.e.,  $N = M$ ) is reached.

Figure 2 provides a graphical illustration of the equilibrium amounts invested in aggregate in equity, deposits and storage as a function of  $R$  in the regions of partial and full inclusion.

## 4 The incidence of binding capital requirements

We now turn to the question how bank capital regulation affects the banks and, by extension, the investors funding the banks. To study this issue, we first establish that our benchmark model provides a market solution that is coincident with what a central planner would choose, so that the banking sector fully internalizes the (social) cost borne by investors in overcoming their aversion to holding bank securities. This will then imply that, since capital regulation would be distortionary, the framework so far provides an efficient benchmark to understand who primarily bears the burden of regulation. The possible benefits of regulation will be discussed below.

To this end, we consider the case where a central planner chooses bank capital  $k_P$  to maximize social welfare (i.e., investors' aggregate returns net of aggregate participation costs),

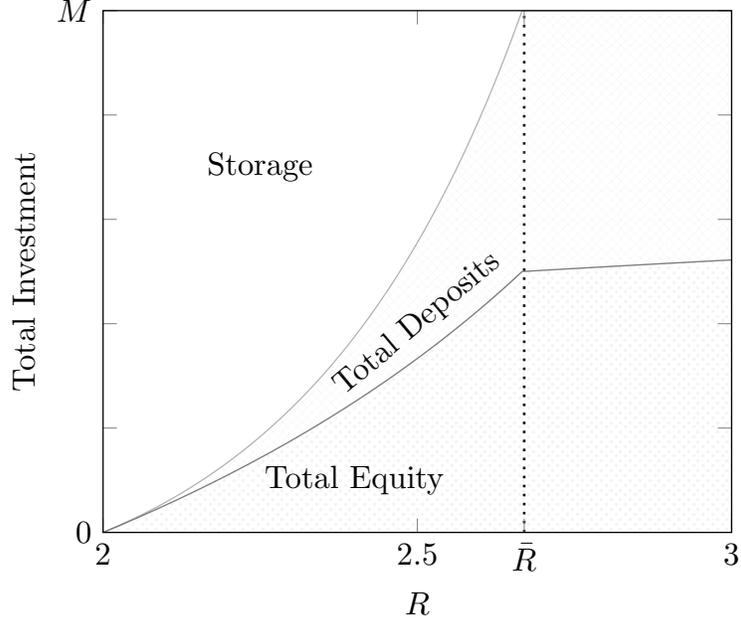


Figure 2: The figure presents the total amount of equity capital in the banking sector,  $K$ , and the total number of bank,  $N$ . The difference between the two is amount of household deposits,  $D = N - K$ . The remainder,  $S = M - N$ , is the total demand for storage. Parametrization:  $C = 1, \underline{c} = 0$ .

while deposit rates are set by the banks in order to maximize their expected excess return.

This means that the planner solves

$$\max_k SW = \rho K + uD - \int_0^{\rho-u} M \frac{c}{C} dc - N\underline{c} \quad (7)$$

subject to

$$r_D = \arg \max_{r_D} \frac{1}{S} \int_{r_D(1-k)}^R (r - (r_D(1-k)))dr - \rho k \quad (8)$$

and (4), (5), and (6). The constraints are as in the market problem except constraint (8), which indicates that the deposit rate  $r_D$  is chosen by the bank to maximize its excess returns.

We can now state the following result.

**Proposition 2.** *The central planner's allocation of bank capital coincides with the competitive capital structure in Proposition 1.*

Although banks behave as price takers in the competitive equilibrium and do not individually consider the impact of their capital structure choices on the equilibrium rate of return on capital, they end up issuing the same amount of bank capital as what a central planner would choose. The reason is that there is no pecuniary externality in the bank equity market in our model. Banks maximize returns to the benefit of bank shareholders and, given the market for capital is competitive, they ultimately internalize investors' costs to become sophisticated and willing to hold bank securities.

Having shown that the market solution is (constrained) efficient, so that there is no friction for capital regulation to overcome, we next study the issue of the incidence of binding capital requirements. Note first that any capital requirement imposing  $k^{reg} \leq k^*$ , where  $k^*$  denotes the market solution for capital from Proposition 1, will not be binding since banks will prefer to choose  $k^*$  over the regulatory minimum. Therefore, we restrict our analysis to cases where  $k^{reg} > k^*$ .

**Proposition 3.** *Suppose that  $k^{reg} > k^*$ , and define  $K^*$  and  $K^{reg}$  as the aggregate amount of capital in the market and regulatory solutions, and  $N^*$  and  $N^{reg}$  as the corresponding number of banks so that  $K^* = N^*k^*$  and  $K^{reg} = N^{reg}k^{reg}$ .*

1. *Suppose  $N^{reg} < M$ , with  $u^{reg} = 1 + \underline{c}$ . Then,  $\frac{d\rho^{reg}}{dk^{reg}} < 0$  and  $\frac{dN^{reg}}{dk^{reg}} < 0$  so that  $K^{reg} < K^*$ .*
2. *Suppose  $N^{reg} = M$ , with  $u^{reg} > 1 + \underline{c}$ . Then,  $\frac{du^{reg}}{dk^{reg}} < \frac{d\rho^{reg}}{dk^{reg}} < 0$  so that  $K^{reg} > K^*$ .*

The proposition establishes that binding capital requirements either lead to fewer banks operating,  $N^{reg} < N^*$ , or to the burden being borne disproportionately by depositors. The first result, which arises when  $N^{reg} < M$ , so that some investors use storage and not all funds are allocated to the banking sector, gives rise to an inefficiency since productive projects are funded through banks. Therefore, reducing the total funds that flow to the banking sector reduces the number of banks in equilibrium. In other words, the deadweight loss associated with capital requirements here is reflected in lower output being produced. Moreover, since

$N^{reg} < N^*$ , this implies that there is a region of parameter values  $R$ ,  $\underline{c}$ , and  $C$  such that for  $N^{reg} < N^* = M$ , depositors are made strictly worse off by binding capital requirements since without regulation they would earn an expected return of  $u^* > 1 + \underline{c}$ , whereas they earn only a return equal to  $u^{reg} = 1 + \underline{c}$  when banks are subject to capital regulation.

The second result highlights how binding capital requirements affect the different classes of investors. Since the market solution also maximizes aggregate output, a binding capital requirement leads to less total surplus. Since the number of projects that are financed remains constant with regulation (for local changes in the amount of capital around the market solution  $k^*$ ), the deadweight loss arises from the increased participation costs borne by the additional investors that need to be induced to hold bank equity. While the equilibrium return to equity holders,  $\rho$ , decreases, the return to depositors,  $u$ , decreases *even more* because the difference between them,  $\rho - u$ , must increase in order for more investors to be willing to hold bank capital. Thus, while shareholders earn a lower return, reflecting the greater deadweight losses and lower aggregate output, the bulk of the losses are borne by depositors, who bear much of the burden of the increased capital requirement.<sup>9</sup> It is therefore the investors who in principle should be protected most by larger capital requirements (as indeed they are since bankruptcy risk is reduced) who ultimately pay for this protection through more than commensurate reductions in the return they earn in equilibrium.<sup>10</sup>

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<sup>9</sup>Throughout the paper we measure the incidence of capital regulation in terms of the difference in the monetary returns between shareholders and depositors, i.e.,  $\rho - u$ . An alternative is to assess the incidence in terms of households' utility, thus comparing the expected utility to shareholders,  $\rho - \bar{c}$ , to the utility  $u$  accruing to depositors, where  $\bar{c}$  represents the average cost paid by households that become shareholders. Using  $\rho - u = \hat{c}$ , we have  $(\rho - \bar{c}) - u = \hat{c} - \bar{c} = \frac{\underline{c}}{2}$ . Clearly, the imposition of capital requirements  $k^{reg} > k^*$  increases the marginal cost more than the average cost of becoming shareholders, so that the wedge in utility goes up as well.

<sup>10</sup>In the appendix we extend the analysis to consider the case where households' limited participation in financial markets stems partly from a demand for safe assets, with households being either suspicious or financially ignorant about more complex products which are subject to default risk, including possibly bank deposits. We show that our results on the incidence of capital regulation extend to this case.

## 4.1 Endogenizing the return to storage

Our baseline model assumes that the return to the storage option is always equal to one, independently of how many households avail themselves of that option. This may be reasonable if one views households' endowments as consisting of nonperishable goods which can be stored "under the mattress", but more generally supply and demand considerations for storage vehicles, which are assets that deliver consumption in the future with certainty, may drive their return. Here, we extend the model to endogenize the return on the storage technology. As we show, in this case depositors lose from the introduction of capital regulation also in the case of partial inclusion, much as they do under full inclusion.

Given that the pool of funds available for investment is  $M$ , recall that  $S = M - N$  represents households' demand for storage since it represents the funds not allocated to productive projects through the banking sector. Storage represents any asset that delivers 1 unit of consumption at the end of the period. We denote the price of a unit of storage as  $P(S)$ , where  $P$  is an increasing function of  $S$ . We can now and calculate a return to the storage asset. Denoting the storage asset's yield by  $\delta$ , so that the gross return is  $1 + \delta$ , this return can be calculated as  $\delta(S) = \frac{1}{P(S)} - 1 = \frac{1-P(S)}{P(S)}$ . Clearly,  $\delta$  will be negative if  $P(S) > 1$ , which would correspond to the case where there is excess demand for storage assets. Also,  $\delta$  is increasing in the demand  $S$  for storage.

As above, denote the market equilibrium by the set  $\{k^*, N^*, \rho^*, u^*\}$ , and consider now the effect of imposing a binding capital requirement,  $k^{reg} > k^*$ . There are two relevant cases corresponding to whether  $N^* < M$  or  $N^* = M$ . Start with the former. In this case, with  $k^{reg} > k^*$ , we must have the number of banks,  $N^{reg}$ , go down relative to the market equilibrium  $N^*$ . This implies that the demand for storage,  $S$ , will increase, raising the price  $P(S)$  and reducing the yield  $\delta(S)$ . In other words, the return to storage will go down, and households that are either users of storage or are depositors will be strictly worse off as a result – i.e.,  $u^{reg}$  will be lower than  $u^*$ , the equilibrium return in the absence of capital regulation.

Next, consider the case where  $N^* = M$ . If, under  $k^{reg} > k^*$ ,  $N^{reg} < N^*$ , then the argument from above applies again and the equilibrium return to storage and deposits goes down. If instead  $N^{reg} = N^* = M$ , there is then no effect on the demand for storage simply because no storage is being used. For this case, all our results with an exogenous return to storage presented in Proposition 3 continue to hold as stated.

Put together, these results highlight that, when the return to storage is endogenized, the finding from Proposition 3 that depositors may be indifferent to capital regulation no longer holds, even for the case where  $N < M$ . Now, depositors are always harmed by the imposition of binding capital regulation since this pushes more investors into using storage, bidding up its price and as a result lowering its return. To push it further, if the price of the storage asset,  $P(S)$ , is sufficiently elastic, meaning that the price reacts a great deal to a change in the demand for storage, the reduction in depositors' return  $u$  may be sufficiently large that  $\rho^{reg} - u^{reg}$  increases relative to what obtains in the unregulated equilibrium,  $\rho^* - u^*$ , even for the case where  $N < M$ . This occurs both because depositors' return  $u$  goes down significantly, and because the drop in  $u$ , by reducing what gets paid out to depositors, leaves more for shareholders and thus tempers the negative effect on output stemming from the increased capital requirement.<sup>11</sup> Therefore, our results are not only robust to endogenizing the return to storage, but they actually strengthen the implications regarding redistribution arising from bank capital regulation.

## 5 Market frictions and bank capital regulation

In the baseline model with no frictions, banks' capital structure decisions are constrained efficient and there is no scope for capital regulation. Now we extend the model to incorporate

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<sup>11</sup>How much the price of the storage asset reacts to changes in demand,  $S$ , may depend on public policies such as monetary policy. For example, if the price of storage is largely determined by the availability of "safe" assets such as Treasury bonds, for instance, the central bank may intervene through open market operations react to increased demand for storage assets through open market operations by increasing the supply of government bonds. This would have the effect of reducing the slope of the price function  $P$ , and compressing the yield changes in the return to storage that accompany changes in demand. Conversely, holding fixed the supply of Treasury bonds would likely maximize the impact of changes in the demand for storage on its yield.

various frictions that can, at least partly, be resolved through capital requirements.

Specifically, we study some canonical market failures associated with financial intermediaries. The first is the presence of externalities in the recovery value of assets that may arise when many banks fail at once – “fire sales” – and that may depress asset values. The second market failure derives from the introduction of deposit insurance, which provides an implicit or explicit subsidy for raising deposits rather than equity, and tilts banks’ capital structures toward being excessively levered. Next, we study a situation where banks impose an externality on other sectors of the economy. Finally, we introduce a moral hazard (or risk shifting) problem induced by limited liability, which leads banks to exert too little effort and take excessive risk. We first establish that in all these instances capital regulation can help improve the market solution and thus increase social welfare. We then show that depositors are not the primary beneficiaries, and may even be hurt, from such regulation.

For simplicity, in what follows we normalize the cost of holding a deposit,  $\underline{c}$ , to zero in order to simplify the expressions.

## 5.1 Fire sale externalities

So far we have assumed that in the case of bankruptcy the entire project return is dissipated. Consider now a modification where liquidation yields a recovery value equal to a fraction  $h < 1$  of the realized cash flow  $r$  and that such a value depends on how many other banks are in default and thus being liquidated. In other words, losses under bankruptcy are equal to  $(1 - h)r$ , where  $h$  decreases in the number of active banks  $N$ . This captures the idea that the failure of many banks at once depresses asset prices for all banks that are being liquidated – a “fire sale” externality.

As before, each bank chooses the amount of capital  $k$  that maximizes its expected excess returns, as given by (3). The only change to the bank’s problem stems from depositors’ participation constraint, which now incorporates that depositors may receive something in

the event of bankruptcy, and is given by

$$E[U_D] = \frac{1}{R} \int_0^{r_D(1-k)} \frac{hr}{1-k} dr + \frac{1}{R} \int_{r_D(1-k)}^R r_D dr \geq u. \quad (9)$$

The recovery under bankruptcy is reflected in the first term,  $\frac{hr}{1-k}$ , while the second term is the promised repayment, which is made whenever  $r \geq r_D(1-k)$ , as before.

As is typical when there is an externality, banks choose their capital structure disregarding the effect of their choice on the equilibrium asset liquidation value. By contrast, a central planner would choose the amount of capital at each individual bank to maximize total surplus as given by

$$SW = N \frac{1}{R} \int_{r_D(1-k)}^R r dr + M - N - \int_0^{\hat{c}} M \frac{c}{C} dc + N \frac{1}{R} \int_0^{r_D(1-k)} hr dr. \quad (10)$$

Denote as  $k^*$  and  $k^{reg}$  the solutions to the decentralized and the central planner's problems and as  $N^*$  and  $N^{reg}$  the number of banks in the respective cases. We then have the following result.

**Proposition 4.** *In the case of fire sale externalities, we have  $k^{reg} \geq k^*$  and  $N^{reg} \leq N^*$ , with the inequalities strict whenever  $N^{reg} < M$ .*

The proposition establishes that there is a social value to requiring banks to hold more capital than what they are inclined to do as a way of reducing the externalities associated with fire sales in asset prices. By requiring banks to hold more capital, not only do banks face lower bankruptcy costs but, more importantly, the central planner succeeds in reducing the number of banks that will operate and, hence, possibly go bankrupt. This contraction in the number of banks reduces bankruptcy costs through greater recovery values as  $h$  is negatively correlated with  $N$ , and increases social welfare.

We now turn again to the question of who benefits from the increased surplus. Define  $\rho^*$  and  $\rho^{reg}$  as the return to shareholders and  $u^*$  and  $u^{reg}$  as the return to depositors in the market and in the central planner solutions, respectively. We have the following.

**Corollary 4.1.** *When there are “fire sale” externalities, we have  $\rho^{reg} > \rho^*$  for  $N^{reg} < M$ .*

The result establishes that shareholders are again the primary beneficiaries of the increased surplus generated by the introduction of capital regulation. Capital regulation increases total surplus, which again must be allocated between households that become depositors and those that become equity holders. When  $N < M$ , the surplus accrues entirely to shareholders since depositors’ expected return is  $u = 1$  and moreover leverage,  $1 - k$ , decreases. Thus,  $\rho$  goes up, which also implies that a greater amount of equity on aggregate ( $K$ ) is employed in the banking sector despite the reduction in the number of banks. Once  $N = M$ , further increases in capital requirements no longer affect the number of banks that operate and can possibly fail, and so there is no further role for binding capital requirements. As a result, in that region the planner’s solution coincides with the market solution.

To sum up, the presence of fire sale externalities that may arise when banks are liquidated provides a rationale for the introduction of capital requirements. The increased social surplus banks generate leads to a greater use of capital in the banking system, although also to fewer active banks. As before, the increased surplus is entirely appropriated by shareholders through higher returns. Households that choose to remain depositors do not benefit from capital regulation that increases aggregate surplus. Thus, the return differential  $\rho - u$  increases as an effect of regulation. Even more, if we allow the return to storage to be endogenous as in Section 4.1, the reduction in  $N$  along with the increase in  $K$  imply a greater demand for storage and thus a lower return. It follows that since depositors’ returns are set by the return to storage in the region where  $N < M$ , the return differential between equity and deposit holders may increase even more as a result of capital regulation aimed to alleviate the typical fire sale problem often discussed in the literature.

## 5.2 Deposit Insurance

In the analysis above we assumed that deposits are not insured, so that the interest rate on deposits fully reflects the bank’s risk of default. Suppose now instead that deposits are

fully insured so that they become safe assets. As has often been argued (see, e.g. Boot and Greenbaum, 1993; Demirgüç-Kunt and Detragiache, 2002), the introduction of deposit insurance encourages excessive risk taking by reducing banks' incentives to raise capital, thus providing a motivation for capital regulation.

Assume therefore that deposits are fully insured, so that even if the bank goes bankrupt depositors are repaid in full by the deposit insurance fund. For simplicity, we also assume that the insurance is financed from the proceeds of non-distortionary lump sum taxes.<sup>12</sup> In this case, the bank chooses capital  $k$  so as to maximize

$$\max_{k, r_D} \frac{1}{R} \int_{r_D(1-k)}^R (r - r_D(1-k)) dr - \rho k \quad (11)$$

subject to

$$E[U] = r_D = u. \quad (12)$$

The provision of insurance distorts banks' capital structure decisions, as shown in the following result, where we indicate with the superscript  $DI$  the market solution in the case of deposit insurance.

**Proposition 5.** *In the presence of deposit insurance, the unique equilibrium is characterized by  $k^{DI} = 0$ ,  $r_D^{DI} = u^{DI} = R$  and  $N^{DI} = M$ .*

In equilibrium banks hold zero capital and promise depositors the highest possible return. Hence, they go bankrupt with probability one and, as a result, deposit insurance always pays a total of

$$DI = N \frac{1}{R} \int_0^{r_D(1-k)} r_D(1-k) dr = MR \quad (13)$$

to depositors. This occurs because the rate on deposits is insensitive to the probability of bankruptcy and thus independent of the amount of leverage banks choose. It follows that banks have no incentives to raise capital as a way of reducing bankruptcy risk since their

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<sup>12</sup>The results continue to hold if deposit insurance is priced such that it is actuarially fair from an ex-post perspective (see Allen, Carletti, and Marquez, 2015).

cost of borrowing (i.e., deposits) will not reflect this reduction in risk. In the end, banks' extreme leverage choices impose a clear social cost and reduce aggregate welfare.

We next show that minimum capital requirements can reduce the deadweight losses due to bankruptcy and increase welfare. To see this, suppose that a planner imposes a minimum bank capital requirement which is chosen as to maximize welfare,

$$\max_{k^{reg}} SW = \rho K + uD - \int_0^{\rho-u} M \frac{c}{C} dc - DI. \quad (14)$$

subject to constraints (4)-(6) and (8). The primary difference between the planner's objective function and that of the banks is that the planner internalizes the cost of providing deposit insurance. Denote as  $k^{reg}$  and  $r_D^{reg}$  the solutions to the planner's problem in (14). We obtain the following.

**Proposition 6.** *When deposits are insured, we have  $k^{reg} > k^{DI} = 0$  and  $u^{reg} < u^{DI}$ .*

The proposition shows that the optimal regulatory solution entails banks holding a strictly positive amount of capital as a way of reducing the capital structure distortion induced by deposit insurance. As a consequence, given that a sufficiently high return has to be paid to create incentives for some investors to hold capital, under regulation the deposit rate will be lower than the maximum return  $R$ . The result implies that capital regulation leads to redistribution of surplus away from depositors and toward households that become equity holders. In other words, higher capital standards reduce bank bankruptcy risk and thus deposit insurance expenditures, but the fall in  $u$  lowers the return to investors with higher participation costs.

Although capital regulation improves welfare relative to an economy with deposit insurance only, the presence of deposit insurance is by itself inefficient. In other words, the presence of deposit insurance has no justification in our framework given that, as shown in Section 4, the unregulated equilibrium without deposit insurance yields the same allocation as would be chosen by a central planner. To justify the role of deposit insurance, we next show that

the combination of capital regulation and deposit insurance improves upon the completely unregulated case (i.e., when there is neither deposit insurance nor capital regulation) even if neither one on its own increases total surplus.

To see this, we denote again the total surplus in the market solution in the absence of deposit insurance described in Section 3 as  $SW^*$  and recall that this coincides with the central planner's solution, so that there is no real scope for capital regulation when deposits are uninsured. We can now state the following result.

**Corollary 6.1.** *Total surplus in an economy with deposit insurance and (optimal) capital regulation is strictly higher than in the absence of either of them:  $SW^{reg} > SW^*$ .*

The corollary establishes that deposit insurance, coupled with minimum capital standards, increases surplus overall relative to the case where the market is entirely unregulated and there are no guarantees on bank liabilities, including deposits. In other words, while deposit insurance by itself introduces a substantial and costly distortion, capital regulation plays an important role and serves to increase total output above the level reached in an unregulated economy.

### 5.3 Social costs of bank failure

Up to now, we have considered cases where the primary rationale for regulation stems from either a type of coordination failure that leads one bank's decisions to spill over onto other banks through fire sales, or through a distortion in the pricing of claims that leads to an inefficient capital structure. In both of these cases, imposing a capital requirement reduces the friction and leads to greater overall output, either at the bank level or at the industry level, leaving more surplus to be divided between the claimants, depositors and equity holders.

However, not all regulation is geared toward improving efficiency and output within the banking sector. Rather, regulation may be (and often is) motivated by a need to correct a social inefficiency that the banking sector does not internalize. For instance, bank failure

may impose externalities on bank customers, who cannot easily shift to obtain credit from other sources and thus bear losses whenever their main bank goes out of business or contracts lending. Similarly, any public funds that are used either in the provision of deposit insurance for failed banks, or in the management of the bank resolution process, are likely to have a higher social cost. In such cases, capital regulation may improve social welfare, while lowering bank profit.

To study this aspect of regulation which is *external* to the output produced directly by the banking sector, we introduce a simple modification to the model that captures in a reduced form various alternative rationales for regulation. In particular, we assume that, upon bankruptcy, the social cost of failure is  $1 + \psi \geq 1$  for every unit that is lost. In other words, while as before bankruptcy destroys the entire project's return  $r$ , there is an additional social cost of  $\psi r$  that a central planner would internalize.<sup>13</sup> With this addition, the central planner's objective function becomes

$$\max_{k_P} SW = N \frac{1}{R} \int_{r_D(1-k)}^R r dr - N \frac{1}{R} \int_0^{r_D(1-k)} \psi r dr - \int_0^{\hat{c}} M \frac{c}{C} dc + M - N. \quad (15)$$

The only difference between (15) and (7) is the addition of the term  $-N \frac{1}{R} \int_0^{r_D(1-k)} \psi r dr$ , reflecting the higher social costs of bank failure than what is borne privately. We can now state the following result.

**Proposition 7.** *For  $\psi > 0$ , we have  $k^{reg} > k^*$  and  $N^{reg} \leq N^*$ , with the inequality strict whenever  $N^{reg} < M$ . Furthermore,  $\rho^{reg} < \rho^*$ , with  $\rho^{reg} - u^{reg} > \rho^* - u^*$  when  $N = M$ .*

The proposition shows that, when capital regulation is necessary to solve problems stemming from externalities created by bank failure the central planner requires banks to hold a higher level of capital, thus pushing down also the number of active banks in the case of

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<sup>13</sup>An alternative setup is to consider that there is a social cost  $r_D(1-k)\psi$  proportional to the investments made by depositors. The parameter  $\psi$  would then capture the “shadow cost” of government funds that may be required to either repay depositors, or to provide unemployment benefits, etc., which may be claimed by the investors that have suffered losses. This alternative setting is qualitatively similar to the analysis in the text.

partial inclusion. As a result, investors that choose to be shareholders would be inclined to oppose any proposals to increase capital requirements since the constraint imposed would reduce their expected return. The reason is that, as in the benchmark case from Section 4, capital requirements reduce overall industry returns and hurt shareholders. Nevertheless, as in Proposition 3, it is still depositors who may bear the main burden of capital regulation. This is evident from the proposition for the case with  $N = M$ , and could be the case also when  $N < M$  if the return to storage is decreasing in the demand for storage, as described in Section 4.1.

## 5.4 Moral hazard induced by limited liability

The final friction we study stems from limited liability, which is often argued to induce moral hazard or risk shifting, thus creating an inefficiency. To study such a setting, we modify the model slightly to allow the bank or, equivalently, bank shareholders to take a privately costly action  $a$  aimed at increasing project returns. Specifically, we assume that by putting in effort  $a$  at time  $t = \frac{1}{2}$ , the bank can increase the project's return by  $a > 0$  but bears a cost of  $\frac{\eta}{2}a^2$ . This action is taken after the bank has chosen its capital structure and financed its project, so that all variables of interest –  $u$ ,  $\rho$ ,  $r_D$ , and  $k$  – are taken as fixed when choosing  $a$ . If the bank fails, there is a social cost  $1 + \psi$  per unit that is lost, as in Section 5.3, so that default risk imposes an externality.

The model is solved backward. At  $t = \frac{1}{2}$ , the bank maximizes

$$\max_a E[\Pi_B] = \frac{1}{R} \int_{\max\{r_D(1-k), a\}}^{R+a} (r - r_D(1-k)) dr - \rho k - \frac{\eta}{2}a^2, \quad (16)$$

taking  $k$ ,  $\rho$ , and  $r_D$  as given. Assuming that  $r_D(1-k) > a$ ,<sup>14</sup> the FOC with respect to the

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<sup>14</sup>This assumption, which is satisfied for values of  $\eta$  large enough, ensures that the action  $a$  that will be chosen will be small relative to the amount of deposits  $1-k$  so that the bank remains subject to bankruptcy risk. Although the derivation is slightly different, the analysis also extends to the case where, for small enough values of  $\eta$ , the bank chooses  $a$  high enough that default never occurs, i.e., so that  $r_D(1-k) < a$ .

action  $a$  is

$$\frac{1}{R} (R + a - r_D(1 - k) - Ra) - \eta a = 0,$$

from which we obtain

$$a^* = \frac{R - r_D(1 - k)}{\eta R - 1}. \quad (17)$$

Note that the optimal action  $a^*$  is increasing in the amount of capital  $k$  at the bank but remains below the level  $a = \frac{R}{\eta R - 1} > a^*$  that maximizes project returns. This is a standard result of the limited liability effect, given that the bank chooses its action  $a$  taking the deposit rate  $r_D$  and the capital structure  $k$  as given – see, e.g., Allen, Carletti, and Marquez (2011).

Turning to  $t = 0$ , the bank chooses its capital  $k$  and the deposit rate  $r_D$  so as to maximize (16) after substituting the expression for  $a^*$  in (17), subject to the depositors' participation constraint now given by

$$E[U_D] = \frac{1}{R} \int_{r_D(1-k)}^{R+a^*} r_D dr \geq u, \quad (18)$$

and the same constraints (5) and (6) as in the baseline model.

Consider now the case where the amount of capital at each individual bank is chosen by a central planner that maximizes total surplus as given by

$$\begin{aligned} SW &= N \frac{1}{R} \int_{r_D(1-k)}^{R+a} r dr - N \frac{1}{R} \int_a^{r_D(1-k)} \psi r dr - \int_0^{\hat{c}} M \frac{c}{C} dc + M - N - \frac{\eta}{2} a^2 \\ &= N \frac{1}{R} \int_{r_D(1-k)}^{R+a} r dr - N \frac{1}{R} \int_a^{r_D(1-k)} \psi r dr - \frac{1}{2C} M \hat{c}^2 + M - N - \frac{\eta}{2} a^2, \end{aligned} \quad (19)$$

where both  $a$  and  $r_D$  are chosen by the banks and thus are equal to, respectively,  $a^*$  as in (17) and the value of  $r_D$  that solves (18) with equality. The term  $-N \frac{1}{R} \int_a^{r_D(1-k)} \psi r dr$ , as above, represents the social losses associated with bank failure, and now incorporates how the bank's effort decision,  $a$ , affects the probability of failure and hence of a social loss. We have the following result, where we define  $k^*$  and  $k^{reg}$  as the market and the central planner solutions, respectively.

**Proposition 8.** *For the case of a bank moral hazard problem, we have  $k^{reg} > k^*$ . Further-*

more,  $\rho^{reg} < \rho^*$ , with  $\rho^{reg} - u^{reg} > \rho^* - u^*$  when  $N = M$ .

The proposition shows that the central planner prefers a higher level of capital  $k$  at each bank than in the market solution as a way of inducing greater effort and reducing the bank's moral hazard problem. As in the previous section, all things equal capital reduces a bank's probability of default, lowering the expected social cost of bankruptcy. Additionally, the accompanying increase in effort further reduces default risk, lowering the social cost yet further. As a consequence, each bank winds up less levered and faces lower bankruptcy costs. Thus, capital regulation is socially valuable as it leads to binding capital requirements that increase social welfare.

In line with the previous section, the primary rationale for regulation here is to reduce the social cost associated with bank failure and the requirement to raise more capital than is individually optimal implies a loss of surplus to the banking sector. As before, such a loss reduces the return accruing to investors. In the region where  $N < M$ , the return to shareholders decreases, while depositors get no benefit from capital regulation. Moreover, while depositors' return remains constant when the return to storage is exogenously specified, it would actually decrease if the return to storage were endogenous, as discussed in Section 4.1, since capital regulation reduces the number of active banks, thus pushing up the demand for storage. In the region where  $N = M$ , both classes of investors see their return reduced as a consequence of capital regulation, but depositors bear the brunt of the decreased bank surplus.

## 6 Capital requirements and recapitalizations

So far we have considered capital requirements that are in place upon inception, so that our analyses represent comparisons of equilibria. From this perspective, our results can be viewed as comparisons across steady states between a banking market that is unregulated and one that is subject to capital regulation. However, the debate concerning increases in

capital standards relates to not only long term changes in banks' risk profiles, in terms of what should be the new status quo, but also to how stakeholders of existing banks are likely to be affected by changing capital requirements in the short term. In other words, increases in capital requirements represent, practically speaking, recapitalization exercises on existing banks which have both long and short term consequences.

While our framework is not explicitly about the effect of recapitalizations in the short run, it can help shed light on this issue. To see how, consider again our baseline model as in Section 2 and the equilibrium allocation as in Proposition 1. For that case, we may view banks as having an initial capital structure  $k_0 = k^*$ , which may need to be adjusted if a regulator imposes a capital requirement  $k^{reg} > k_0$  in an interim period, such as at time  $t = \frac{1}{2}$ . To the extent that the capital requirement is anticipated by financial institutions, banks in our model would react by complying with the requirement from inception at  $t = 0$ , rather than delaying to implement the requirement at the interim period. The reason is that there is no value in delaying in the model. Anticipating future recapitalization, investors willing to hold equity would initially require a higher return than in the case of no recapitalization. This would avoid potential conflicts between existing and new shareholders in the future, as well as between shareholders and depositors. In other words, the analysis of an anticipated capital requirement to be applied at  $t = \frac{1}{2}$  would mirror our steady state analysis above.

By contrast, an unanticipated recapitalization, or one which is not very likely, raises additional issues. Now, banks would be forced to raise capital at  $t = \frac{1}{2}$ , requiring them to convert some existing depositors into equity holders. This process raises an interesting dynamic that we have not studied in the analysis above. Specifically, existing stakeholders would be required to relinquish part of their claims in order to be able to convince reluctant investors – namely some current depositors – to become shareholders. Importantly, these would be investors who, under the initial pricing of claims, which we denote  $\rho_0$  and  $u_0$ , did not find it optimal to hold equity. In other words, these are investors with higher participation costs and who, consequently, would require an expected return  $\rho_1 > \rho_0$  in order to participate,

setting up a conflict between existing shareholders and new shareholders, as well as possibly between shareholders and depositors. If deposit contracts can be reset at  $t = \frac{1}{2}$ , reflecting the demandable and short term nature of much bank debt, then recontracting will lead deposit rates to come down, reducing the burden that must be borne by the initial shareholders. If, by contrast, the promised deposit payments cannot be renegotiated as would be the case for term contracts, then any recapitalization would lead to a debt overhang problem. In this case, at least part of the benefit from the reduction in bankruptcy risk will accrue to the banks' depositors and will come largely at the expense of existing shareholders. As a consequence, existing shareholders would oppose leverage reductions, perhaps even to a greater extent than what is suggested by the comparison of equilibria we conduct above, in the spirit of the "leverage ratchet effect" analyzed in Admati et al..

## 7 Conclusion

This paper presents an analysis of bank optimal capital structure in a setting where investors may be reluctant to participate in financial markets and have to be induced to do so through the promise of higher returns. The equilibrium amount of market participation in the banking sector is thus endogenous, and depends on the distribution of returns associated with the investment opportunity set available to banks. We use this framework to study the incidence of capital regulation, and shed light on whether requirements geared toward reducing bank failure and absorbing losses that would otherwise accrue to depositors and by extension the deposit insurance fund affect various classes of investors differently.

We show throughout the paper that investors are asymmetrically affected by the introduction of capital regulation. In particular, depositors reap little or no benefit from capital regulation even when this leads to greater surplus in the banking sector, and they tend to bear the brunt of the cost when regulation reduces bank profitability. In this sense, our results suggest that capital regulation may be seen as a channel to widen the return differen-

tials between sophisticated and unsophisticated investors, thus amplifying income inequality, as argued in the recent literature on limited market participation and household wealth accumulation (e.g., Lusardi et al., 2017).

Our focus throughout the paper has been on the impact of capital regulation on the sources of financing for the bank, studying which types of investors primarily bear the brunt, or reap the benefits, of regulation. In other words, we have emphasized the right hand side of the bank's balance sheet. Of course, there are other parties that interact with the bank which are also likely affected by regulation. A salient example is bank borrowers, particularly those that are dependent on their main bank for most financing, and who ultimately may bear part of the cost (or benefit) of regulation through changes in interest rate margins, or through the availability of credit. Likewise, some of the costs and benefits may fall on bank employees. While these aspects are likely important for understanding the full consequences of changes to regulatory requirements, our main findings related to how the wedge between shareholder and depositor returns responds to stricter bank capital standards should remain.

We have limited the analysis to the case where investors' only alternative to storage is to hold deposits or invest in the financial sector through banks. In practice, of course, there are other institutions, including non-financial firms, with needs for funding and who may wish to raise debt or issue equity. Studying how capital regulation for banks affects the equilibrium distribution of investment and the returns to various financial instruments when such firms are included seems like a useful avenue for future research. Nevertheless, we expect that our main finding concerning the incidence of regulation should continue to hold given it is primarily driven by the need to compensate investors more heavily the greater the need for equity financing is.

Finally, in our analysis, we have explicitly sidestepped issues related to the interaction of risk and leverage that are present when systematic risk is priced by assuming risk neutrality. Therefore, the standard results stemming from the work by Modigliani and Miller (1958) are not present, allowing us to isolate the effects stemming from limited market participation

and capital regulation. An interesting issue, however, would be to consider how risk aversion, coupled with the existence of systematic risk, interact with the results we obtain here. At present, investors' reluctance to invest in equities implies in our framework that greater need or desire to issue capital (i.e., equity) by banks requires that investors earn a higher return in order to induce them to participate. By contrast, the usual logic of risk aversion and systematic risk implies that greater leverage makes equity riskier on a systematic basis, and increases its required return. The study of this issue introduces additional complexities to understand the exact source of households' unwillingness to participate in financial markets, and is left for future research.

## A Proofs

**Proof of Proposition 1:** We start by analyzing the bank's problem. Solving (4) with equality for  $k$  gives

$$k = 1 - \frac{r_D - u}{r_D^2}. \quad (20)$$

Substituting this back into (3), differentiating it with respect to  $r_D$  and solving for  $r_D$  gives

$$r_D = \frac{u(2\rho - u)}{\rho}. \quad (21)$$

Substituting again (20) and (21) into (3) again gives

$$E[\Pi_B] = \frac{\rho^2 R}{2u(2\rho - u)} - \rho.$$

Given there is free entry, excess returns are zero in equilibrium. Solving for  $\rho$  yields

$$\rho = \frac{2u^2}{4u - R}. \quad (22)$$

Substituting this expression into (21) and (20) yields

$$r_D = \frac{R}{2} \quad (23)$$

and

$$k = \frac{4u}{R} - 1. \quad (24)$$

Turning now to market clearing, from (1) the equilibrium in the bank capital market commands that  $Nk = \frac{M}{C}(\rho - u)$  for  $0 \leq \rho - u < C$ , where  $\rho - u = \hat{c}$ . Using this, we have

$$Nk = \frac{M}{C}\hat{c}, \quad (25)$$

or, alternatively,

$$\rho - u = \frac{NCk}{M}. \quad (26)$$

Let us now assume that there is full inclusion, i.e., no storage is used and all investors invest in either equity or deposits so that  $N = M$ . Substituting (24) and (22) into (26) and solving for  $u$ , we find

$$u = \frac{R}{4} \left( 1 + \sqrt{\frac{R}{R+8C}} \right) \quad (27)$$

as the unique solution that, once is substituted back into (24), is consistent with a non-negative capital stock of  $k = \sqrt{\frac{R}{R+8C}}$ . Combining these results with (26), we find  $\rho = \frac{R}{4} \left( 1 + \sqrt{\frac{(R+4C)^2}{(R+4C)^2 - 16C^2}} \right)$ .

If the expression in (27) is such that  $u \geq 1 + \underline{c}$ , the full inclusion equilibrium also satisfies the depositor participation constraint and is the unique equilibrium of the game. However, in case (27) is such that  $u < 1 + \underline{c}$ , the full inclusion equilibrium is not feasible because it violates the depositor participation constraint: at full participation returns, depositors would strictly prefer storage over deposits. The unique equilibrium must in this case exhibit partial inclusion, with some funds going to storage, and the return of deposits matching the storage outside option, thus  $u = 1 + \underline{c}$ . Substituting this into (22), (24) and (25), we then obtain the equilibrium variables for  $k, \rho$  and  $N$  as in part i) of the proposition. It is easy to see from the expressions of the returns  $\rho$  and  $u$  that  $\rho > E[r] = \frac{R}{2} > u$  in either full or partial inclusion.

The equilibrium with partial inclusion exists for values of  $R$  below the threshold value such that the total amount of funds used in the banking sector, as given simply by  $N$ , equals  $M$  with  $u = 1 + \underline{c}$ . Thus, equating the expression for  $N$  given in part i) of the proposition to  $M$  and solving it for  $R$  gives the threshold value  $\bar{R}$  as

$$\bar{R} = \frac{(1 + \underline{c}) \left[ (1 + \underline{c}) - 4C + \sqrt{(1 + \underline{c})(1 + \underline{c} + 8C)} \right]}{1 + \underline{c} - C}.$$

This concludes the proof. □

**Proof of Corollary 1.1:** We divide the proofs in the two regions of partial and full inclusion. For  $R \leq \bar{R}$ , it is immediate from the respective derivatives that  $\frac{\partial k}{\partial R} < 0$ ,  $\frac{\partial \rho}{\partial R} > 0$ , and  $\frac{\partial N}{\partial R} > 0$ , while

$$\frac{\partial K}{\partial R} = \frac{2(1 + \underline{c})^2 M}{C(R - 4\underline{c} - 4)^2} > 0,$$

where  $K = Nk$ . Finally, the probability of bankruptcy, as given by  $\Pr(r < r_D(1 - k)) = 1 - \frac{2}{R}$ , is increasing in  $R$ .

For the case of full inclusion where  $R > \bar{R}$ , it can be seen immediately that  $\frac{\partial k}{\partial R} > 0$ . Given  $N$  is constant, this implies  $\frac{\partial K}{\partial R} > 0$ . Moreover, because  $u = \frac{R}{4}(1 + k)$  and  $M(\rho - u)/C = Nk$ , we must have  $\frac{\partial u}{\partial R} > 0$  and  $(\frac{\partial \rho}{\partial R} - \frac{\partial u}{\partial R}) = C \frac{\partial k}{\partial R}$ , hence  $\frac{\partial \rho}{\partial R} > \frac{\partial u}{\partial R} > 0$ . Finally, the probability of bankruptcy, as given now by  $\Pr(r < r_D(1 - k)) = \frac{1}{2} - \frac{1}{2\sqrt{R(R+8C)}}$ , is decreasing in  $R$ .  $\square$

**Proof of Corollary 1.2:** We again divide the proofs in the two regions of partial and full inclusion for each part of the corollary.

Concerning part i) for  $R > \bar{R}$  the expressions for  $k, \rho, u, N$  and thus  $K = Nk$  do not depend on the participation cost  $\underline{c}$ . Thus, they are unaffected by changes in  $\underline{c}$ . As for the region where  $R \leq \bar{R}$ , it is immediate to see from the relative expressions that  $\frac{\partial k}{\partial \underline{c}} > 0$ ,  $\frac{\partial u}{\partial \underline{c}} > 0$  and

$$\frac{\partial \rho}{\partial \underline{c}} = \frac{\partial \rho}{\partial u} \frac{\partial u}{\partial \underline{c}} = -\frac{4(R - 2u)u}{(R - 4u)^2} < 0,$$

given  $r_D = \frac{R}{2} \geq u$  and thus  $R - 2u > 0$ . Furthermore,

$$\frac{\partial K}{\partial \underline{c}} = \frac{\partial K}{\partial u} \frac{\partial u}{\partial \underline{c}} = -\frac{M((R - 2u)^2 + 4u^2)}{C(R - 4u)^2} < 0,$$

from which it follows that  $\frac{\partial N}{\partial \underline{c}} < 0$ .

Concerning part ii), for  $R \leq \bar{R}$ , it is immediate that  $k, \rho$ , and  $u$  are unaffected so that  $\frac{\partial k}{\partial C} = \frac{\partial \rho}{\partial C} = \frac{\partial u}{\partial C} = 0$ , while

$$\frac{\partial N}{\partial C} = -\frac{MRu(R - 2u)}{C^2(R - 4u)^2} < 0.$$

It follows  $\frac{\partial K}{\partial C} = \frac{\partial(Nk)}{\partial C} < 0$ .

For the case  $R > \bar{R}$ ,  $\frac{\partial k}{\partial C} = -\left(\frac{R}{R+8C}\right)^{3/2} < 0$ ,  $\frac{\partial K}{\partial C} < 0$  and, since  $u = \frac{R}{4}(1+k)$ ,  $\frac{\partial u}{\partial C} < 0$  must hold. By contrast,  $\frac{\partial \rho}{\partial C} = \frac{4C}{R}\left(\frac{R}{R+8C}\right)^{3/2} > 0$ .

Regarding part iii) on the comparative statics of  $\bar{R}$ , note that  $\bar{R}$  is obtained from the condition  $N = M$  at  $u = 1 + \underline{c}$ . Defining  $z = 1 + \underline{c}$ , the condition reads

$$M\left(\frac{Rz(R-2z)}{C(R-4z)^2} - 1\right) = 0.$$

Differentiating the expression in brackets with respect to  $z$  yields  $\frac{R^3}{C(R-4z)^3}$ , which must be negative since  $R-4z = R-4u < 0$ . On the other hand, the derivative of the same expression with respect to  $R$  is equal to  $2u^2 \frac{4u-3R}{C(R-4u)^3}$  which must be positive because both its denominator and its numerator are negative:  $4u-3R = 4(u-\frac{3}{4}R) < 4(u-\frac{1}{2}R) = 4(u-r_D) < 0$ . Therefore, an increase in  $\underline{c}$  must result in a reduction in the bracket term of (??), which must be offset by an increase in  $R$  as to keep the equation intact, hence  $\frac{\partial \bar{R}}{\partial \underline{c}} > 0$ . Finally,  $\frac{\partial \bar{R}}{\partial C} > 0$  follows directly from the fact that  $N$  is decreasing in  $C$  but increasing in  $R$ , which means that the defining condition of  $\bar{R}$ ,  $N = M$ , can only be met for greater  $R$  if  $C$  is increased.  $\square$

**Proof of Proposition 2:** To establish the result, it is useful to start with the problem of bank excess return maximization, which can be expressed as

$$\max_k E[\Pi_B] = \frac{1}{R} \int_{r_D(1-k)}^R (r - r_D(1-k)) dr - \rho k$$

subject to the same constraints as above for the social planner's problem. Given that depositors' participation constraint will always be satisfied with equality, we can substitute it into the bank's maximization problem to obtain

$$\max_k E[\Pi_B] = \frac{1}{R} \int_{r_D(1-k)}^R r dr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - (\rho - u) = 0,$$

where  $\frac{\partial r_D}{\partial k}$  is obtained from the constraint. This first order condition must be satisfied in equilibrium whatever the values for  $\rho$  and  $u$ , which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned} \max_{k_P} SW &= N \frac{1}{R} \int_{r_D(1-k)}^R r dr + M - N - \int_0^{\hat{c}} M \frac{c}{C} dc - N \underline{c} \\ &= N \frac{1}{S} \int_{r_D(1-k)}^R r dr + M - N - \frac{1}{2C} M \hat{c}^2 - N \underline{c}, \end{aligned}$$

reflecting the fact that maximizing the return to all stakeholders is equivalent to maximizing aggregate output,  $N \frac{1}{S} \int_{r_D(1-k)}^R r dr$ , since all output is allocated to either depositors or capital holders. The next term,  $M - N$ , represents the funds that are not invested in the banking sector but rather held as storage, to the extent that  $M$  may be strictly greater than  $N$ . The last two terms capture investors' total participation cost. Recall now the market clearing condition (25),  $M \frac{\hat{c}}{C} = kN$ , which implies that  $\hat{c} = \frac{C}{M} kN$ , or that  $\hat{c}^2 = \left( \frac{C}{M} kN \right)^2$ , and which is taken into account by the social planner. We can thus write the maximization problem above as

$$\begin{aligned} \max_{k_P} SW &= N \frac{1}{R} \int_{r_D(1-k)}^R r dr + M - N - \frac{1}{2C} M \left( \frac{C}{M} kN \right)^2 - N \underline{c} \\ &= N \frac{1}{R} \int_{r_D(1-k)}^R r dr + M - N - \frac{1}{2} \frac{C}{M} k^2 N^2 - N \underline{c}. \end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$N \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - \frac{C}{M} k N^2 + \frac{\partial N}{\partial k} \frac{1}{R} \int_{r_D(1-k)}^R r dr - \frac{C}{M} k^2 N \frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} (1+\underline{c}) = 0.$$

Grouping terms obtains

$$N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - \frac{C}{M} k N \right) + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr - \frac{C}{M} k^2 N - (1+\underline{c}) \right) = 0.$$

Since at equilibrium  $\rho - u = \hat{c} = \frac{C}{M} k N$ , we can further rewrite as

$$N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - (\rho - u) \right) + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr - k(\rho - u) - (1+\underline{c}) \right) = 0$$

Now observe that  $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1 + \underline{c}$  since, if  $u > 1 + \underline{c}$ , all funds are being used in the banking sector, so a marginal increase in  $k$  cannot change  $N = M$ .

Consider first the case that  $u > 1 + \underline{c}$ , so that  $\frac{\partial N}{\partial k} = 0$ . We are then left with only

$$N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - (\rho - u) \right) = 0,$$

which is the same condition as must be satisfied for the bank's problem.

Alternatively, suppose that  $u = 1 + \underline{c}$ , which allows us to express the first order condition for the social planner as

$$\begin{aligned} N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - (\rho - (1+\underline{c})) \right) \\ + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr - k\rho - (1+\underline{c})(1-k) \right) = 0. \end{aligned}$$

Note now that the term in the parentheses of the second line is simply equal to  $E[\Pi_B]$  for the case where  $u = 1 + \underline{c}$ , which in equilibrium is equal to zero, with all rents going to

shareholders through  $\rho$ . This leaves the term below, after eliminating the  $N$ :

$$\frac{1}{R} \left( r_D^2 (1 - k) - \frac{\partial r_D}{\partial k} (1 - k)^2 r_D \right) - (\rho - (1 + \underline{c})) = 0,$$

again exactly as in the bank's problem for the case  $u = 1 + \underline{c}$ . Therefore, for all cases the necessary condition to be satisfied is identical to that which maximizes excess returns. Given that the market clearing condition that pins down the equilibrium returns  $u$  and  $\rho$  is the same across both maximization problems, we can conclude that both problems must have the same solution.  $\square$

**Proof of Proposition 3:** To see that  $\frac{d\rho}{dk} < 0$  for both cases (1) and (2) in the proposition, we first show that  $\rho$  is maximal at the market solution and thus  $\frac{d\rho}{dk} < 0$  must hold for  $k^{reg} > k^*$ :

Bank excess returns are given by  $E[\Pi_B] = \frac{1}{R} \int_{\hat{r}_D(1-k)}^R (r - \hat{r}_D(1-k)) dr - \rho k$ , where we use  $\hat{r}_D$  to represent the deposit rate that satisfies depositors' participation constraint, (4). Clearly,  $\hat{r}_D$  is a function of  $k$ . For ease of notation, define  $F(k) = \frac{1}{R} \int_{\hat{r}_D(1-k)}^R (r - \hat{r}_D(1-k)) dr$ , and note that  $F$  is increasing and strictly concave in  $k$ . This allows us to write bank excess returns as  $E[\Pi_B] = F(k) - \rho k$  where, as always,  $\rho$  is chosen such that  $E[\Pi_B] = 0$  in equilibrium. The implicit function theorem tells us that

$$\frac{d\rho}{dk} = -\frac{\frac{\partial E[\Pi_B]}{\partial k}}{\frac{\partial E[\Pi_B]}{\partial \rho}} = -\frac{F'(k) - \rho}{-k} = \frac{F'(k) - \rho}{k}. \quad (28)$$

The first order condition for the market solution is  $F'(k^*) = \rho$ , and from strict concavity of  $F$  it follows that  $F'(k) > \rho$  for all  $k < k^*$ , and  $F'(k) < \rho$  for all  $k > k^*$ . Combined with (28) this proves that  $\rho$  is maximal at  $k = k^*$ .

To establish the rest of the result, consider first the case where  $N < M$ . Market clearing implies

$$\rho - u = Ck \frac{N}{M}.$$

Given the result above that  $\rho$  is lower for larger  $k$ , and the fact that  $u = 1 + \underline{c}$ , the RHS

must decrease to satisfy market clearing. Given that we are considering an increase in  $k$ ,  $N$  must fall, and it must fall enough that even  $kN$  must be lower. Since  $K = kN$ , it follows that  $K^{reg} < K^*$ .

Consider next the case where  $N = M$  and therefore  $u > 1 + \underline{c}$ . For a marginal increase in  $k$ , market clearing simplifies to

$$\rho - u = kC.$$

The RHS must be larger than in the market solution because  $k > k^*$ . Given that  $\rho$  is lower, per the argument above, we must have that  $u$  decreases more than proportionally. That is,  $0 > \frac{d\rho}{dk} > \frac{du}{dk}$ , as desired. Finally,  $K^{reg} = Nk > Nk^* = K^*$ , which concludes the proof.  $\square$

**Proof of Proposition 4:** The proof follows a similar approach as that of Proposition 2. Since at equilibrium (9) will be satisfied with equality, we can rewrite (3) as

$$\max_k E[\Pi_B] = \frac{1}{R} \int_{r_D(1-k)}^R r dr + \frac{1}{R} \int_0^{r_D(1-k)} hr dr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - (\rho - u) + h \frac{1}{R} \left( \frac{\partial r_D}{\partial k} (1-k)^2 r_D - r_D^2 (1-k) \right) = 0,$$

or

$$\frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) (1-h) - (\rho - u) = 0,$$

where  $\frac{\partial r_D}{\partial k}$  is obtained directly from (9). This first order condition must be satisfied in equilibrium whatever the values for  $\rho$  and  $u$ , which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively

be written as

$$\begin{aligned}\max_{k^{FB}} SW &= N \frac{1}{R} \int_{r_D(1-k)}^R r dr - \int_0^{\hat{c}} M \frac{c}{C} dc + M - N + N \frac{1}{R} \int_0^{r_D(1-k)} hr dr \\ &= N \frac{1}{R} \int_{r_D(1-k)}^R r dr - \frac{1}{2} \frac{C}{M} k^2 N^2 + M - N + N \frac{1}{R} \int_0^{r_D(1-k)} hr dr.\end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$\begin{aligned}N \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) - \frac{C}{M} k N^2 + \frac{\partial N}{\partial k} \frac{1}{R} \int_{r_D(1-k)}^R r dr - \frac{C}{M} k^2 N \frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} \\ + N h \frac{1}{R} \left( \frac{\partial r_D}{\partial k} (1-k)^2 r_D - r_D^2 (1-k) \right) + \frac{\partial N}{\partial k} \frac{1}{R} \int_0^{r_D(1-k)} hr dr + N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr = 0.\end{aligned}$$

Grouping terms obtains

$$\begin{aligned}N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) (1-h) - \frac{C}{M} k N \right) \\ + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr + \frac{1}{R} \int_0^{r_D(1-k)} hr dr - \frac{C}{M} k^2 N - 1 \right) + N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr = 0.\end{aligned}$$

Since at equilibrium  $\rho - u = \hat{c} = \frac{C}{M} k N$ , we can further rewrite as

$$\begin{aligned}N \left( \frac{1}{R} \left( r_D^2 (1-k) - \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) (1-h) - (\rho - u) \right) \\ + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr + \frac{1}{R} \int_0^{r_D(1-k)} hr dr - k(\rho - u) - 1 \right) + N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr = 0\end{aligned}$$

Now observe that  $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$  since, if  $u > 1$ , all funds are being used in the banking sector, so a marginal increase in  $k$  cannot change  $N = M$ .

Consider now the market solution for the case that  $u > 1$ , so that  $\frac{\partial N}{\partial k} = 0$ . In that case,  $\frac{\partial h}{\partial k} = 0$  as well, and there is thus no scope for capital regulation.

Alternatively, suppose that under the market solution  $u = 1$ , and again substitute these

values into the social planner's first order condition. This would give us

$$N \left( \frac{1}{R} \left( r_D^2 (1-k) + \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) (1-h) - (\rho - 1) \right) \\ + \frac{\partial N}{\partial k} \left( \frac{1}{R} \int_{r_D(1-k)}^R r dr + \frac{1}{R} \int_0^{r_D(1-k)} hr dr - k\rho - (1-k) \right) + N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr.$$

Under the  $k$ ,  $r_D$ ,  $u$ , and  $\rho$  implied by the market solution, the term in the parentheses,  $\frac{1}{R} \int_{r_D(1-k)}^R r dr + \frac{1}{R} \int_0^{r_D(1-k)} hr dr - k\rho - (1-k)$ , is simply  $E[\Pi_B]$  for the case where  $u = 1$ , which in equilibrium is equal to zero, with all rents going to shareholders through  $\rho$ . This leaves the term below:

$$N \left( \frac{1}{R} \left( r_D^2 (1-k) + \frac{\partial r_D}{\partial k} (1-k)^2 r_D \right) (1-h) - (\rho - 1) \right) + N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr.$$

At the market solution, the first term is equal to zero. This means that, evaluated at the market solution, the derivative with respect to  $k$  of the social planner's objective function is  $N \frac{1}{R} \int_0^{r_D(1-k)} \frac{\partial h}{\partial k} r dr > 0$ . Therefore, the market solution involves banks holding too little capital relative to what a social planner would like and, consequently, too many banks.  $\square$

**Proof of Corollary 4.1:** Since  $k^{reg} > k^*$  but  $N^{reg} < N^*$ , the comparison of  $K^{reg}$  relative to  $K^*$  is at first glance ambiguous. However, since the increased capital requirement leads to less leverage at each bank, bankruptcy costs at each bank are lower. In addition, the reduction in the number of banks to  $N^{reg}$  further reduces bankruptcy costs through the greater recovery:  $h' < 0$ . Thus, a decrease in  $K$  and  $\rho$  would be inconsistent with an increase in  $SW$  since all the surplus goes to either depositors or shareholders. For  $N < M$ , depositors cannot be getting the surplus since  $1 - k$  goes down at each bank, and  $u = 1$  remains fixed. Hence, shareholders must be capturing the increase in surplus, implying that  $\rho$  and  $K$  increase.  $\square$

**Proof of Proposition 5:** Note that the optimization problem in (11) can be rewritten as

$$E[\Pi_B] = \frac{1}{2R} (R - r_D(1-k))^2 - \rho k, \quad (29)$$

which is a convex function of  $k$  (as verified by  $\frac{d^2}{dk^2} E[\Pi_B] = \frac{r_D^2}{R} > 0$ ) with a minimum at  $k = 1 - R \frac{1 - (\rho/r_D)}{r_D}$  which is no less than 1 for  $\rho \geq r_D = u$ . Since  $\rho \geq u$  holds for any  $k \geq 0$ , the unique maximum of  $E[\Pi_B]$  is attained by choosing the lowest capital level in the  $[0, 1]$  interval. Therefore,  $k^{DI} = 0$ , and all  $M$  investors become depositors:  $N^{DI} = M$ . Since bank excess returns (29) in equilibrium are zero, we finally obtain  $u^{DI} = r_D^{DI} = R$ .  $\square$

**Proof of Proposition 6:** The social welfare function can be written as

$$\begin{aligned} SW &= -N(1-k) \frac{1}{R} \int_0^{r_D(1-k)} r_D dr + \rho K + uD - M \int_0^{\rho-u} \frac{c}{C} dc \\ &= \underbrace{-Nr_D(1-k)}_{=Dr_D} \underbrace{\frac{r_D(1-k)}{R}}_{=\text{Pr}(\text{bankr.})} + \rho K + uD - M \frac{(\rho-u)^2}{2C}. \end{aligned}$$

Applying the market clearing condition for capital,  $K = M \frac{\rho-u}{C}$ , to the last term and substituting  $r_D = u$  from the depositor participation constraint we obtain

$$SW = uD(1 - \text{Pr}(\text{bankr.})) + \frac{K}{2}(\rho + u).$$

Taking derivatives wrt  $k$  at the point of the unregulated equilibrium  $k = 0$ , we get

$$\begin{aligned} \frac{d}{dk} SW &= \frac{dD}{dk} \underbrace{u(1 - \text{Pr}(\text{bankr.}))}_{=0} + D \frac{d}{dk} [u(1 - \text{Pr}(\text{bankr.}))] \\ &\quad + \frac{dK}{dk} \underbrace{\left(\frac{\rho+u}{2}\right)}_{\geq 0} + \underbrace{K}_{=0} \frac{d}{dk} \left(\frac{\rho+u}{2}\right). \end{aligned}$$

The term  $D \frac{d}{dk} [u(1 - \text{Pr}(\text{bankr.}))]$  must be positive because

$$\frac{d}{dk} [u(1 - \text{Pr}(\text{bankr.}))] = \frac{du}{dk} \underbrace{(1 - \text{Pr}(\text{bankr.}))}_{=0} + u \underbrace{\left(-\frac{d}{dk} \text{Pr}(\text{bankr.})\right)}_{>0} > 0,$$

and hence social welfare must increase strictly in  $k$  at  $k = 0$ . The planner therefore optimally

chooses a strictly positive amount of  $k$  to maximize welfare.  $\square$

**Proof of Corollary 6.1:** To establish the result, consider again the case where there is no deposit insurance as in Section 3 and denote the allocation for capital as  $k^*$ , which is strictly positive, with corresponding  $u^*$ ,  $\rho^*$ , and  $N^*$ . Suppose now that, when there is deposit insurance, the social planner were to set  $k^{reg} = k^* > 0$ . This capital requirement will be binding since, as shown in Proposition 5 banks prefer to hold zero capital when deposits are insured:  $k^{DI} = 0$ . There are two cases now to consider:

1) Suppose that  $N^* = M$ . In this case, it must be that  $\rho^* - u^* = \rho^{reg} - u^{reg}$  since  $K^* = Mk^* = Mk^{reg} = K^{reg}$ . It follows that either  $\rho^* \geq \rho^{reg}$  or  $\rho^* < \rho^{reg}$  must hold. Suppose first that  $\rho^* \geq \rho^{reg}$  and thus  $u^* \geq u^{reg}$ . This implies  $r_D^* > r_D^{reg}$ , since  $r_D^* > u^* \geq u^{reg} = r_D^{reg}$ , so that bankruptcy costs are strictly lower with deposit insurance. Hence,  $SW^{reg}$  must be higher than  $SW^*$ , leading to a contradiction in  $\rho^* \geq \rho^{reg}$ , so that we must have  $\rho^* < \rho^{reg}$ , and  $u^* < u^{reg}$ .

2) Suppose instead that  $N^* < M$  so that  $u^* = 1$ . It follows that  $u^{reg} \geq u^*$  must hold. Suppose now that  $N^{reg} \leq N^*$ . Then  $u^{reg} = u^* = 1$ , and  $r_D^{reg} < r_D^*$ . But then expected bankruptcy costs must be lower at each bank when deposits are insured and, since  $N^{reg} \leq N^*$ , they must be lower in aggregate. This implies that  $\rho^{reg} > \rho^* \Rightarrow K^{reg} > K^*$ . But since  $k^{reg} = k^*$ , it would then have to be that  $N^{reg} > N^*$ , which is a contradiction to  $N^{reg} \leq N^*$ . Therefore, it must be that  $N^{reg} > N^*$ , and  $SW$  is higher under deposit insurance when  $k^{reg} = k^* > 0$ .

Finally, since this is true for  $k^{reg} = k^*$ , it must a fortiori be true that  $SW$  will be higher when capital regulation is set optimally. Clearly, choosing  $k = 0$  is not optimal, so that optimally we must have  $k^{reg} > 0$ .  $\square$

**Proof of Proposition 7:** We first note that the solution to (7), which does not incorporate an externality, is the same as the market solution. The first order condition to the problem now contains the additional term  $\frac{d}{dk} \left( -N \frac{1}{R} \int_0^{r_D(1-k)} r \psi dr \right)$ , which is clearly positive since

more capital reduces the deposit rate  $r_D$  as well as the bankruptcy threshold directly, thus lowering the social cost of bankruptcy. Therefore,  $k^{reg} > k^*$ .

The argument that  $\rho^{reg} < \rho^*$  follows directly from Proposition 3: since binding capital requirements do not lead to increased output from the banking sector, they must reduce the return to shareholders, exactly as in the prior result. Likewise, the argument that  $\rho^{reg} - u^{reg} > \rho^* - u^*$  when  $N = M$  derives from the fact that  $K = kN$  must increase.  $\square$

**Proof of Proposition 8:** For  $\psi = 0$ , a minor extension of the result from Proposition 2 continues to hold, and the bank's problem is equivalent to that of the central planner, implying that the solutions for  $k$  are the same as well. For  $\psi > 0$ , the social planner's problem differs from the bank's problem by the term

$$-N \frac{1}{R} \int_{a^*}^{r_D(1-k)} r \psi dr,$$

which can be seen to be increasing in  $k$  directly because the upper limit  $r_D(1-k)$  is decreasing in  $k$  (the entire term is negative), and because  $a^*$  is increasing in  $k$ . This leads to  $k^{reg} > k^*$ , as desired. The remainder of the argument follows the exactly same logic as in the previous proposition.  $\square$

## B Household demand for safe assets

In the baseline model presented in the paper, we assume that households view bank deposits as equivalent to storage as long as the deposits earn an equilibrium expected return at least as high as the return to storage. One possible concern, however, is that in the absence of insurance, deposits offered by a bank may be subject to default, and indeed our primary focus is on the parameter region where in equilibrium banks choose risky capital structures. As such, an argument can be made that households that are averse to participating in financial markets may view any instrument, even a bank deposit, as exposing them to risk, and may

demand additional compensation for being exposed to this risk. In other words, households may exhibit a demand for *safe*, or at least safer, assets (Golec and Perotti, 2017; Gorton, 2017).

To study this issue, we extend the model as follows. Consider that a deposit contract can be viewed as an investment with a binary return: with  $\Pr(\text{no default})$ , the investment pays off  $r_D$  per unit of deposit. With complementary probability,  $\Pr(\text{default})$ , it repays zero. We model households' aversion to default as a disutility that they incur whenever the deposit contract defaults, and we denote this disutility as  $\alpha$ . Higher values of  $\alpha$  can therefore be interpreted as households demanding safer assets, and in the limit, as  $\alpha \rightarrow \infty$ , households would only be willing to hold deposits that are perfectly safe. With this, we can write the depositors' expected utility as

$$\begin{aligned} E[U_D] &= \Pr(r \geq r_D(1-k))r_D - \alpha \Pr(r < r_D(1-k)) \\ &= \int_{\max\{R-S, r_D(1-k)\}}^R r_D dr - \alpha \int_{R-S}^{\max\{R-S, r_D(1-k)\}} dr, \end{aligned}$$

with the usual constraint that the expected utility must be at least as high as the opportunity cost:  $E[U_D] \geq u$ . Note, however, that  $u$  is no longer the expected return to the deposit contract, but rather the expected utility for depositors given that they incur a cost  $\alpha$  whenever the deposit contract defaults.

We can now state the bank's maximization problem as maximizing abnormal returns as in (3), subject to the usual constraints that  $E[\Pi_B] \geq 0$ ,  $0 \leq k \leq 1$ , and  $E[U_D] \geq u$ , where  $E[U_D]$  is as defined above. The solution to this problem will determine, as before, the bank's optimal capital structure  $k^*$ , the optimal deposit rate  $r_D^*$ , the equilibrium returns to equity holders and depositors,  $\rho^*$  and  $u^*$ , respectively, and the number of banks that operate in equilibrium,  $N^*$ .

Since our primary goal is to show that the main implications concerning the incidence of capital regulation are unchanged by introducing a demand for safe assets on the side of

households, we omit the characterization of equilibrium and instead focus on the incidence of regulation. Suppose therefore that capital regulation is imposed, which is binding:  $k > k^*$ .

We divide our analysis into two cases:

1. Suppose that  $R$  is sufficiently low that  $N < M$ . In that case,  $E[U_D] = 1$  since the expected utility of depositors must equal the return to storage given that some households,  $M - N > 0$ , choose to hold their wealth in storage rather than in the banking sector. For this region, the results are analogous to those studied in previous sections: a marginal increase in the capital requirement does not change  $E[U_D] = u = 1$ . Therefore, if there is a benefit to regulation, it must accrue to shareholders. Conversely, if there is a cost, it must also be borne by shareholders, so that the number of banks that are formed must decrease.
2. If  $R$  is sufficiently large that  $N = M$ , then we must have that  $E[U_D] = u > 1$  since all households strictly prefer to hold claims against the banking sector. Now, an increase in the capital requirement at each bank requires that  $K = kN$  increase. In order for this to happen,  $\rho - u$  must increase, similar to before. So either depositors benefit less from the tightened capital requirement, or they bear more of the cost, whichever the case may be. This is true even if the increased capital requirement for each bank reduces the bank's default risk and thus, ceteris paribus, increases the expected utility of depositors.

**Conclusion:** The results on the incidence of capital regulation obtained in the baseline model, as well as in our extensions with frictions, extend to the case where households demand safe assets and view risky deposits as somewhat inferior to (safe) storage.  $\square$

## C An alternative: Wealth heterogeneity

In this section, we show that our model is isomorphic to one where, rather than assuming heterogeneity in households' sophistication cost, we allow for households to be heterogeneous

in their initial wealth. Suppose therefore that each household has the same cost  $c$  of becoming sophisticated, but initial wealth  $w_i$  varies across households:  $w_i \sim G(\cdot)$ , with support in  $[\underline{w}, \bar{w}]$ .

As above, banks' need for capital is obtained from maximizing their expected profits,

$$\max_{k, r_D} E[\Pi_B] = \frac{1}{R} \int_{r_D(1-k)}^R (r - (r_D(1-k))) dr - \rho k$$

subject to

$$E[U_D] = \frac{1}{R} \int_{r_D(1-k)}^R r_D dr \geq u.$$

From this, we obtain the usual expression for capital supply by each bank,  $k = \frac{4u}{R} - 1$ . For conciseness, we will restrict development of the model to the case where  $N < M$ , so that  $u = 1$  and  $k = \frac{4}{R} - 1$ . Substituting it into  $E[\Pi_B] = 0$  gives  $\rho = \frac{2u^2}{4u-R}$ , which in the case where  $N < M$  and therefore  $u = 1$ , yields  $\rho = \frac{2}{4-R}$ .

We can now calculate the demand and supply of capital. Assuming households put all their wealth into either deposits/storage or equity (which is indeed optimal given risk neutrality), household  $i$  will choose to hold equity if  $\rho w_i - c > u w_i$ , or, equivalently, if  $\frac{c}{w_i} < \rho - u$ . Rewriting, for given returns  $\rho$  and  $u$ , any household with wealth  $w_i > \hat{w} \equiv \frac{c}{\rho-u}$  will invest in equity, while those with  $w_i < \hat{w}$  will choose to either hold deposits or invest in the storage asset. It follows that the total household demand for bank equity will be equal to

$$K^D = \int_{\frac{c}{\rho-u}}^{\bar{w}} w g(w) dw,$$

where  $g(\cdot)$  is the density function of  $G$ . This implies that the total funds available for investment, as given by the summation of all households' initial wealth, equals  $M = \int_{\underline{w}}^{\bar{w}} w g(w) dw$ , the total pool of funds held in either deposits or storage is  $M - \int_{\frac{c}{\rho-u}}^{\bar{w}} w g(w) dw = M - K^D$ ,

and the total participation costs of households are given by

$$\int_{\frac{c}{\rho-u}}^{\bar{w}} cg(w) dw = c \left( 1 - G \left( \frac{c}{\rho-u} \right) \right).$$

Aggregating across all  $N$  banks, we get the supply of capital  $K^S = N \left( \frac{4}{R} - 1 \right)$ . Equating capital supply to demand then gives

$$K^S = N \left( \frac{4}{R} - 1 \right) = \int_{\frac{c}{\rho-u}}^{\bar{w}} wg(w) dw = K^D.$$

. Substituting into the market clearing condition obtains

$$N \left( \frac{4}{R} - 1 \right) = \int_{\frac{c}{4-R^{-1}}}^{\bar{w}} wg(w) dw,$$

or

$$N = \frac{\int_{\frac{c}{4-R^{-1}}}^{\bar{w}} wg(w) dw}{\left( \frac{4}{R} - 1 \right)}.$$

We can now see that, as in the framework of the baseline model with participation cost heterogeneity, we have qualitatively the same features: (1)  $\rho > u \geq 1$ ; (2)  $\frac{dk}{dR} < 0$  for  $N < M$ ; (3)  $\frac{d\rho}{dR} > 0$  for  $N < M$ ; and (4)  $\frac{dN}{dR} > 0$ . In other words, a model where wealth rather than participation costs is heterogeneous delivers qualitatively the same results as in our framework above. Allowing for heterogeneity in participation costs, however, has the added flexibility of allowing one to view our results through the lens of financial market sophistication or literacy, rather than purely through initial disparities in wealth.

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