Belief Polarization and Investment*

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July 29, 2019

*We thank seminar participants at the UBC Economics Brownbag Series, the 2018 Pacific Northwest Finance Conference and the ASU Sonoran Winter Finance Conference for helpful comments and discussions. Financial support from the Social Science and Humanities Research Council of Canada (SSHRC) is gratefully acknowledged.

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Abstract

We study a canonical real option model where the decisions to acquire and subsequently abandon an asset are made sequentially by a group of agents with heterogeneous beliefs. Casting the decisions as a voting game, we show that inefficient underinvestment can occur when the group mediates disagreement through majority rule: Although each group member would acquire the option if they had the exclusive right to choose the abandonment time, the group votes against acquisition. We show that this inefficiency occurs when group members' beliefs are polarized in that they form two opposing factions with sufficiently large belief discrepancy. Given the pervasive nature of group decisions in the life of organizations and institutions, our theory is particularly relevant for the behavior of venture capitalists, financing syndicates, corporate boards, and committees at large.

Keywords: group decisions, dynamic voting, real investment.

1 Introduction

From a legal standpoint, a corporation is formally an individual decision-making entity. It is not surprising, therefore, that the theory of corporate finance typically models a corporation as a single 'manager' or 'entrepreneur'. Yet, the vast majority of corporate decisions are group decisions where diverse views must be aggregated into a single corporate decision. Boards of directors, venture capital groups, financing syndicates, and public policy committee such as the Board of Governors of the Federal Reserve System, are obvious examples of the ubiquitous nature of group decisions in all aspects of economic life. Despite the pervasiveness of group decisions within corporations, the finance literature has devoted little attention to the possible coordination frictions that can emerge in group decision making.

In this paper we study the implications of group decision making by considering a canonical investment problem, the decisions to invest in, and subsequently abandon or continue, a project. To this problem, extensively studied in the real options literature, we add a key ingredient: the investment and abandonment decisions are undertaken by a group of individuals rather than an "entrepreneur" or a "representative manager." We further assume that group members have heterogeneous beliefs and that the group makes decisions based on a strict-majority voting rule. To focus explicitly on the coordination frictions that this governance rule entails, we explicitly abstract away from modeling informational asymmetries, learning and bargaining among group members.

We borrow from the political-economy literature on voting by treating the investment decision as a dynamic *voting* game. We find that agents' disagreement about the optimal management of the investment opportunity can be an important source of coordination friction within the group. Specifically, we focus on majority voting and identify *decisive voters* for the investment or the abandonment decisions as those (i) whose vote determines the outcome of the group decision and (ii) whose preferences for the decision are the weakest among all members who support that decision. We define *polarization* as the difference in beliefs held by decisive voters in a sense to be made precise later in the paper. We then show that when decisive voters' beliefs are sufficiently *polarized*, the equilibrium of the voting game exhibits *inefficient underinvestment*: a project that is perceived as valuable by each group member acting individually is nevertheless rationally rejected by a group governed by a majority voting rule. This result is not restricted to the majority voting rule but extends to a large class of self-governing voting mechanisms adopted by the group. For ease of exposition, we keep referring throughout the rest of this introduction to the case of strict majority rule. Section 5 extends our analysis to other voting mechanisms.

Our result that polarization can lead to investment inefficiency rests on the following four key assumptions. First, corporate investment management decisions are sequential in nature. Second, group members have heterogeneous beliefs about the cash flows delivered by the investment opportunity. Third, group members settle their disagreement through a voting procedure that is stable over time, and cannot abstain from voting. Fourth, group members cannot solve their disagreement by trading votes.

Under these assumptions, we study the decisions of a group of agents faced with the following real-option problem. At an initial time, the group has the opportunity to pay a fixed cost to invest in a project that generates a continuous stream of cash flow until the group decides to abandon it for a fixed and certain cash liquidation value. Although group members agrees on the volatility of the underlying cash flow, they disagree on the expected growth rate and, as a consequence, on the optimal time to abandon and on the overall value of the project.

Intuitively, polarization and underinvestment due to majority voting emerge because the decisive voter in the investment decision may differ from that of the abandonment decision. When evaluating an investment opportunity, all agents recognize that the project consists of a perpetual growing cash flow plus an abandonment option. Agents with heterogeneous beliefs about cash flow growth rates, disagree on the time to optimally abandon the project: a pessimist is more likely to abandon earlier than an optimist. Because disagreement about abandonment is resolved through majority voting, the decisive voter is the one whose preferred abandonment time is the earliest that can induce a majority support for abandonment. The group will then adopt the optimal abandonment timing of the decisive member. For example, in a group of four members where member 1 is the most pessimist and member 4 is the most optimist, there will be disagreement on when to abandon: Member 1 wants to abandon early and member 4 wants to abandon late. The coalition supporting abandonment is thus built over time with member 1 being the first join and member 4 the last. With majority voting, the smallest coalition that can enable the group to abandon is formed by three members: 1, 2 and 3. Member 3, being the last one to join the coalition, triggers the group to abandon at his optimal abandonment timing. Therefore, at the time when the group abandons, member 3 is marginally indifferent between abandoning and continuing, and has the weakest preferences for abandonment relative to member 1 and member 2. Member 3 is a thus the decisive member for abandonment and the group adopts member 3's optimal abandonment timing.

When evaluating the initial investment opportunity, group members know whether they will be decisive for the future abandonment decision. To the non-decisive members, the abandonment decision of the group appears as suboptimal. If the difference in beliefs between decisive and non-decisive members widens the perceived costs of implementing a suboptimal abandonment policy can be sufficiently high that non-decisive members would optimally vote against investment and possibly lead the group to reject investment. *Underinvestment* occurs then when a project that is attractive to each agent individually, is forgone when agents act collectively within a group. For example, in a group of four members where member 1 is the most pessimist and member 4 is the most optimist, Member 1 (resp. member 4) has the lowest (resp. highest) valuation for the project. The smallest coalition enabling the group to invest is thus formed by members 4, 3, and 2. Member 2, being the most pessimistic within that coalition, has the weakest preferences for investing relative to members 3 and 4. Therefore, member 2 is decisive for the investment decision. If the difference of beliefs between the decisive voter for investment, member 2, and the decisive voter for the abandonment member 3 is wide enough, the optimal abandonment policy of member 3 may be too undesirable to member 2. Member 2 will then vote against investment thus leading the group to reject investment.

We identify three key general properties of underinvestment. First, underinvestment occurs when the decisive voter for the abandonment decision is different from the decisive voter for the investment decision. In our specific context, this switch in the decisive voter happens because the group's decision involves converting an agreed upon value (i.e., the investment amount) into a value that is not agreed upon (i.e., the risky cash flow) which can then be converted back to an agreed upon value (i.e., the liquidation value). More broadly, we view the switch in decisive voters as a rule rather than an exception and it can occur for motives that are not modeled in our framework such us dynamic bargaining, status quo endogeneity or alternative governance rule. Second, underinvestment is more likely to happen when beliefs are more polarized. When the beliefs are similar, although some members of the group see the group's abandonment decision as suboptimal, the perceived sub-optimality of the future abandonment decision is not sufficient to change their overall favorable view of the investment. As polarization increases, the perceived abandonment sub-optimality grows to the point where the decisive voter finds the initial investment unattractive in the first place and leads the group to reject investment.

Third, in our setting disagreement is uni-dimensional, that is, agents disagree about the optimal time to abandon. This implies that the size of the group is critical for the likelihood of investment inefficiency. In particular, with an odd number of members in the group, all with different beliefs, there will be a *single* decisive voter who effectively controls both the investment and abandonment decision and acts therefore as a dictator for the group. This result, which is a direct consequence of the median voter theorem (Black, 1948), is consistent with the widely used rule of thumb in practice that calls for boards with an odd numbers of seats. As we will show however, this result is not robust to minor deviations in voting rules since underinvestment can occur with odd number of group members if some members have veto power. More importantly, as with the median voter theorem, the odd-number result does not hold when the there is more than one dimension to the decision problem since the group behavior is unlikely to be duplicated with an individual dictator when the single-dimension assumption is relaxed.

Our analysis shows further that polarization within a group is an important economic force that interacts with and is not subsumed by the effect of volatility. While an increase in volatility, by increasing the option value of waiting, may accelerate investment and delay abandonment, an increase in polarization may lead to underinvestment, *at any level of volatility*. Due to the coordination friction caused by polarization within a group, projects may not be undertaken in the first place, irrespective of how valuable they may be to each individual group member.

The voting protocol is a key determinant of the investment inefficiency. We focus on majority voting, where underinvestment requires sufficient disagreement between the two agents whose beliefs straddle the median belief of the group. In contrast, with unanimity, underinvestment requires that the range of beliefs in the group is sufficiently dispersed. We also explore the implication of other voting protocols such as super-majority and majority with vetoers and show that even under these more general protocols, underinvestment requires some degree of polarization within the group.

Our work contributes to a relatively new literature on group decision making in financial economics. Garlappi, Giammarino, and Lazrak (2017) also show that inefficient underinvestment can occur in group decision making. However, their mechanism relies on agents learning about the value of a project overtime and on a utilitarian governance rule that aggregates individual beliefs for the purpose of decision making. In contrast, in our setting underinvestment occurs even in the absence of learning and without a utilitarian aggregation rule. The only requirement is that decisions are made sequentially and that disagreements are mediated through voting.

Our paper is also related to a recent political economy literature on dynamic voting that rationalizes the emergence of inefficient gridlock and status quo. Dziuda and Loeper (2016) show that, in a dynamic voting game where preferences evolve over time and where the previous decision becomes the next status quo, inefficiency and deadlock can arise. Building on this intuition, Donaldson, Malenko, and Piacentino (2017) analyze and extend the problem of gridlock within corporate boards who select a CEO.

The key mechanism for obtaining gridlock in these papers is the change of agents' preferences overtime and the endogeneity of the status quo. Agents oppose current proposals even if they are perceived as better than the status quo. If a current proposals is accepted, it becomes the status quo for future decisions. Agents who hope for the arrival of their preferred proposal in the future will therefore block any current proposal. Although our result is similar in that it shows the possibility of an inefficient group choice, our mechanism differs along several dimensions. First we emphasize a new coordination friction due to polarization of beliefs between group members. Second, unlike the previous papers, we do not rely on changing preferences. We therefore show that inefficiencies can also occur naturally in a standard real option framework where preferences do not change over time and where there is no explicit role for the endogenous status quo.

Our paper also contributes to the real-option literature (Brennan and Schwartz, 1985; McDonald and Siegel, 1986; Dixit and Pindyck, 1994), which we extend to allow for group decision making. To the best of our knowledge, ours is the first study to formally analyze the exercise of a real investment option by a group of agents with heterogeneous beliefs. Our analysis provide a general tractable framework to study dynamic investment decisions undertaken collectively by a group of agents. One of the messages of the real option literature is to emphasize the impact of volatility on the timing of option exercise. To that literature, our paper delivers the news insight that polarization interacts with volatility in affecting the dynamics of option exercise. In particular we show that volatility can either mitigate or exacerbate the underinvestment problem, making volatility and group polarization two distinct and independent economic forces.

Our paper is also related to a literature in macroeconomics that investigates the effect of political uncertainty and polarization on aggregate investment. Most notably, Azzimonti (2011) builds a neoclassical growth model in which agents are polarized along a "political dimension," captured by the size of the government. The model rationalize the empirical observation that, for a cross section of countries, greater polarization is typically associated with lower growth and lower private investment. The main mechanism for underinvestment in this model is however different from ours. While our model provides a micro-foundation for individual underinvestment, in Azzimonti (2011), underinvestment occurs due to political uncertainty. In particular, governments are shortsighted and tend to engage in overspending financed by distortionary taxation that crowds out private investment. Azzimonti (2018) provides corroborating evidence of this channel by constructing a partisan conflict index from textual analysis of newspaper articles.

Finally, our paper is also related to the theoretical literature on experimentation by groups. Strulovici (2010) studies the problem of how a group of individual agents with heterogeneous preferences experiments with new opportunities in a two-arm bandit framework. The main result of his analysis is that incentives for experimentations are always weaker when decision power is shared among group members as opposed to concentrated in the hands of a dictator. As in our model, the coordination friction imposed by the necessity to mediate across different beliefs, may give rise to underinvestment. In particular, the concept of "option value", i.e., the ability of a decision maker to react to news, may be muted in a context with multiple decision makers.

Our paper proceeds as follows. In the next section we present the intuition of our theoretical results through a simple two person example of a continuous time, dynamic voting game. Section 3 provides a generalization of these results while Section 4 explores the quantitative implications of our results. We consider how our results would change under other voting rules in Section 5 and conclude the paper in Section 6. Appendix A contain proofs.

2 An illustrative example

Consider a group composed of two agents, P (for pessimist) and O (for optimist), who form a "Decision Making Group". At time zero the group must decide, on behalf of the corporation, whether or not to invest in a project that generates a potentially infinite uncertain stream of cash flows (*the investment decision*). Subsequent to this initial decision, the group must also decide whether and when to abandon the project and reallocate the resources to an alternative use with a fixed redemption value (*the abandonment decision*).

Technology. At time $t = 0$ the group has access to a take-it-or-leave-it project that requires a sunk cost I to be undertaken. Upon investment, the project generates a random cash flow, X_t, per unit of time until it is abandoned, in which case the project is liquidated for a certain cash recovery amount D. Agent P and O disagree on the underlying cash flow dynamics of the project. To formally model disagreement, we assume that each member $n \in \{P, O\}$ of the group believes that the cash flow of the project X_t is governed by a geometric Brownian motion with volatility σ and drift μ_n, that is

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0, \quad \mu_P < \mu_O, \tag{1}$$

where $W_{n,t}$ is a standard Brownian motion under agent n's belief. Group members share equally the sunk cost payment I, the project cash flows X_t, and the value D when the project is liquidated.

Group decisions. The investment and abandonment decisions are each determined by separate votes, each subject to strict majority rule (unanimity in a group with two members). The investment vote takes place at time zero while the abandonment vote can happen at *any* time afterwards. In particular, following the initial investment, any member of the group can, at any time, propose that the project be abandoned, thereby triggering a vote. Abandonment will take place at that point in time only if the majority, i.e., both parties, agree. Each group member anticipates how the subsequent abandonment decision will be made when voting on the initial investment decision.

Group members. We assume that agents are risk-neutral and discount the cash flow X at the risk free rate r. Ignoring the option to abandon at a future time, the subjective value of the individual share of cash flow stream X in (1) at any date t under the belief of member n is

$$\mathbb{E}^n \left[\int_t^\infty e^{-r(s-t)} \frac{X_s}{2} ds \middle| X_0 = x \right] = \frac{1}{2} \frac{x}{r - \mu_n}, \quad n \in \{P, O\}. \tag{2}$$

Since $\mu_P < \mu_O$, to insure that subjective valuations are properly defined for each agent, we impose that $\mu_O < r$. It is important to emphasize that although agents have different beliefs, they have the same information. In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it.

Individual abandonment decisions. After investment is made at time zero, each group member n has the right to propose to the group that the project be abandoned. Before studying the group abandonment decision it is helpful to examine the optimal abandonment decision of each individual group member *as if* they they have full control rights. In this setting, the individual abandonment decision is a simple optimal stopping problem that involves choosing the time τ_n that maximizes the present value of cash flow X_t, given the redemption value $D > 0$. Formally, agent n solves the following stopping time problem

$$J_n^*(x) = \max_{\tau_n} \mathbb{E}^n \left[\int_0^{\tau_n} \frac{X_t}{2} e^{-rt} dt + \frac{D}{2} e^{-r\tau_n} \middle| X_0 = x \right]. \tag{3}$$

To solve agent n's problem, consider the strategy of abandoning at the first time X_t reaches a threshold $\widehat{X}$ from above and denote by $\tau^n_{\widehat{X}}$ the random time in which this happens. Let us denote by $J_n(x, \widehat{X})$ the value of a claim to (i) the stream of cash flow X_t in the interval $(0, \tau^n_{\widehat{X}})$ and (ii) the redemption value D at time $\tau^n_{\widehat{X}}$. The value $J_n(x, \widehat{X})$ can be thought of as the value of (a) receiving X over $(0, \infty)$ plus (b) a perpetual option to exchange at time $\tau^n_{\widehat{X}}$ the value of the cash flow stream X over $(\tau^n_{\widehat{X}}, \infty)$ with the certain cash flow D. Formally, denoting by $\pi^n(x, \widehat{X})$ the value to agent n of a security that pays \$1 at the first time X_t hits $\widehat{X}$, we have that

$$J_n(x, \widehat{X}) = \frac{1}{2} \frac{x}{r - \mu_n} + \frac{1}{2} \left(D - \frac{\widehat{X}}{r - \mu_n} \right) \pi^n(x, \widehat{X}), \quad x > \widehat{X}, \tag{4}$$

where $\pi^n(x, \widehat{X})$ is given by[1]

$$\pi^n(x, \widehat{X}) = \left(\frac{x}{\widehat{X}} \right)^{-m_n}, \quad \text{with} \quad m_n = \frac{(\mu_n - \frac{1}{2}\sigma^2) + \sqrt{(\mu_n - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 0. \tag{5}$$

[1]To prove (5) denote by $f(X_t) \equiv \pi^n(x, \widehat{X}) = \mathbb{E}^n_t[e^{-r(\tau^n_{\widehat{X}} - t)} | X_t = x]$, where $\tau^n_{\widehat{X}} = \inf\{s \geq t : X_s \leq \widehat{X}\}$, for all $t \geq 0$. The process $e^{-rt} f(X_t)_{t \geq 0}$ is a martingale and therefore its drift is null. Using the dynamics (1) and Itô's formula we obtain that f must satisfy the ODE

$$\mu_n x f'(x) + \frac{1}{2}\sigma^2 x^2 f''(x) = r f(x),$$

whose general solution is of the form $f(x) = a x^{\beta_1} + b x^{\beta_2}$, with $\beta_1 < 0$ and $\beta_2 > 1$ solutions of the quadratic equation $\mu_n \beta + \frac{1}{2}\sigma^2 \beta(\beta - 1) - r = 0$. Imposing the boundary conditions $f(\widehat{X}) = 1$ and $\lim_{x \to \infty} f(x) = 0$, we obtain that $b = 0$ and $a = \widehat{X}^{-\beta_1}$, thus yielding $f(x) = \left(\frac{x}{\widehat{X}} \right)^{\beta_1}$. Equation (5) follows by setting $m_n \equiv -\beta_1$.

Hence, the value at time zero to agent n of abandoning at a generic threshold $\widehat{X}$ is given by

$$\begin{aligned} J_n(x,\widehat{X}) &= \mathbb{E}^n\left[\int_0^{\tau^n_{\widehat{X}}} \frac{X_t}{2} e^{-rt} dt + \frac{D}{2} e^{-r\tau_{\widehat{X}}} \middle| X_0 = x\right] \\ &= \begin{cases} \frac{D}{2}, & \text{for } x \leq \widehat{X} \\ \frac{1}{2}\frac{x}{r-\mu_n} + \frac{1}{2}\left(D - \frac{\widehat{X}}{r-\mu_n}\right)\left(\frac{x}{\widehat{X}}\right)^{-m_n}, & \text{for } x > \widehat{X} \end{cases}, \end{aligned} \tag{6}$$

Notice that $J(x,0) = \frac{x}{r-\mu}$. This is because the stopping policy is $\tau_0 = \infty$ almost surely and thus the abandonment option will never by exercised. Also, $J(x,\widehat{X}) = D$ for $x \leq D$, as the abandonment option is immediately exercised.

The solution of the optimal stopping problem (3) can be then obtained by finding the threshold X_n^* that maximizes (6). Doing so, leads to the following threshold

$$X_n^* = \frac{m_n}{m_n+1} D(r-\mu_n), \quad n \in \{P,O\}. \tag{7}$$

Therefore, acting individually, the optimal abandonment time for agent n, is given by the stopping time

$$\tau_n^* = \inf\{t \geq 0 \ : \ X_t \leq X_n^*\}.$$

From Equations (5) and (7), it can be shown that $\mu_P < \mu_O$ implies $m_P < m_O$ and $X_P^* > X_O^*$. That is, the pessimist has a strictly higher abandonment threshold than the optimist. Therefore, when acting individually, P will abandon the project earlier than O, $\tau_P^* < \tau_O^*$.

Figure 1 illustrates the values $J_P(\cdot,\widehat{X})$ and $J_O(\cdot,\widehat{X})$ as a function of the current cash flow x for different values of $\widehat{X}$. As the figure shows, the values defined in (6) are uniformly ranked, that is,

$$J_P(x,\widehat{X}) \leq J_O(x,\widehat{X}), \text{ for all } \widehat{X} > 0$$

with the inequality being strict for $x < \widehat{X}$. Thus

$$J_P(x,X_O^*) < J_P(x,X_P^*) < J_O(x,X_P^*) < J_O(x,X_O^*), \text{ for all } x < X_O^* \tag{8}$$

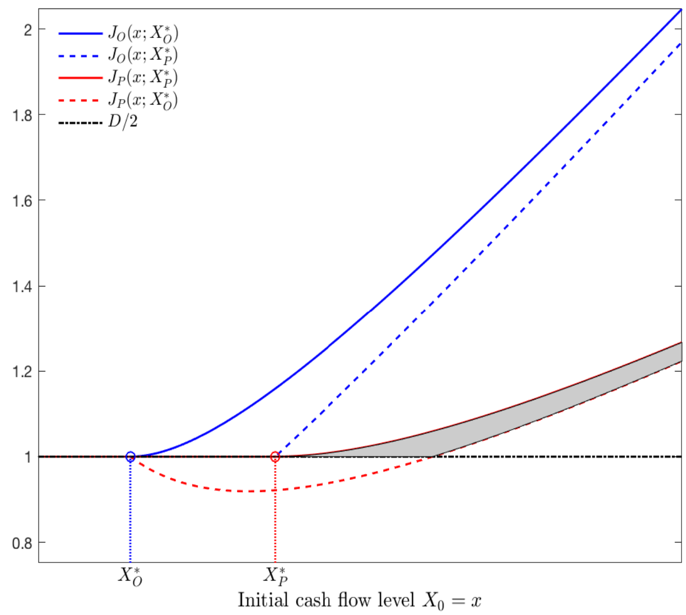

Figure 1: Value Functions

The figure depicts the value functions of $J_P(\cdot, X_P^*)$ (solid red), $J_P(\cdot, X_O^*)$ (dashed red), $J_O(\cdot, X_O^*)$ (solid blue), $J_O(\cdot, X_P^*)$ (dashed blue). Parameter values: $r = 0.05$, $\sigma = 0.3$, $D/2 = 1$, $\mu_P = 0.01$, $\mu_O = 0.03$.

At time $t = 0$ the investment cost is I. This cost is equally borne by both agents. Hence the NPV to agent n of acquiring the license and investing at the threshold $\widehat{X}$ is

$$V_n(x, \widehat{X}) = J_n(x, \widehat{X}) - \frac{I}{2}. \tag{9}$$

Equilibrium investment. Because both the vote to invest at time zero and the vote to abandon requires a majority (i.e., unanimity in a group with two agents), we note that this sequential voting problem may exhibit inefficient investment due to the fact that the decisive voter in the investment decision differs from the decisive voter in the abandonment decision.

To see this, consider the pessimist's voting expectations and assume that the current level of cash flow x is larger than X_P^*. The pessimist understands that, if X declined from the current level to X_P^*, the level at which the pessimist would optimally abandon, the optimist would *not* vote in favor of abandonment. The reason for this is that the optimist knows that, if she rejects abandonment at X_P^* and waits until cash flows fall to the lower level X_O^*, the pessimist will support her abandonment proposal. Although the pessimist would prefer to abandon sooner, he is still better off supporting abandonment as soon as the optimist is willing to do so. That is, the optimist is decisive for the abandonment decision. As a result, the expected abandonment policy of the group must be $\tau_O^* = \inf\{t : X_t \le X_O^*\}$, that is the optimal policy for the optimist.

Anticipating this, the pessimist understands that the time zero value of the subsequent exercise decision will be given by $J_P(x, X_O^*)$, the value of the investment under the pessimist's beliefs and assuming the the group adopts the optimist's preferred abandonment threshold. From the ranking property (8), we have that

$$J_P(x, X_O^*) < J_P(x, X_P^*) < J_O(x, X_O^*) \text{ for all } x < X_O^*. \tag{10}$$

When $I/2 \le J_P(x, X_O^*)$, both P and O will both vote for investment and there will be no underinvestment.

We can see from inequality (10), however, that there are alternative values of the initial investment cost I satisfying $J_P(x, X_O^*) < I/2 \le J_P(x, X_P^*)$. In such a case, although both P and O would invest if they were able to unilaterally pick the subsequent abandonment timing, the group rejects investment. While O votes for investment, P votes against it, and by the strict majority rule, the group rejects investment. In fact, the decision to invest is blocked by P who is decisive for the investment decision. We refer to this situation where investment is blocked by an extreme (pessimistic) view as *underinvestment*. It is important to observe that underinvestment is inefficient: both group members would invest if they had unilateral control of the investment and abandonment decision. The pessimist would abandon at τ_P^* and he would invest because $I/2 \le J_P(x, X_P^*)$. The optimist would abandon at τ_O^* and she would invest because $I/2 \le J_O(x, X_O^*)$.

A compromise timing would facilitate investment but majority voting does not allow compromise. Any value I for which

$$J_P(x, X_O^*) < \frac{I}{2} \le J_P(x, X_P^*) < J_O(x, X_O^*), \tag{11}$$

would give rise to underinvestment.

The shaded area in Figure 1 represents the underinvestment region. For a given level of cash flow value x, there is an interval of values for the investment cost $[\underline{I}(x), \overline{I}(x)]$ for which underinvestment occurs. These bounds are given by

$$\frac{\underline{I}(x)}{2} = J_P(x, X_O^*), \quad \frac{\overline{I}(x)}{2} = J_P(x, X_P^*).$$

When $I \leq \underline{I}(x)$, the investment cost is so low that the pessimist will agree to invest even if the optimist determines the abandonment decision. When $I > \overline{I}(x)$, the investment cost is so high that the pessimist will reject investment. That decision is, however, efficient since the pessimist would not invest even is he had full control of the abandonment decision as $I/2 > J_P(x, X_P^*)$.

Discussion. Both group members would benefit from giving all the group decision power to investor P. Under this alternative governance arrangement, the group will abandon at the threshold policy X_P^*, resulting in a perceived value $J_P(x, X_P^*)$ to investor P and $J_O(x, X_P^*)$ to investor O. Since

$$\frac{I}{2} \leq J_P(x, X_P^*) < J_O(x, X_P^*),$$

letting investor P rule the group for the abandonment decision Pareto improves the equilibrium outcome of a strict majority decision rule. To see this, note that under strict majority, underinvestment occurs when condition (11) is satisfied, and the investment is not undertaken resulting in a value of $I/2$ to both P and O. Hence giving decision power to P will eliminate underinvestment and generates a payoff that is greater than $I/2$ to *both* agents, a Pareto improvement.

At a broader level, the inefficient investment that we describe in this example requires a change of the identity of the decisive voters for the investment decision and the abandonment decision. Being more pessimistic, it is agent P who is more reluctant to invest and thus he is decisive for the investment decision. On the other hand, the more optimistic O is more reluctant to abandon and therefore she is decisive for the abandonment decision. Since (i) P

is decisive in the investment decision, (ii) he will lose his status of decisive voter in the future abandonment vote, and (iii) the beliefs of P and O are sufficiently different (polarized), P will vote against investment. When there is no disagreement between group members, both investors are decisive as they are identical and, as a result, underinvestment cannot occur.

Similarly, if the decision rule is dictatorial, that is, when all the voting power is in the hand of agent O (resp. P), the decisive voter remains agent O (resp. P) for both the investment and abandonment decision. There is therefore no threat of change of identity of the decisive voter and as a result underinvestment does not occur.

It is also important to notice that the change of identity of the decisive voter is only possible because we have a switch in the nature of the alternatives that group members vote for. The first vote (investment) is on a proposal to convert an agreed upon quantity (the investment amount) into an asset about which there are differences in beliefs (the risky cash flow stream). Since the pessimistic investor P places a lower value on the risky cash flow, he is decisive for the investment vote. The second vote (abandonment) is a proposal to convert the risky cash flow, about which there are differences in beliefs, to a risk free cash flow with an agreed upon value. As the optimistic investor O is more reluctant to abandon the risky cash flow, she will be decisive for the abandonment vote. The mechanism that we describe would not occur for example in a real option model when an investment decision is followed by an expansion decision since the conversion is from a risky cash flow to a larger risky cash flow, both of which are subject to different beliefs. In this setting, P remains the decisive voter for both the investment and the abandonment decisions and as a result there is no change in the identity of the decisive voter.

Perhaps less intuitive is the fact that, as we formally prove in the next section, inefficient underinvestment cannot occur with a group that includes *three* or, in general, an odd number of members and is governed by a strict majority rule. In this case, notice that the group holding the median belief is the decisive voter for the abandonment timing decision as his preferred timing is the earliest time that can generate a majority vote for abandoning.

It turns out that the median belief member is also decisive for the investment decision and consequently underinvestment cannot occur with a group with an odd number of voters.

If the median belief member votes for investment, then all members who are more optimistic vote for investment. The coalition voting for investment forms a majority and enable the group to invest. If the median belief member votes against investment, group members who are more pessimistic than him also vote against investment and therefore the group does not invest. Although the group does not invest, this is not an inefficient outcome because the median voter is decisive in both the investment and abandonment decision. Therefore not investing is optimal for the median voter.

3 A general model of investment decision by groups

3.1 The framework

A group $\mathcal{N} = \{1, \ldots, n, \ldots, N\}$ of N infinitively lived members collectively decides whether to invest in a project and subsequently when to abandon the project and receive a liquidation value D.

Technology and information. Investment is irreversible and the project cash flows X follows a geometric Brownian motion. Group members agree on the volatility of the cash flow process but disagree on its expected growth. Specifically, member 1 perceives the cash flow process as

$$dX_t = \mu_1 X_t + \sigma X_t dW_{1,t}, \quad X_0 = x > 0$$

where $\mu_1 \geq 0$ and $\sigma > 0$, and where $(W_{1,t})_{t\geq 0}$ is a standard Brownian motion under a given probability $\mathbb{P}_1$ representing member 1's' belief. Under the probability measure $\mathbb{P}_1$, the cash flow process is a geometric Brownian motion with drift μ_1.

Member $n = 2, \ldots, N$ disagree with member 1 on the drift of the value process. Their perceived drift is μ_n, where, without loss of generality, we assume that

$$\mu_1 \leq \mu_2 \leq \ldots \leq \mu_N, \tag{12}$$

that is, group members are ranked based on how optimistic they are with regard to the expected growth of the cash flow process X_t. Member n's belief, for each $n = 2, \ldots, N$, is given by a probability measure $\mathbb{P}_n$, equivalent to $\mathbb{P}_1$ under which the cash flow process satisfies[2]

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0$$

where W^n is a standard Brownian motion under the probability $\mathbb{P}_n$ and given by

$$W_{n,t} = W_{1,t} - \eta_n\, t,$$

where $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

It is important to emphasize that although group members have different beliefs, they have the same information.[3] In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it. Furthermore, note that disagreement about the drift is only possible if $\sigma > 0$. If $\sigma = 0$ then $X_t = xe^{\mu t}$ and therefore group members cannot observe X_t and x, as we assumed, and at the same time disagree on the value of μ.

Agents' preferences, payoffs and strategies. Agents are risk neutral and discount future cash flows at the risk free rate r. We assume that

Assumption 1. *Agents are sufficiently impatient, that is,*

$$r > \mu_N.$$

Given condition (12), this assumption implies that for each investor, there are states of the world where it is optimal to exercise the abandonment option. Without this assumption, the patient investors will always oppose exercise of the abandonment option.

The group make two consecutive decisions. The first decision is whether to pay a cost I and invest or not in the project at time 0. If the group agree to invest, at any future

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[2]By the Girsanov Theorem (Karatzas and Shreve, 2012), the probability $\mathbb{P}_n$ is formally defined as $\left(\frac{d\mathbb{P}_n}{d\mathbb{P}_1}\right)_{\mathcal{F}_t} = \xi_{n,t}$, where the process $\xi_{n,t}$ is the non-negative $(\mathbb{P}_1, \mathbb{F})$-martingale defined by $d\xi_{n,t} = -\eta_n \xi_{n,t} dW_{1,t}, \quad \xi_{n,0} = 1$, with $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

[3]This is mathematically captured by the fact that the Brownian filtration generated by the Brownian motion $W_{n,t}$ for each $n = 1, \ldots, N$ is identical to the natural filtration of the commonly observed process X_t.

time $t > 0$ any group member can propose to abandon the project in exchange for a fix cash flow D, resulting in a payoff of D/N to each group member. The group abandons if the strict majority of its member accept the proposal. If no member propose to abandon or if a proposal to abandon is rejected by the group, the abandonment option remains available for exercise at future dates.[4]

The timing strategies that investors follow when they propose abandonment are assumed to be of the threshold type, that is,

Assumption 2. *Group members submit a proposal to the group when the observed value of the cash flow is smaller than a threshold value* $\widehat{X} \in [0, \infty]$. *The proposal date is the stopping time* $\tau_{\widehat{X}}$ *at which the threshold* $\widehat{X}$ *is first reached, that is,*

$$\tau_{\widehat{X}} = \inf\{t \geq 0 : X_t \leq \widehat{X}\}.$$

This class of timing decisions is standard in the real option literature and natural, given the stationary structure of our complete information game. This formulation greatly simplifies our equilibrium voting outcome and allows us to discuss our economic insight in a transparent way. If group expects to accept a proposal at date $\tau_{\widehat{X}}$ the value to investor $n = 1, \ldots, N$ defined as

$$J_n(x, \widehat{X}) = \frac{1}{N}\mathbb{E}^n \left[\int_0^{\tau_{\widehat{X}}} X_t e^{-rt} dt + D e^{-r\tau_{\widehat{X}}} \right]$$

is given by,

$$J_n(x, \widehat{X}) = \begin{cases} \frac{1}{N} D, & \text{for } x \leq \widehat{X} \\ \frac{1}{N} \left[\frac{x}{r-\mu_n} + \left[D - \frac{\widehat{X}}{r-\mu_n} \right] \left(\frac{x}{\widehat{X}} \right)^{-m_n} \right], & \text{for } x > \widehat{X} \end{cases}, \tag{13}$$

[4]Note that it is possible to engage in "take-it-or-leave it options," that is, group members that propose a vote to abandon, could potentially threat the group to withdraw his support in the future if the proposal does not go through. However, this threat is not credible because it amounts to sub-optimally give up an option to abandon at future dates.

with m_n defined in equation (5). It can be shown that the values J_n are ordered, that is,

$$J_n(x, \widehat{X}) \leq J_{n+1}(x, \widehat{X}) \text{ for all } (x, \widehat{X}) \in (0, \infty)^2, \; n = 1, .., N-1. \tag{14}$$

If we rename member P (resp. O) from Section 2 member 1 (resp. 2), Figure 1 where the dashed red curve represents $J_1(\cdot, X_2^*)$ and the solid red curve represents $J_2(\cdot, X_2^*)$ illustrate inequality (14). The figure shows that the graph of the two functions are uniformly weakly ordered and thus inequality (14) is satisfied for $n = 1$, $\widehat{X} = X_2^*$, and for any $x > 0$.

It is also possible to prove that the function $J_n(x, \widehat{X})$ is maximized at

$$J_n(x, X_n^*) = \sup_{\widehat{X}} J_n(x, \widehat{X}),$$

where X_n^* is the bliss point for group member n defined in equation (7). Because pessimistic members find it optimal to abandon at higher level of the cash flow levels, the sequence of bliss points satisfies

$$X_1^* \geq \ldots \geq X_N^*. \tag{15}$$

In our baseline analysis we assume that at the current level of the project value x, no group member finds it optimal to propose an immediate exercise of the abandonment option. This amounts to insuring that each agent faces a non-trivial investment decision ruling out situation where the investment cost is so low that some investors find it optimal to buy a project and liquidate it immediately, an unlikely equilibrium opportunity.

Assumption 3. *At the current level $X_0 = x$ of the project value of the abandonment option is out of the money for all investors, that is,*

$$x > X_1^*.$$

Lastly, we assume that investors' preferences and the group decision rules (described below) are common knowledge and that investors anticipate the future abandonment decisions of the group when voting both in the initial investment stage and in the abandonment stage. Individual voting of any group member cannot thus alter the belief of other group members.

Consequently, group members vote *sincerely*, that is, they always vote for the alternative that gives the highest payoff given their expectation about future abandonment policies. This implies that, if the group is expected to abandon at time $\tau_{\widehat{X}}$, the value to investor n of the project at time zero is given by

$$V_n(x, \widehat{X}) = J_n(x, \widehat{X}) - \frac{I}{N}. \tag{16}$$

At time zero each group member n decides whether to vote for investment, if $V_n(x, \widehat{X}) \geq 0$ or reject it, otherwise.[5]

3.2 Decisions rules, equilibrium and inefficient underinvestment

The group makes decisions with simple majority voting at both the investment and abandonment stage. Formally,

Assumption 4. *The group makes decisions based on a simple majority rule, that is, a decision is undertaken by the group if at least k members accept it, with k the smallest integer larger than $N/2$.*

An equilibrium outcome for the voting game consists of a pair $s = (\lambda, \widehat{X}) \in \{0, 1\} \times [0, \infty]$, where $\lambda = 1$ ($\lambda = 0$) denotes that the group undertakes (rejects) investment, and the variable $\widehat{X}$ denotes the threshold at which the group abandons the project. Equivalently, abandonment takes place at the stopping time $\tau_{\widehat{X}}$. We adopt the convention that $\widehat{X} = \infty$ means that the group never abandons. The strategy of forgoing the investment is denoted by $s_{\mathcal{N}} \equiv (0, \infty)$. We denote by $\mathcal{S} \equiv \{0, 1\} \times [0, \infty]$ the set of possible strategies.

In order to state the equilibrium conditions, it is convenient to define the set of *feasible* thresholds $\mathcal{X}_f$, as the set of abandonment thresholds $\widehat{X}$ such that a proposal to invest at time $\tau_{\widehat{X}}$ generates a majority vote.

The following definition describes the equilibrium in our dynamic voting game:

[5] The weak inequality breaks down the tie in favor of investment: when indifferent between investing and not investing, group members vote for investment. This is without loss of generality as the result on underinvestment also hold when we break down the tie in favor of not investing.

Definition 1. *The strategy* $s = (1, \widehat{X})$ *is an equilibrium outcome of the dynamic voting game if and only if*

1. *The abandonment threshold* $\widehat{X}$ *is the largest feasible threshold:* $\widehat{X} = \sup_X\{X : X \in \mathcal{X}_f\}$.
2. *Conditional on* $\widehat{X} \in \mathcal{X}_f$ *being implemented in the second stage vote, the investment alternative* $\lambda = 1$ *generates a majority vote of the group against* $s_{\mathcal{N}}$.

The strategy $s = s_{\mathcal{N}} \equiv (0, \infty)$ *is an equilibrium outcome of the dynamic voting game if none of the strategies* $(1, \widehat{X})$ *with* $\widehat{X} \in \mathcal{X}_f$ *generates majority support of the group in an election against* $s_{\mathcal{N}}$.

In our voting game there is no private information. The beliefs of group members are given and cannot be altered by making proposals. This implies that the success of a proposal is known in advance and there is not value in making a proposal that fails. We therefore assume that no proposal is made unless it is expected to generate a majority vote. We additionally assume that the equilibrium investment threshold is the largest element of the set of feasible strategies. This means that whoever moves first with a proposal that can generates majority will determine the abandonment policy of the group.

In equilibrium, all group members vote to maximize the value of their project share while forming correct expectations about other group members' future voting policies. The equilibrium we describe is therefore subgame perfect. Specifically, at the investment stage investors internalize their expectation on the abandonment policy in their decision of how to vote. For strategic motives, investors could vote against investment even if investment is the alternative they would favor when acting individually.

3.3 Analysis of the voting game

We now solve for the equilibrium of the dynamic voting game and the resulting behavior of the group. To state our results, it is convenient to identify two particular group members: the *left pivot*, n_L, and the *right pivot*, n_R defined as follows.

Definition 2. *When the number of group members N is odd, the left pivot and the right pivot are the same individual*

$$n_L = n_R = \frac{N+1}{2}$$

When the number of group members is even, the left and right pivot are defined by

$$n_L = \frac{N}{2}, \quad n_R = \frac{N}{2} + 1.$$

The labeling of "left" and "right" pivot refers to the ranking of their assessment of the cash flow growth rate, $\mu_{n_L} < \mu_{n_R}$.[6] As we will explain shortly, the right pivot n_R acts as a decisive voter for the abandonment decision whereas the left pivot n_R will be the decisive voter for the investment decision. The next lemma characterizes the set of feasible investment thresholds

Lemma 1. *The set of feasible investment is given by*

$$\mathcal{X}_f = [0, X^*_{n_R}].$$

The lemma implies that the bliss point threshold for the right pivot n_R, given by $X^*_{n_R}$ is the first threshold that generates a majority vote. The coalition supporting abandonment at the proposal time $\tau_{X^*_{n_R}}$ is given by $\{1, \ldots, n_R\}$. The lemma also implies that a candidate equilibrium should be of the form $s = (1, X^*_{n_R})$ or of the form $s = s_{\mathcal{N}}$. The next proposition characterize the equilibrium outcome of the dynamic voting game.

Proposition 1. *The equilibrium outcome of the voting game is unique and consists of*

1. *Not investing, i.e., $s = s_{\mathcal{N}}$, if*

$$J_{n_L}(x, X^*_{n_R}) < \frac{I}{N} \tag{17}$$

2. *Investing and abandoning at the threshold $X^*_{n_R}$, i.e., $s = (1, X^*_{n_R})$, when*

$$J_{n_L}(x, X^*_{n_R}) \geq \frac{I}{N}. \tag{18}$$

[6] Note that optimal abandonment threshold $X^*_{n_L}$ of the left pivot, is located to the *right* of the optimal abandonment threshold $X^*_{n_r}$ of the right pivot.

The proposition implies that the preference of the left pivot is binding for the decision to invest, that is, member n_L is decisive for the investment decision. When condition (17) holds, the coalition formed by group members from the set $\{1, \ldots, n_L\}$ vote against investment. This is due to the inequalities

$$J_1(x, X^*_{n_R}) \leq J_2(x, X^*_{n_R}) \leq \cdots \leq J_{n_L}(x, X^*_{n_R}) < \frac{I}{N}$$

and, by definition of the left pivot in a majority vote, the coalition is able to block investment. When Condition (18) holds, the coalition formed by group members from the set $\{n_L, n_R, \ldots, N\}$ vote for investment. This is an implication of the inequalities

$$\frac{I}{N} \leq J_{n_L}(x, X^*_{n_R}) \leq J_{n_R}(x, X^*_{n_R}) \leq \cdots \leq J_N(x, X^*_{n_R})$$

and, by definition of the pivots, the coalition $\{n_L, n_R, \ldots, N\}$ forms a majority supporting investment and leads the group to invest.

The proposition also shows that the equilibrium outcome for a group with N investors is identical to the equilibrium outcome for a group formed with only members n_L and n_R, revealing the importance of the illustrative example from Section 2. In fact our results could also apply for group governed with alternative rules than majority as long as they give rise to two *influential* members, that is, the group with N members behavior can be duplicated with a group that only includes the two influential members.

3.4 Underinvestment

In this section, we show that the voting equilibrium may generate inefficient underinvestment. Formally,

Definition 3. *Inefficient underinvestment occurs when the equilibrium outcome is not to invest (s_N) despite the fact that all investors prefer to invest if they were acting individually, that is,*

$$\frac{I}{N} \leq J_n(x, X^*_n) \quad \textit{for } n = 1, \ldots, N. \tag{19}$$

The decision to not invest is inefficient under condition (19). If the investors could commit to the most pessimistic abandonment threshold policy $\widehat{X} = X_1^*$, investing would generate a unanimous vote. The inequality (19) and condition (14) imply

$$0 \leq J_1(x, X_1^*) - \frac{I}{N} \leq J_2(x, X_1^*) - \frac{I}{N} \leq \ldots \leq J_N(x, X_1^*) - \frac{I}{N}.$$

Therefore, each investor get a positive payoff from investing and they will all vote for investing against $s_{\mathcal{N}}$ provided they can commit to abandon at the threshold X_1^*.

Notice that inefficient underinvestment cannot occur if the all the voting power is attributed to a single investor (a *dictator*) for both elections. In that case, the group will follow the dictator's preferred investment timing policy and condition (19) implies that the dictator will undertake the investment at date $t = 0$. Thus inefficient underinvestment cannot occur when the group is ruled by a self interested dictator.

Similarly, inefficient underinvestment cannot occur with a utilitarian planner who perceive a value process following a geometric Brownian motion with the average drift

$$\mu_{\text{pool}} = \frac{\sum_{n=1}^{N} \lambda_n \mu_n}{N}$$

with weights satisfying $\lambda_n \geq 0$ and

$$\sum_{n=1}^{N} \lambda_n = 1.$$

Such a planner will adopt the investment threshold X^*_{pool} defined by equation (7) and (5) with μ_{pool} replacing μ_n. Because $\mu_1 \leq \mu_{\text{pool}}$, we must have

$$J_1(x, X_1^*) \leq J_{\text{pool}}(x, X^*_{\text{pool}}).$$

This inequality implies that if all individual investors would invest if they were the sole owner of the project, that is, condition (19) holds, then the planner also decides to invest. Hence underinvestment does not occur when such a utilitarian planner is in charge of the group decisions at both round.[7]

The next proposition provides a characterization of inefficient underinvestment.

Proposition 2. *If the number of group member is odd, inefficient underinvestment cannot occur in equilibrium. If the number of group members is even, inefficient underinvestment occurs if and only if*

$$J_{n_L}(x, X^*_{n_R}) < \frac{I}{N} \leq J_1(x, X^*_1). \tag{20}$$

Underinvestment takes place because the investors $\{1, \ldots, n_L\}$ dislike the abandonment policy represented by the threshold $X^*_{n_R}$. Inefficient underinvestment is an implication of strategic voting in the investment decision: investors anticipate the abandonment threshold taken by the group and when some of them dislike the anticipated threshold, they vote against investment. When at least half the group members dislike the anticipated threshold, they preclude those who favor investing, that is the investors $\{n_R, \ldots, N\}$, to constitute a majority, leading the group to reject investment.

Proposition 2 also shows that the underinvestment problem we highlight does not occur with an odd number of voters. The reason is that the left pivot coincides then with the right pivot giving rise to a single median voter $n_m = n_R = n_L$. If investment is undertaken, the group follows the median voter abandonment threshold $X^*_{n_m}$. Even though many pessimistic investors may dislike the anticipated abandonment strategy $X^*_{n_m}$, they will not be able to block investing since those in favor of investment, the investors $\{n_m, \ldots, N\}$, form a majority.

Another useful intuition to understand the impossibility of inefficient underinvestment with an odd number of investors is to realize that the median voter n_m acts as a dictator of the group. This observation is reminiscent of the median voter theorem from the canonical spatial voting models. In fact, for both elections, if an alternative is taken by the group it must be that the median voter adhere to that alternative: With ranked single peaked preferences and an odd number of voters, any majority should include the median voter.

[7]Note however that the planner outcome may not Pareto dominate the equilibrium outcome. Member 1 for example may not be satisfied with the planner solution when $J_1(x, X^*_{\text{pool}}) < I/N$. Thus not all members would agree to delegate the group's decision power to a utilitarian planner.

Therefore, the group choice will not change if the median voter was the dictator of the group. We know however that inefficient underinvestment cannot occur if the group is ruled by a dictator. The reason is that if all group members would invest if they act individually, the dictator will not reject investment because the dictator is by definition acting individually.

Notice that in the underinvestment condition (20) only three agents matter: the two pivots, n_L and n_L, and the most pessimistic agent 1. In particular, the equilibrium outcome would not change if the optimistic agent were to become even more optimistic. This result is due to the fact that voting does not allow to express the intensity of preferences. With a majority vote where each person can only cast one binary vote, the group ignores the intensity of preferences over alternatives.

4 Quantitative implications

4.1 Impact of polarization

Figure 2 describes the underinvestment conditions for a group of $N = 2$ agent. The left pivot is agent P (pessimist) and the right pivot is agent O (optimist). It follows that for a two-agent group, condition (20) will always be satisfied as long as the two agents have different beliefs about the growth rate of X_t. The light gray-shaded area in between the curves $J_P(x; X_O^*)$ and $J_P(x, X_P^*)$ represents the combination of current cash flow x and investment cost I for which there is underinvestment. Intuitively, for any given level of x, $I/2$ falls within this region then agent P, aware that the optimal abandonment threshold is X_O^*, will find unattractive to invest in the project and therefore will vote against it. Because investment requires strict majority, the project will not be funded, even if, under the optimal abandonment policies, each agent would agree to fund it.[8]

Figure 2 also illustrate the effect of polarization of beliefs on underinvestment. We consider the case in which agent O entertains an even more optimistic forecast of cash flow growth. This is reflected in a lower abandonment threshold $X_O^{*\prime}$. As the figure shows, such

[8]Note that we do not include values of investment falling below the abandonment value D. These values are not economically relevant because they would be generating a "money pump": if $I < D$ then the optimal strategy would be to invest and immediately abandon.

a lower threshold further lowers the value that agent P attaches to a project. Overall the effect is to enlarge the underinvestment region by adding the darker-shaded area.

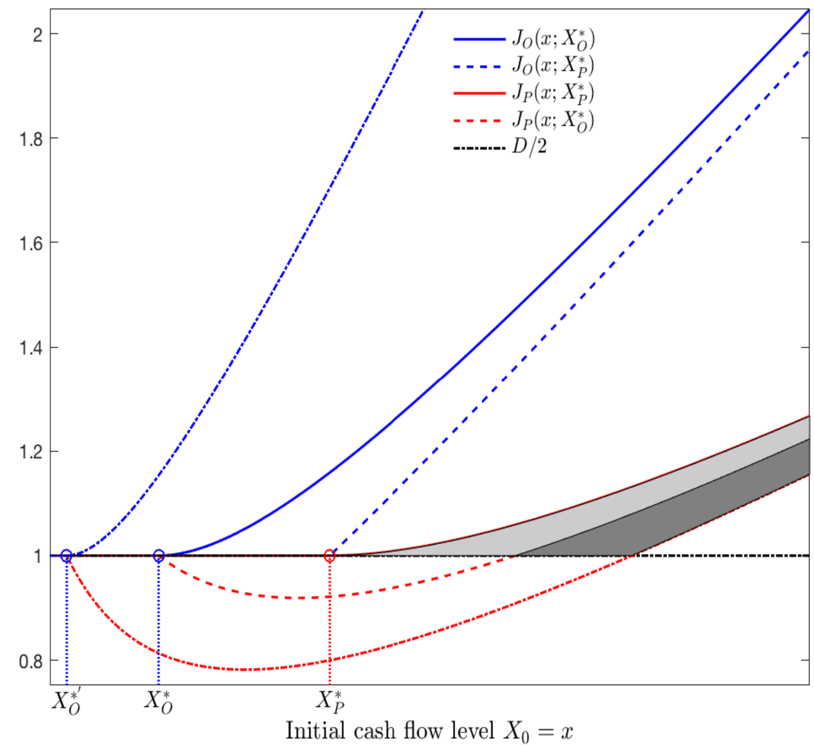

Figure 2: Belief polarization and underinvestment in a two-agent group.
The figure reports the value function of both agents as a function of the initial cash flow x under different abandonment policies. Parameter values: $r = 0.05$, $\sigma = 0.3$, $D = 1$, $\mu_P = 0.01$, $\mu_O = 0.03$, $\mu_{O'} = 0.039$.

Figure 3 describes the underinvestment conditions for a group of $N = 4$ agent. Agent 2 and 3 are, respectively, the right and left pivot. The shaded area represents the set of cash flow and investment cost (x, I) for which the underinvestment condition (20) occurs. From the figure we can infer that as the right and left pivot assessment of the drift of X_t diverge, the underinvestment region increases. For example, as agent 3 becomes more optimistic, the value $J_2(x, X_3^*)$ deteriorates thus causing the gray-area to enlarge. Similarly for the case in which agent 2 were to become more pessimistic.

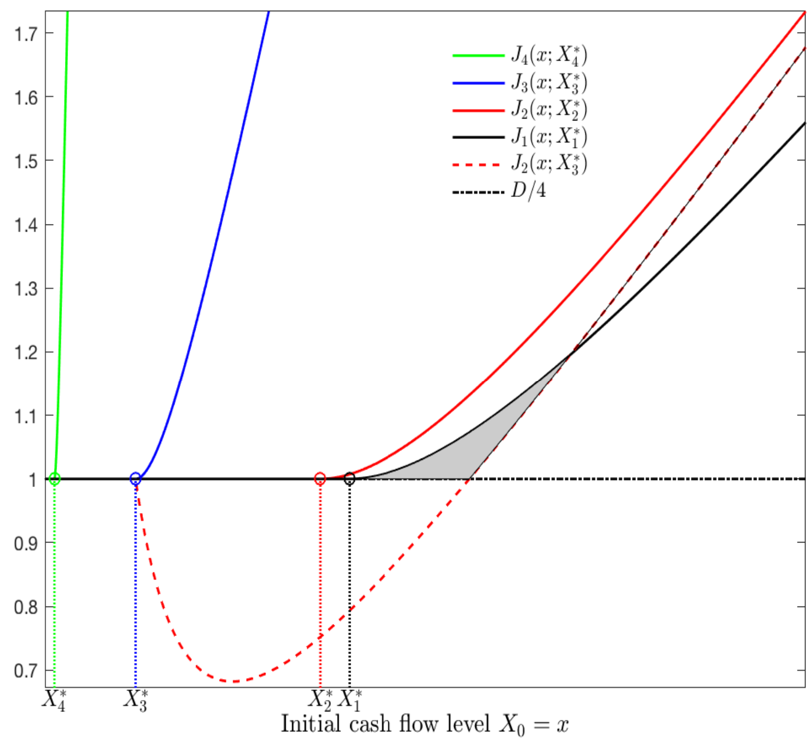

Figure 3: Belief polarization and underinvestment in a four-agent group.
The figure reports the value of the four agents as a function of the initial cash flow x for different abandonment policies. Parameter values: $r = 0.05$, $\sigma = 0.3$, $D/4 = 1$, $\mu_1 = 0.01$, $\mu_2 = 0.015$, $\mu_3 = 0.04$, $\mu_4 = 0.049$.

The figures also makes it clear that polarization—that is, the divergence between agent 2 and agent 3's beliefs about the expected growth of the project value—is a necessary requirement for underinvestment to occur. To see this, suppose that agent 2's belief μ_2 increases towards μ_3. As μ_2 increases, the distance between X_2^* and X_3^* in the figure shrinks, implying that there is a smaller region of cash flow and investment costs for which condition (20) is satisfied. Similarly, as agent 3's beliefs, μ_3, decrease towards μ_2 the underinvestment region will disappear.

4.2 Project uncertainty and underinvestment

The investment and abandonment decisions analyzed in the previous sections are the workhorse framework of the real option approach to project valuations. A common and well understood insight from that literature is that volatility makes the option to continue receiving cash flows more valuable and consequently delays the exercise of the abandonment option. In this section we investigate the effect of volatility on inefficient underinvestment. The main conclusion is that if polarization is sufficiently strong, then inefficient underinvestment is always possible, independent of the level of volatility. This result is important in that it highlights a unique channel through which underinvestment can occur, that is, the polarization of beliefs.

As in the standard real option setting, high volatility will increase the option value of abandoning (a put option) and lower the abandonment threshold for each player. However, when beliefs are sufficiently polarized, inefficient underinvestment can occur *at any level of volatility*. It is important to emphasize that inefficient underinvestment is conceptually different from "delayed" investment. In our setting the agents face a take-it-or-leave-it decision at time 0 and therefore there is no notion of investment delay.

We illustrate the effect of uncertainty on underinvestment in the context of a four-agent groups analyzed in the previous sections. In Figure 4 we take as given the beliefs μ_1 and μ_3, and report, for a range of values of the initial cash flow x, the *minimum* amount of disagreement, between agent 2 and 3, that would lead to the occurrence of underinvestment. Formally, we are reporting, for different value of volatility the difference $\mu_3 - \overline{\mu}_2$ where $\overline{\mu}_2$ is such that

$$J_2^{\overline{\mu}_2}(x; X_3^*) = J_1(x; X_1^*),$$

and where $J_2^{\overline{\mu}_2}(x; X_3^*)$ is the value function of a fictitious member with beliefs $\overline{\mu}_2$, assessed under member 3's threshold X_3^*. Geometrically, $\overline{\mu}_2$ represents the beliefs for which the curve $J_2(x; X_3^*)$ in Figure 3 intersects $J_1(x, X_1^*)$ on the righ-most side of the graph. When $\mu_2 < \overline{\mu}_2$, the gap in beliefs between agent 2 and agent 3 is sufficiently wide. We then have $J_2(x; X_3^*) < J_2^{\overline{\mu}_2}(x; X_3^*) = J_1(x; X_1^*)$ and underinvestment can occur at the cash flow level x, for some value of I. When $\mu_2 \geq \overline{\mu}_2$, the dispersion in beliefs between agent 2 ande agent 3 is narrow and we have $J_2(x; X_3^*) \geq J_2^{\overline{\mu}_2}(x; X_3^*) = J_1(x; X_1^*)$. As a result, underinvestment cannot occur at the cash flow level x.

Figure 4 shows two important points. First, each of the curves reported are increasing in the cash flow x. This means that for any given level of volatility, a higher level of initial cash flow requires a higher belief discrepancy between agent 2 and agent 3. To understand this result, consider the region of high cash flow level in Figure 3 and recall that for underinvestment to occur, it must be that agent 2 value when using the suboptimal group policy (red dashed curve) must fall below agent 1's optimal value (black curve). At that region, the abandonment put option is out of the money and, for all agents, the option component of the investment value is very small compared to the perpetuity component. As the cash flow level x grows, the option value for each agent asymptotically converges to the value of the perpetuity given by $x/(r - \mu_n)$ for member n. As can be visualized in Figure 3, this implies that the discrepancy between the optimal value of agent 2 (red curve) and the optimal value of agent 1 (black curve) increases as the cash flow level x grows. This means that for high level of cash flow x, in order to offset the wedge between the optimal value of member 1 and the optimal value of member 2, using the suboptimal policy X_3^* must drastically penalize member 2's value. Only extreme difference of opinions between member 2 and member 3 can penalize member 2 value and give rise to underinvestment.

Second, for low level of cash flows, the disagreement needed to generate underinvestment is *higher* when underlying volatility is high. In other words, the effect of volatility on underinvestment is not uniform across different level of cash flow. There are two forces at play. The first force operates through the "signal-to-noise" ratio. Intuitively, a difference of one percent in the growth rate of cash flow is much less relevant in a in a high volatility environment than in a small volatility environment. So for the disagreement of two agents to matter in a high volatile environment, their opinions need to be sufficiently polarized. This explains why the minimal belief spread required for underinvestment under high volatility (red line) is highest for low level of cash flows.

However, as the cash flow increases, the relationship between the two lines in the figure gets reversed. In the example of the figure, the difference of beliefs required for underinvestment to occur is actually *higher* in the low volatility environment. In general, the high level of cash flow dampens the “signal-to-noise” effect and reduces the difference between the belief spreads across the two curves. In summary, Figure 4 shows that underinvestment can occur at any level of volatility and that volatility mitigates underinvestment—i.e., higher belief discrepancies are needed for underinvestment to occur—when the option to abandon is in the money (low x) and exacerbates underinvestment—i.e., lower belief discrepancies are needed for underinvestment to occur—when the option to abandon is out of the money (large x).

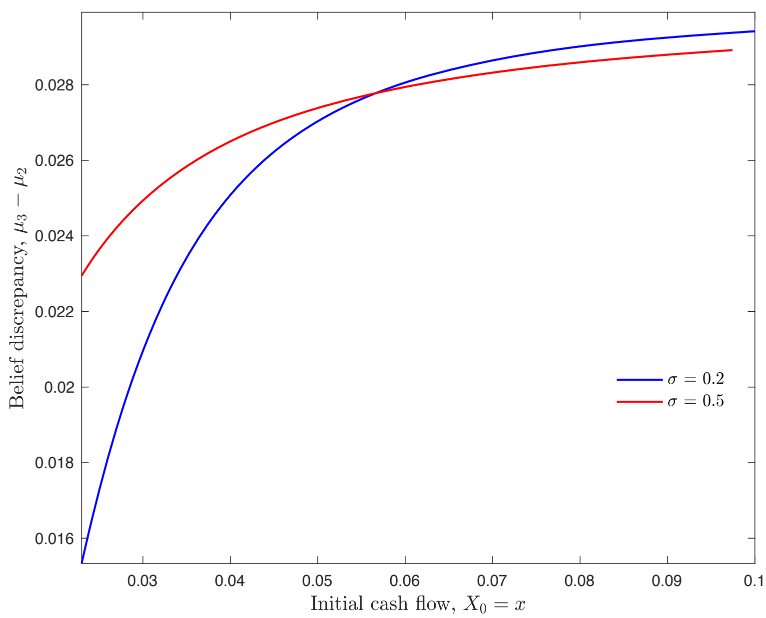

Figure 4: Uncertainty and underinvestment in a four-agent group.

The figure reports the minimum discrepancy between agent 2 and agent 3’s beliefs, $\mu_3 - \mu_2$ that lead to the existence of underinvestment, given an initial level of cash flow $X_0 = x$. Each curve corresponds to a different level of underlying volatility. Parameter values: $r = 0.05$, $D = 1$, $\mu_1 = 0.01$, $\mu_3 = 0.04$, $\mu_4 = 0.049$.

5 Other governance rules

In this section, we discuss whether the underinvestment problem we highlight occurs under different governance rules within a group. In Section 5.1 we consider groups in which agent have veto power and groups who mediate disagreement using super-majority rules. In Section 5.2 we consider the possibility of bargaining between disagreeing agents within the group.

5.1 More general voting protocols

As noted earlier, there cannot be inefficient underinvestment if the group delegates all the decision power to a single agent—a "dictatorial" governing rule. Unanimity is an alternative governance rule that we will also explore. Dictatorship and unanimity are polar cases of voting protocols within the class of *non-collegial* governance rules, that is, rules in which some members of the group have veto power, *the vetoers*. Majority and super-majority are voting protocols within the class of *collegial* rules, that is, rules in which no group member has veto power. In the next two subsections we explore whether the inefficient underinvestment problem can be mitigated by changing the voting rule. The main result from this analysis is that the inefficiency will either be unaffected or worsened by the alternative voting procedures that we explore.

5.1.1 Voting rules

Given a group of $\mathcal{N} = \{1, \ldots, N\}$, a voting rule is defined as a set of *decisive coalitions* $\mathcal{D} \subseteq 2^N/\{\varnothing\}$ (Austen-Smith and Banks, 1999). If a proposal is made and all members of a decisive coalition $\mathcal{C} \in \mathcal{D}$ votes to accept (reject) it, then the group accepts (rejects) the proposal. Therefore, defining the voting rules is equivalent to fixing the set $\mathcal{D}$ of decisive coalitions.

The next definition formally characterizes a majority governance rule in which some some group member have veto power.

Definition 4. *A governance rule is called majority with vetoers if and only of there is a set of investors* $\mathcal{V} = \{v_1, v_2, \ldots, v_m\}$ *with* $m \leq N$ *and* $v_1 < v_2 < \ldots < v_m$ *such that*

$$\mathcal{D} = \left\{ \mathcal{C} \subseteq \mathcal{N} :\mid \mathcal{C} \mid > \frac{N}{2} \text{ and } \mathcal{V} \subseteq \mathcal{C} \right\}$$

Majority rule with a single vetoer captures the idea that the group gives special veto power to a single member, such as the chair of the board of directors or the founder of the corporation. Unanimity is a polar example of such rule, where every agent has veto power, that is $\mathcal{V} = \mathcal{N}$ and hence the set of decisive coalitions is the singleton $\mathcal{D} = \{\mathcal{N}\}$.

A group ruled by a dictator $n_d \in \mathcal{N}$ is defined by the set

$$\mathcal{D} = \{\mathcal{C} \subseteq \mathcal{N} : n_d \in C\}$$

The next definition formally characterizes the super-majority or quota non-collegial rule.

Definition 5. *A governance rule is called super-majority or quota if there exist* q *with* $\frac{N}{2} + 1 < q < N$ *such that*

$$\mathcal{D}_q = \{C \subseteq \mathcal{N} :\mid \mathcal{C} \mid \geq q\}.$$

5.1.2 Unanimity rule

When the group make decisions based on Unanimity, the following proposition holds

Proposition 3. *Assume that the group utilizes the unanimity governance rule. Inefficient underinvestment occurs if and only if*

$$J_1(x, X_N^*) < \frac{I}{N} \leq J_1(x, X_1^*). \tag{21}$$

With unanimity rules, the group will abandon only when all investors agree to abandon. Thus the group will abandon at the lowest threshold X_N^*. Anticipating this, the more pessimistic investor 1 will vote against investment if and only if the left inequality of (21) holds. With the unanimity rule, the group will thus not invest since investor 1 votes against investment. This result is very similar to the underinvestment with majority that we describe in Proposition 2. The equivalence can be made clearer by observing that with a unanimity rule, the left pivot is investor 1 and the right pivot is investor N. With this observation in mind, it is possible to realize that the underinvestment region described in Figure 3 with four investors, will expand under unanimity relative to majority. This means that the underinvestment problem is worsened under unanimity.

5.1.3 Majority rule with vetoers

We assume in this section that group makes decisions based on a majority rule with vetoers. The following proposition characterizes the equilibrium outcome relative to the equilibrium outcome in the absence of vetoers.

Proposition 4. *Assume that the group utilizes the majority with vetoers governance rule. Let $p = \min(v_1, n_L)$ and $q = \max(v_m, n_R)$, then inefficient underinvestment occurs if and only if*

$$J_p(x, X_q^*) < \frac{I}{N} \leq J_1(x, X_1^*). \tag{22}$$

It is important to observe than condition (22) is less stringent than condition (20) as a result, when group utilizes a majority governance rule, the underinvestment region is larger in presence of vetoers than in the absence of vetoers. This is due to the fact that $p \leq n_L$, $q \leq n_R$, and $X_{n_R}^* \leq X_q^*$ and hence $J_{n_p}(x, X_{n_q}^*) < J_{n_L}(x, X_{n_R}^*)$. This can be seen more easily in Figure 3 which describe the underinvestment for a group of four investors. Assuming a majority rule with the vetoer $\mathcal{V} = \{4\}$, the group will then apply the threshold policy X_4^*. We see then that introducing the vetoer 4 to a majority governance rule has the effect of increasing the shaded underinvestment area describing the the interval of investment costs that give rise to inefficient underinvestment. Similarly to the unanimity rule, introducing a veto rule to a simple majority rule can worsen the inefficient underinvestment problem.

The above results also implies that the presence of veto power can generate underinvestment also in a group with an odd number of members. To see this, consider, for example, the case of a three-member group. According to our analysis above, in the absence of veto power, the group will behave according to the preference of the median group member, agent 2. If however veto powers rests in the hand of agent 3, then abandonment will occur at the threshold X_3^* and therefore the underinvestment condition is $J_2(x, X_3^*) < J_1(x, X_1^*)$. In general underinvestment will occur in a odd-numbered group if veto power is in the hands of agents that are more optimistic than the median agent.

5.1.4 Non collegial rules: super-majority

Proposition 5. *Assume the group follows a super-majority rule with q being an integer satisfying $N/2 + 1 < q < N$. Inefficient underinvestment happens when*

$$J_{N-q+1}(x, X_q^*) < \frac{I}{N} < J_1(x, X_1^*). \tag{23}$$

With a super-majority rule, underinvestment will occur independently of whether the number of group members is odd or even. This is because the investment policy adopted by the group is X_q^*. For the super-majority rule, the left pivot becomes the member $n_R = N-q+1$ and the right pivot is the member $n_L = q$. Super-majority creates a natural wedge between the left and the right pivot without requiring an even number of investors. Notice that condition (23) is naturally less stringent than (20) because $X_q^* > X_{n_R}^*$ and $J_{N-q+1}(x, X_q^*) < J_{n_L}(x, X_q^*) < J_{n_L}(x, X_{n_R}^*)$. We thus conclude that the inefficient underinvestment problem is worsened under a super-majority rule.

5.2 Bargaining in presence of disagreement

So far , we assumed that under majority voting, if the group voting outcome is a tie, the group does not invest or does not abandon. This implies that not investing and not shutting down are the (exogenous) the status quo for our voting game.

In this section, we consider without loss of generality the case of a two-agent group (P and O) in which disagreeing investors engage in bargaining. Specifically,

Assumption 5. *When the two group members disagree on the investment threshold a bargaining game ensues in which agent O succeeds with probability p.*

The above assumption capture the implications of bargaining without modeling the details of the negotiations. The assumption states that the outcome desired by the optimistic voter is obtained with an exogenous independent probability p. The parameter p represents the bargaining power of the most optimistic agent who desires a late abandonment policy. When $p = 1$, we obtain the case described in the illustrative example of Section 2.

The following proposition shows that, as long as the parameter $p \neq 0$, underinvestment can occur.

Proposition 6. *Assume $x > X_P^*$ and that Assumption 5 holds with $p \neq 0$. If*

$$(1-p)J_P(x, X_P^*) + pJ_P(x, X_O^*) < \frac{I}{2} < J_P(x, X_P^*)$$

then inefficient under-investment occurs with probability $1 - p$.

When $p = 1$ the underinvestment is most severe, as can be seen from Figure 2. As $p \to 0$, agent P becomes the dictator and therefore no underinvestment will be possible. Although the underinvestment area is reduced, underinvestment problem is still an issue in this setting.

6 Conclusion

In this paper, we have examined coordination frictions and consequent investment inefficiencies that arise when a group instead of an individual makes corporate decisions. More specifically, we examine the acquisition and subsequent management of a real option by a group whose members have heterogeneous beliefs. We show that in such a setting the group may reject an investment opportunity on behalf of the corporation even though each member of the group sees the opportunity as valuable. This happens when views of group member are polarized. In the dynamic setting we analyze, inefficiencies arises when the decisive voters change over the life of the project.

We also characterize and contrast the role of polarization and volatility in investment inefficiency. Volatility is an important determinant of the value and management of a real option. Increases in volatility can lead to a delay in investment as the value of waiting increases. Such delays, however, are optimal and do not constitute inefficiencies. In contrast, polarization of beliefs leads to inefficiencies in that investments are not just delayed but lost altogether. Our result apply generally to groups that mediate conflicts according to either majority or super-majority voting rules and to groups in which agents have veto power. While descriptive of actual final decision making in groups, in reality votes are cast in the context of dynamic pre-vote interactions. Developing theories that are more descriptive of the political economy of corporate finance will be the subject of future research.

A Appendix: Proofs

Proof of Lemma 1

Any proposal to abandon at time $\tau_{\mathcal{X}}$ with $\mathcal{X} \in [0, X^*_{n_R}]$ will receive majority support. Because $\mathcal{X} \leq X^*_{n_R}$, all group members $n \leq n_R$ will vote for abandonment leading the group to abandon. ∎

Proof of Proposition 1

All group members expect the group's abandonment decision to be taken at the threshold $X^*_{n_R}$. When condition (17) holds, member n_L votes against investment. Furthermore all group members $n < n_L$, being more pessimistic than member n_L, also vote against investment. Therefore the coalition $\{1, \ldots, n_L\}$ blocks investment and the group does not invest. On the other hand, when condition (18) holds, member n_L vote to invest. All group members $n \geq n_L$, being more optimistic than member n_L, also vote to investment. The coalition $\{n_L, n_R, \ldots, N\}$ forms thus a majority supporting investment and thus leading the group to invest. ∎

Proof of Proposition 2

The right inequality of (20) is in fact equivalent to (19). To see this, recall that condition (14) implies

$$J_1(x, X_1^*) \leq J_2(x, X_1^*) \leq J_2(x, X_2^*) \leq J_3(x, X_2^*) \leq J_3(x, X_3^*) \leq \ldots \leq J_N(x, X_N^*).$$

Therefore, $J_1(x, X_1^*) = \min_n J_n(x, X_n^*)$ and the equivalence of the right inequality in (20) with condition (19) obtains. The right inequality of (20) means that all group members would undertake the investment if they were acting individually. If the investment is undertaken by the group, member n_R is the decisive voter for the abandonment decision and the abandonment threshold policy $X^*_{n_R}$ is utilized by the group. The left inequality of (20) shows that the left pivot n_L rejects investment. As a result, all investors who are more pessimistic than the left pivot, that is investors $\{1, \ldots, n_L - 1\}$, will also reject investment. With majority rule, the coalition $\{1, \ldots, n_L\}$ vote against investment leads the group to reject investment. The left inequality of (20) implies that inefficient underinvestment as described in Definition 3 occurs. ∎

Proof of Proposition 3

With unanimity, the decisive voter for the abandonment decision is member N: the group abandons only when everyone wants to abandon at the lowest threshold X_N^*. The decisive voter for investment is member 1: Investment is undertaken by the group only when everyone wants to invest. Member 1, being the more pessimistic, is then decisive for the investment decision. The right inequality of (21) says that all group members would invest if they can optimally choose the abandonment policy. The right inequality of (21) says that member 1 votes against investment leading the group to reject investment. Therefore, underinvestment in the sense of Definition 3 occurs. ∎

Proof of Proposition 4

In order for the group to abandon, a majority vote in favor of abandonment is required and, in addition, all vetoers must be part of that majority coalition. The decisive voter for the abandonment decision is thus member q. The group abandons the projet then at the threshold X_q^*. As in condition (20), the right inequality of (22) says that all group members would invest if they control the abandonment decision. The decisive member for the investment decision is member p: a majority vote in favor of investment is required and, in addition, all vetoers must be part of that majority. The right hand side of (22) says that the decisive member for the investment decision votes against investment. Therefore the group does not invest and we obtain inefficient underinvestment. ∎

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