(a preliminary draft)
Regional Differences in Economic Growth:
A Political Economy Approach

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Abstract

This paper proposes a new political-economy model to shed light on cross-regional disparity in economic growth. The growth model is constructed in an overlapping generation economy with heterogeneous agents such that the spatial difference in growth shall be attributed to region-specific features of the local economy. The political conflict emerges over the government expenditure program within a voting game framework. The model especially highlights the institutional channels that shape, in a non-linear manner, the growth of capital and local public infrastructure. The endogenous distribution of the agents acts as a bridge between the institutions and economic growth. The calibration exercises with cross-state U.S. data show that, in most of the American states, the market institution causes the most remarkable impact on growth whereas the political institution exhibits moderate impact.

1 Introduction

Regional difference in economic growth can be observed in many countries whereas the theoretical framework to account for the difference has not been firmly established. A conventional approach to tackle the spatial issue is growth accounting. Suppose a production function with the inputs of capital, labor as well as public productive good (i.e. local public infrastructure), where all the other inputs are aggregated to total factor productivity. Growth accounting allows researchers to identify the contribution of individual input to growth in a decomposition fashion. However, this approach gives very limited economic implication because it cannot answer more essential question that immediately arises. The question is: what accounts for the regional difference in the productive inputs including capital, labor, the public good or total productive factors? This paper tackle this question from a political economy point of view. In particular, I propose an endogenous growth model embedded fundamental and institutional characteristics. Both of the characteristics are region-specific natures that yield different level of growth. The former includes

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innate labor productivity whereas the latter contains market entry costs for entrepreneurs and individual preference in ideology. In essence, this paper seeks for one-step deeper factorization, on top of growth accounting, with those region-specific channels to shed light on spatial disparity in economic growth.

A seminal work, Acemoglu (2008) emphasizes the importance of political economy approach to deal with spatial-difference issues in macroeconomics. It points out that capturing “fundamental causes” of growth allows researchers to understand spatial variation in economic performance. The fundamental causes are equivalent to underlying driving forces to characterize the level of production inputs in local economy, including capital, labor, public infrastructure or any other productive factors. It also argues that “institution” is the most likely important channel among fundamental causes. The concept of this terminology encompasses “economic institution” and “political institution”. The former is economic arrangements that determines the efficiency in production performance including the security of property rights, contracting institutions and entry barriers. The latter is social choice including rules, laws and policies affecting economic activities.

In fact, the region-specific characteristics in my model can be assorted with the typology of Acemoglu (2008). The innate productivity is labeled as a non-institutional fundamental cause, the market entry cost as an economic institution, and the ideological preference as a political institution. I stress that the institutional channels are especially important in this study because they would significantly vary among local regions. First, in any of local societies, there would be region-specific entry barrier to affect the capital accumulation. In the model, the entry barrier is introduced as the market entry costs for entrepreneurs. Second, since a government policy is obviously implemented through a political decision process, there would be potentially region-specific political conflicts that differentiate the local infrastructure expenditures. Furthermore, given a democratic election system introduced across the entire regions, the region-specific conflicts would generally stem from difference in political preference (i.e. individual preference in ideology)\textsuperscript{1}. From this perspective, I describe the ideological difference using a spatial voting model where political utility in individual preference serves to characterize the local political conflicts over economic policy. This voting model is referred to as probabilistic voting game. Thus, taking the spirit of Acemoglu (2008), the proposed model seeks to reveal the politico-economic interactions via the “fundamental causes” to drive cross-regional variation in growth. Note that I do not consider spatial spillover effects that describe collateral interactions between neighboring regions. The ignorance of the spillover effects is justified, given the objective of this study because the effects should be some fallouts generated by difference in the fundamental causes. At least, they shall not be essential for a model offering the primary approximation for the cross-region disparity.

In the model, two key mechanisms are embedded in an overlapping generation economy with no population growth. The mechanisms are: \textit{ex ante} choice and democratic decision-making process. The former mechanism characterizes the endogenous distribution of two types of agents: capitalists

\textsuperscript{1}An instance of the regional difference in ideology can be seen in the U.S. It is well known that American states are labeled with red and blue state in a political context. In the red states, residents predominantly vote for the Republican party. In the blue states, residents predominantly vote for the Democratic party.
who can access the asset market (i.e. able to make savings) and workers who cannot. In addition to the aspect of saving technology, the agents differ in labor productivity, viz., that capitalists are more productive than workers. This productivity difference represents the non-institutional fundamental factor. Ex ante individuals choose either of the agent types, given the expected lifetime payoffs and their own ability assigned by nature. At this stage, the market entering cost is borne to the agents who choose to be capitalists. This pecuniary cost represents the region-specific aspect of market institution. The innate productivity and market institution shape the savings, and hence the capital accumulation in the local economy. They then cause cross-regional variations in growth of capital stock. The other mechanism is the electoral competition that determines the government expenditure policy, given fixed tax rate. The government policy is two-fold: infrastructure expenditure that enhances growth, and welfare payment (i.e. income transfer) for the agents who cannot access the asset market. The policy choice is processed in an election where conflicting interests over the government policy emerge among the individuals, i.e. capitalists are in favor of spending more in infrastructure because it enhances the return of savings whereas workers are in favor of spending more in income transfer. This model component is equivalent to the probabilistic voting game aforementioned, where the ideological preference is defined on a liberal-conservative space. Therefore, the growth of the public infrastructure is shaped by the ideological preference.

The dynamic equilibrium of the model is characterized such that the agents interact in a non-linear fashion within the key double mechanism (i.e. the ex ante choice and the electoral competition). The non-linearity arises because of the endogenous distribution of the heterogeneous agents. On one hand, the ex ante choice process characterizes the distribution, given the expected electoral outcomes. On the other hand, the the electoral outcome is determined, given the distribution. Mathematically, this aggregated consequence is jointly formulated as a system of non-linear equations. Recalling that the ex ante decision process forms the capital growth whereas democratic election forms the infrastructure growth, the endogenous distribution, in turn affects both of the growth. The equilibrium outcomes vary from one region to another due to the difference in the institutions. Taking the opposite view, the cross-regional variations in institutional channels result in the disparity in growth. In particular, the exogenous difference in market entry costs and political preference leads to disparity in growth.

This paper investigates the equilibrium behavior through numerical computation. The model is calibrated with cross-state data including 26 American states. I then study how key parameters shape the regional difference in the steady-state equilibrium; and what is the most influential channel causing the disparity in growth. There is a unique aspect in the calibration process for the ideological preference parameters. The parameters are labeled as political bias. Unlike matching the parameters with some moment conditions, this study estimates the political bias with a political science data, and calibrate with the estimates. This estimation ensures the model to fit with the individual-level dynamics of political bias in actual society. The main finding of the calibration exercise is that, in most of the selected American states the market institution channel possesses
remarkable effect on growth whereas the political institution and innate productivity channel exhibit moderate effect.

This paper also gives a new sight in growth models with productive public good or public capital such as Barro (1990). As Agénor (2012, Ch.7) points out, the growth models with explicit political channels have been very limited. The proposed model would provide a pioneering idea to capture the relationship between growth and the public good in political economy context.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 conducts the calibration with state-level data of the U.S. Section 4 performs comparative statics with the calibrated model to investigate the equilibrium behavior. Section 5 shows how the spatial difference across 26 American states in growth can be explained, given the political economy structure. Finally section 6 concludes.

2 Model

Consider a world of $J$ regional economies with no population growth. Every regional economy is closed to capital and labor flow as well as commodity trade, but share technology knowledge such that the total factor productivity (TFP) is identical among regions. Each local economy is populated by a continuum individuals (households) who live for two periods; a government; and firms, and evolves independently of each other. Within a local economy, a single good is produced. The good can be either consumed in the period whereby it is produced; or invested to produce the good at the beginning of the following period. An individual is endowed with one unit of labor time in young age and zero units in old age. Individuals are divided into two types: capitalists and workers. At the beginning of young age, individuals choose their agent types. The most important difference between those two is accessibility to asset market viz. that capitalists can make savings to ensure the income in the old age whereas workers cannot. There are three sources of income that serve to finance consumptions: labor income, saving return and income re-distribution from government. Labor income is the unique source for all young individuals. Savings can be held only by capitalists in the first period of living in the form of capital stock. The saving return is paid in the following period. Thus capitalists ensure the income in old age by saving, but workers cannot ensure their future income by themselves, and hence shall rely on the income redistribution in old age. The income redistribution therefore plays a role of social security to support individuals with no access to asset market. Firms produce the goods using capital, labor and productive public good. Government taxes labor and capital income and balances the budget in each period. It allocates the revenue for the income redistribution that is transferred to old workers in a lump-sum manner; and the productive public good. The latter is non-excludable and non-rivalrous service flow that can be thought of as an infrastructure good. Given the tax rates fixed, the expenditures for the redistribution and the infrastructure good are residually determined at each period via a majority voting where the old agents cast votes.

\footnote{In this paper, the terms “households”, “individuals” and “agents” are used interchangeably.}

\footnote{This paper uses the terms “productive public good” and “infrastructure good” for the same meaning.}
In what follows, we consider a local economy, suppressing a subscript \( j \in \{1, \ldots, J\} \) for representing the region unless otherwise stated. We here introduce some notations. Let time be indexed by \( t \in \mathbb{N} = \{0, 1, \ldots\} \). Let superscript \( y \) denote for the young, \( o \) for the old, \( k \) for capitalists and \( l \) for workers. Let \( \mu_i^t \in (0, 1) \) for \( i \in \{y, o\} \) denote the fraction of capitalists in \( i \)-aged cohort individuals at time \( t \). All the necessary proofs for lemmas or propositions are shown in appendix.

### 2.1 Households

In the beginning of every period, identical individuals are born with one unit of labor time endowment, and an innate ability index \( e_t \) that represents a time cost for being a capitalist. Suppose \( e_t \) is i.i.d. and uniformly distributed with the support \([0, 1]\) \( \forall t \in \mathbb{N} \). Each individual then chooses either one of two agent types: capitalists or workers. Every individual is allowed to be a capitalist with certainty by investing \( e_t \) units of labor time and paying pecuniary fixed cost, \( \gamma_t \) that is non-taxable. The fixed cost is assumed to augment as the economy grows. In particular, it is given by a linear function of the state variable, \( \gamma_t := \rho K_t \) where \( K_t \) is capital stock and \( \rho > 0 \) is a constant. As shown later, this linearity assumption serves to shape the balanced growth path. Capitalists can be thought as more skillful agents than workers because they shall learn more know-how for economic activity, taking the advantage in access to the asset market. From this perspective, capitalists have high productivity, and hence provide one efficiency unit of labor per unit of labor time. In contrast, workers provide only \( \theta \in (0, 1) \) efficiency units of labor per unit of labor time. The details of the agent-type choice are left until section 2.1.3.

After the agent-type choice is realized, a young capitalist allocates the effective \( 1 - e_t \) units of labor time for labor supply whereas a young worker allocates the one unit of labor time for labor supply. Let \( c_{i,m}^t \) denote, for \( i \in \{y, o\}, m \in \{k, l\} \), consumption of an \( i \)-aged, \( m \)-type agent at time \( t \). The individual’s preference is given in a log-linear form as

\[
\ln c_{y,k}^t + \beta \ln c_{o,k}^{t+1} = u^m \left( c_{y,m}^t, c_{o,m}^{t+1} \right) := \ln c_{y,m}^t + \beta \ln c_{o,m}^{t+1}
\]

where \( \beta \in (0, 1) \) is a type-common time discount factor.

#### 2.1.1 Capitalists

Period-specific budget constraints of a capitalist are given by

\[
s_t + c_{y,k}^t = (1 - \tau^l)(1 - e_t) w_t - \gamma_t,
\]

\[
c_{o,k}^{t+1} = (1 - \tau^k)(1 + r_{t+1}) s_t.
\]

where \( s_t \) is savings, \( w_t \) wage, \( r_{t+1} \) rental rate of capital, \( \tau^l \in (0, 1) \) time-invariant tax rate on labor income, and \( \tau^k \in (0, 1) \) time-invariant tax rate on capital income. The individuals seek to maximize their utility given the budget constraints. The budget constraints (1a) and (1b) are

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4This heterogeneous assumption on labor productivity is similar to Razin et al. (2002).

5The inequality in the each budget constraint is relaxed to the equality because the utility function is strictly increasing and strictly concave, and the budget set \( \{c_{o,k}^t, c_{o,k}^{t+1}\} \) is convex \( \forall t \in \mathbb{N} \).
recast as a single life-time budget constraint,
\[
c_t^{y,k} + \frac{c_{t+1}^{o,k}}{(1 - \tau^k)(1 + r_{t+1})} = (1 - \tau^l)(1 - e_t) w_t - \gamma_t. \tag{2}
\]
Given this single constraint, the maximization problem yields the Euler equation,
\[
\frac{c_{t+1}^{o,k}}{c_t^{y,k}} = \beta(1 - \tau^k)(1 + r_{t+1}). \tag{3}
\]
Equation (3), (2) as well as (1a) give the optimal savings,
\[
s_t = \frac{\beta}{1 + \beta} \left[ (1 - \tau^l)(1 - e_t) w_t - \gamma_t \right]. \tag{4}
\]
Notice that the optimal savings do not depend on the rental rate. This indicates intertemporal income and substitution effects of variation in \(r_{t+1}\) on lifetime consumption cancel each other out. From the optimal savings (4) and budget constraints (1a), (1b), the optimal consumptions are derived as
\[
c_t^{y,k} = \frac{1}{1 + \beta} \left[ (1 - \tau^l)(1 - e_t) w_t - \gamma_t \right], \tag{5}
\]
\[
c_{t+1}^{o,k} = \beta \frac{(1 - \tau^k)(1 + r_{t+1})}{1 + \beta} \left[ (1 - \tau^l)(1 - e_t) w_t - \gamma_t \right]. \tag{6}
\]

2.1.2 Workers

The period-specific budget constraints of workers are given by
\[
c_t^{y,l} = (1 - \tau^l) \theta w_t, \tag{7a}
\]
\[
c_{t+1}^{o,l} = c_{g,t+1}, \tag{7b}
\]
where \(c_{g,t+1} > 0\) is the lump-sum transfer from government.

2.1.3 Ex-ante decision

This subsection details how each individual chooses whether to be a capitalist or a worker, and how the aggregated choices characterize the distribution of agent types, \((\mu_t^y, 1 - \mu_t^y)\). Suppose time \(t\) is divided into two sub-periods: \(t_a\) at which a unitary continuum of individuals are born with idiosyncratic ability \(e_t\), and each individual then chooses either of the agent types; and \(t_b\) at which the choices are accordingly realized, and finally \(\mu_t^y\) is determined. At the beginning of time \(t_a\), capital stock is given by \(K_t\). We then consider how \textit{ex ante} individuals evaluate the type-dependent lifetime payoffs. Given fixed costs \(\gamma_t\) and a set of government policies \(\{\tau^k, \tau^l, g_t, c_{g,t}\}\) and prices \(\{w_t, r_t\}\), the \textit{ex post} lifetime payoff of capitalists is contingent on \(K_t\) and \(e_t\). The payoff is given by the following value function.
\[
U^k(K_t, e_t) := u^k(c_t^{y,k}(K_t, e_t), c_{t+1}^{o,k}(K_t, e_t)) = \ln c_t^{y,k}(K_t, e_t) + \beta \ln c_{t+1}^{o,k}(K_t, e_t), \tag{8}
\]
where $c_{t}^{y,k}$ is given by equation (5), and $c_{t+1}^{o,k}$ given by (6). Likewise, the value function for workers is given by

$$U^l(K_t) := u^l(c_{t}^{y,l}(K_t), c_{t+1}^{o,l}) = \ln c_{t}^{y,l}(K_t) + \beta \ln c_{t+1}^{o,l}$$

where $c_{t}^{y,l}$ is given by equation (7a), and $c_{t+1}^{o,l}$ given by (7b). Since the lifetime payoff of workers is identical among the cohorts regardless of individual ability, $U^l$ does not depend on $e_t$ (see equation (7a) and (7b)).

Lemma 1 Given $K_t$, a unique cutoff level of ability index, $e_t^* \in (0,1)$ is ensured in the agent-type choice such that $U^k(K_t, e_t^*) = U^l(K_t)$.

Figure 1 depicts the unique $e_t^* \in (0,1)$ in lemma 1.

Now suppose the ex ante individual makes a rational decision, given $K_t$ and their own ability $e_t$ assigned by nature. Lemma 1 then induces the outcomes: an individual with ability $e_t \geq e_t^*$ chooses to be a capitalist; and one with $e_t < e_t^*$ chooses to be a worker. Since $e_t$ is uniformly distributed on [0, 1], the cutoff level turns out to equal the fraction of (young) capitalists, i.e.

$$e_t^* = \mu_t^y$$

From this outcome, the aggregated labor supply by capitalists is given by

$$l_t = \int_0^{e_t^*} N(1-z)dz = N\mu_t^y \left(1 - \frac{\mu_t^y}{2}\right)$$
where $z$ is a dummy variable for $e_t$. The aggregated labor supply by workers is given by $N(1 - \mu_t^\theta)\theta$.  

2.2 Firms

Suppose there is a unitary continuum of identical firms with the aggregated production in a Cobb-Douglas form $F(K, L, g) := AK^\alpha [Lg]^{1-\alpha}$, where $A > 0$ the world-common TFP, $K$ capital stock, $L$ the aggregated labor input, $g$ productive public good provided by government, and $\alpha \in (0, 1)$ output share of capital. Note that $g$ is treated as a flow variable. The labor supply is perfectly inelastic such that the aggregated labor supply is equal to the labor input in the production function. The aggregated labor supply is given by

$$L_t = l_t + N(1 - \mu_t^\theta)\theta.$$  

(11)

The markets for capital and labor are perfectly competitive such that the aggregated firms seek to maximize the profit $\Pi_t := Y_t - w_tL_t - (r_t + \delta)K_t$, where $\delta \in (0, 1)$ is the depreciation rate of capital stock. The first order conditions for the profit maximization problem give the optimal level of prices:

$$w_t = \frac{(1 - \alpha)Y_t}{L_t},$$  

(12)

$$r_t = \frac{\alpha Y_t}{K_t} - \delta.$$  

(13)

Given capital stock at initial period $K_0$, the capital accumulation is driven by

$$K_{t+1} = (1 - \delta)K_t + I_t$$  

(14)

where $I_t$ is investment.

2.3 Government

The gross revenue for the government budget is given by

$$T_t := \tau^l w_tL_t + \tau^k (1 + r_t)S_{t-1}$$  

(15)

where $S_{t-1} := N \int_0^{C_t-1} s_{t-1}(z)\, dz$ is the aggregated savings in the last period. Government allocates the available resource for the productive public good, $g_t$ and the income redistribution, $N(1 - \mu_t^\phi)c_{g,t}$. The government balanced budget is then given by

$$g_t = T_t - N(1 - \mu_t^\phi)c_{g,t}.$$  

(16)
This budget constraint implies double rolls offered by infrastructure projects in the actual economy: a local productivity enhancement, and a social security for some individuals. In the actual economy, the gross expenditure in infrastructure projects that is equivalent to \( g_t + N(1 - \mu^o_t)c_{g,t} \) in the model would be observable, but the expenditure composition would be unobservable. Note that equation (16) shows that the expenditure for \( g_t \) is equal to the net revenue. For what follows, we here define a policy space for \( g_t \) and \( N(1 - \mu^o_t)c_{g,t} \) as \( G_t := [0, T_t] \).

### 2.4 Market clearing condition

Since the labor market is always in equilibrium due to the inelastic labor supply, we consider the market clearing conditions for good and asset markets. Combining individual’s budget constraints (1a), (1b), (7a), (7b) yields the aggregated consumption as

\[
C_t := \int_0^{E_t} c_t^{y,k}(z)dz + (1 - \mu^y_t)c_t^{y,l} + \int_0^{E_{t-1}} c_t^{o,k}(z')dz' + (1 - \mu^o_t)c_t^{o,l}
= (1 - \tau^l)L_tw_t - (S_t + N\mu^y_t\gamma_t) + (1 - r_t)S_{t-1} + N(1 - \mu^o_t)c_{g,t}.
\]

Define then the equilibrium conditions: for the good market, \( Y_t = C_t + I_t + g_t \), and for the asset market, \( I_t = S_t \). These two equilibrium conditions with the aggregated consumption (17) and balanced government budget (16) identify the market clearing condition as

\[
(r_t + \delta)K_t = (1 + r_t)S_{t-1} - N\mu^y_t\gamma_t.
\]

The condition indicates that, at the aggregate level, the rental costs of capital must be equal to the net return of savings, i.e. the gross return of savings deducted by the fixed costs for access to the saving technology.

### Definition 1

A competitive equilibrium is a sequence of relative prices \( \{w_t, r_{t+1}\}_{t=0}^{\infty} \), consumption and savings \( \{c_t^{y,k}, c_t^{y,l}, c_t^{o,k}, c_t^{o,l}, s_t\}_{t=0}^{\infty} \), capital stocks \( \{K_{t+1}\}_{t=0}^{\infty} \), given the initial capital stocks \( K_0 \) and a sequence of productive public good \( \{g_t\}_{t=0}^{\infty} \), fixed costs \( \{\gamma_t\}_{t=0}^{\infty} \) and constant tax rates \( \tau^k, \tau^l \), such that:

(i) individuals rationally choose their agent types and maximize utility;

(ii) firms maximize profits;

(iii) all markets are cleared; and

(iv) government budget is balanced.

The competitive equilibrium characterizes the optimal consumption of an old worker and the aggregated savings as

\[
c_t^{o,l} = \frac{T_t - g_t}{N(1 - \mu^o_t)},
\]

\[
S_t = \beta \frac{(1 - \tau^l)l_tw_t - N\gamma_t\mu^y_t}{1 + \beta}.
\]

\[8\] A political science work, Estévez-Abe (2008) also focuses attention on the aspect of infrastructure projects as a social security program, and analyzes the public policy within a job security context.
2.5 Political conflict of interest

This subsection considers preferences over government policies in the competitive equilibrium. Given the constant tax rates \( \tau^k \) and \( \tau^l \), the remaining policy instruments are \( g_{t+1} \) and \( c_{g,t+1} \). The government balanced budget \(^{(16)}\), however implies that either of the two shall be residually determined once the other is fixed. Therefore, we focus on the preference over \( g_{t+1} \) in what follows.

The indirect utility functions of old agents over the public good policy are given by

\[
V^k(K_{t+1}, g_{t+1}, e_t) := \beta \ln c_{o,k}^{o,k}(K_{t+1}, g_{t+1}, e_t),
\]

\[
V^l(K_{t+1}, g_{t+1}) := \beta \ln c_{o,l}^{o,l}(K_{t+1}, g_{t+1}).
\]

Notice that \( V^k(K_{t+1}, g_{t+1}, \cdot) \) varies on the domain \([0, e^*_t]\) because saving returns are contingent on the innate ability.

**Lemma 2** Given \( K_{t+1} \) and \( e_t \), every individual exhibits a single-peaked preference over \( g_{t+1} \in G_{t+1} \) in the competitive equilibrium such that

\[
\frac{\partial^2 V^m}{\partial g_{t+1}^2} < 0, \quad \text{for each } m \in \{k, l\}.
\]

The bliss points for capitalists is given by:

\[
g_{t+1}^{*k} = T_{t+1}, \tag{21}
\]

whereas that for workers is by

\[
g_{t+1}^{*l} \in G_{t+1} \quad \text{such that} \quad \frac{\partial V^l}{\partial g_{t+1}} \bigg|_{g_{t+1}=g_{t+1}^{*l}} = 0 \tag{22}
\]

First, equation \((21)\) indicates that the capitalists are consistently in favor of more \( g_{t+1} \) because an increment in \( g_{t+1} \) raises the rate of capital rent, thus enhancing the saving returns, and hence their consumption. In fact, the most desired electoral outcome for them is that the government spends all the resource in \( g_{t+1} \). On the other hand, equation \((22)\) implies that the workers are in favor of more \( g_{t+1} \) as long as it is under the bliss point level because the marginal increase in \( g_{t+1} \) raises the tax revenue enough to enhance the lump-sum transfer to them i.e. \( c_{g,t+1} \). However, once it is beyond the bliss point level, they become against more \( g_{t+1} \) because the increment in \( g_{t+1} \) decreases \( c_{g,t+1} \), and hence lowers their consumption.\(^9\) The following proposition is immediately derived to ensure the conflicting preference of the agents over the policy.

**Proposition 1** Given \( K_{t+1} \), the conflicting preference arises if

\[
\eta_{t+1} := \frac{K_{t+1}}{g_{t+1}} \in \left( \eta_{t+1}, \overline{\eta}_{t+1} \right) \tag{23}
\]
where

\[ \eta_{t+1} \text{ satisfies } \Lambda A L_{t+1}^{1-\alpha} \tau_k^{1-\alpha} + \tau^k \mu_{t+1} \rho_{t+1} \eta_{t+1} - 1 = 0 \]

\[ \bar{\eta}_{t+1} := \left[ \frac{1}{(1-\alpha) A L_{t+1}^{1-\alpha}} \right]^{\frac{1}{\alpha}}, \]

\[ \Lambda := \tau^l (1-\alpha) + \tau^k \alpha \]

Note that \( \eta_{t+1} \) pins down the upper limit of \( g_{t+1} \) when \( T_{t+1} = g_{t+1} \) whereas \( \eta_{t+1} \) is the lower limit when \( g_{t+1} = g^*_{t+1} \).

### 2.6 Electoral competition

This subsection introduces a probabilistic voting game where the policy variables are endogenously determined. The voting model introduces ideological preference that plays an important role for the equilibrium characterization unlike voting games where the median voter theorem can be applied. This feature of the probabilistic voting game is well fitted with the purpose of this study because it explicitly introduces ideological preference to characterize the policy instrument choices.

Suppose that equation (23) holds to ensure the conflicting preference. In every period, an election is held where all the old individuals vote to elect the politician who implements the policy after the election. There are two candidates, \( A \) and \( B \) who maximize the expected voting share. Candidate \( C \in \{A, B\} \) announces his policy platform \( g^C_{t+1} \). The candidates differ in “ideology” that is certain in other dimensions unrelated to policy space \( G_{t+1} \). The ideology is a permanent feature that cannot be modified as a part of the policy. In general, it may include the candidate’s impression given to voters, political leaderships, and perceived policy stance on another policy space, but this study defines it in the sense of political philosophy. In particular, suppose candidate \( A \) is a liberal (or a left wing) and candidate \( B \) a conservative (or a right wing). Individuals cast votes based on the announced policies and the ideological characteristics. Given the elected candidate \( C \in \{A, B\} \), let the payoff of an \( m \)-type voter be given by

\[ W^{C,m}(K_{t+1}, g^C_{t+1}, x) = V^m(K_{t+1}, g^C_{t+1}, x) + \varepsilon^{C,m}_{t+1} \text{ for } m \in \{k, l\}, \]

where

\[ x := \begin{cases} e_t & \text{if } m = k, \\ \text{none} & \text{otherwise.} \end{cases} \]

\( \varepsilon^{C,m}_{t+1} \) is an agent-specific random utility bias that can take negative and positive values. In this study, we refer to this random bias as “political bias”. The political bias is also known as ideological bias (see e.g. Persson and Tabellini 2002). Let the distribution of the difference between the two

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10 See e.g. Banks and Duggan (2005) for a general framework of probabilistic voting game.
random variables $\varepsilon_{t+1}^{A,m}$ and $\varepsilon_{t+1}^{B,m}$ be given by

$$
\varepsilon_{t+1}^{B,m} - \varepsilon_{t+1}^{A,m} \sim \Phi_{t+1}^m(\cdot)
$$

where $\Phi_{t+1}^m(\cdot)$ is a cumulative distribution function that is non-decreasing and concave i.e. strict quasi-concave. $\varepsilon_{t+1}^{B,m} - \varepsilon_{t+1}^{A,m}$ measures a $m$-type agent’s relative political bias in favor of candidate $B$. More precisely, given that two candidates are mutually at the opposite end of ideological scale, it maps the voter’s bias in a conservative-liberal-ideology space. Define a three-state space for the political bias given by $S := \{s_1 := 1, s_2 := 0, s_3 := -1\}$ where $s_1$ represents the conservative (right-wing) bias; $s_2$ neutral (i.e. no bias); and negative element $s_3$ the liberal (left-wing) bias. Suppose that all individuals face an idiosyncratic shock on their bias at the beginning of the old age. The transition of the individual-level bias follows a Markov chain. The Markov chain for $m$-type agents then is given by

$$
\Phi_{t+1}^m = \Phi_t^m \lambda^m,
$$

$$
\Phi_t^m := (\phi_t^m(s_1), \phi_t^m(s_2), \phi_t^m(s_3)),
$$

$$
\lambda^m := \begin{bmatrix}
\lambda^m(1) \\
\lambda^m(2) \\
\lambda^m(3)
\end{bmatrix}
= \begin{bmatrix}
\lambda_{11}^m & \lambda_{12}^m & \lambda_{13}^m \\
\lambda_{21}^m & \lambda_{22}^m & \lambda_{23}^m \\
\lambda_{31}^m & \lambda_{32}^m & \lambda_{33}^m
\end{bmatrix},
$$

$$
\lambda^m(j) := (\lambda_{j1}^m, \lambda_{j2}^m, \lambda_{j3}^m) \quad \text{for} \quad j = 1, 2, 3,
$$

$$
\lambda^m_{xy} > 0 \quad \forall(x, y) \in \{1, 2, 3\} \times \{1, 2, 3\}, \quad \sum_{y=1}^{3} \lambda^m_{xy} = 1 \quad \forall x \in \{1, 2, 3\},
$$

where $\lambda^m$ is the transitional matrix and $\phi_t^m(\cdot)$ is the distribution function corresponding to $\Phi_t^m(\cdot)$. Given the stability of the Markov process, the distribution of the political bias converges to a unique stationary distribution $\phi^\infty_m$ for each $m \in \{k, l\}$ in the long-run. The stationary distribution is given by

$$
\Phi^\infty := \lim_{t \to \infty} \Phi_t^m (\lambda^m)^t := (\phi^\infty_m(s_1), \phi^\infty_m(s_2), \phi^\infty_m(s_3)) = (\phi^\infty_m(1), \phi^\infty_m(0), \phi^\infty_m(-1))
$$

The timing of events in the election is as follows: [1] Two candidates simultaneously announce their policies $g_{t+1}^A$ and $g_{t+1}^B$. At this stage, they know the voters’ policy preferences $V^m(K_{t+1}, \cdot, x)$ and the distributions of political bias, $\Phi_{t+1}^m$ for each $m \in \{k, l\}$, but not yet its realized values; [2] The actual value of the political bias is realized and the uncertainty from candidates’ perspective is resolved; [3] Voters cast votes; [4] The elected candidate implements his or her announced policy. Figure 2 shows a flow chart of the events.

At stage [2], the probability that an $m$-type individual votes for candidate $A$ is given by

$$
P_r [ (W^{A,m} - W^{B,m}) > 0 ] = \Phi_{t+1}^m [ V^m (K_{t+1}, g_{t+1}^A, x) - V^m (K_{t+1}, g_{t+1}^B, x) ]
$$

Given $K_{t+1}$ and $\epsilon_t$, the candidate $A$’s expected vote share is then given by

$$
\pi_A (K_{t+1}, g_{t+1}^A, g_{t+1}^B, \epsilon_t) = N \mu_t^o \Phi_{t+1}^k [ V^k (K_{t+1}, g_{t+1}^A, \epsilon_t) - V^k (K_{t+1}, g_{t+1}^B, \epsilon_t) ]
$$
Young agents choose their agent type.

Capital $K_t$; Distribution ($\mu^y_t, 1 - \mu^y_t$) realized.

Old agents votes for $g_{t+1}$ realized.

Young age

$g_t^*$ realized

Old age

Distribution ($\mu^y_{t+1}, 1 - \mu^y_{t+1}$) realized.

Policy platform $g_t^A, g_t^B$ announced.

Political bias $X_{t+1}^m$ realized.

Given

Voters' preference over policy $V^m(K_{t+1}, \cdot, x)$; Distributions of political bias $\Phi_{t+1}^m(\cdot)$.


Figure 2: Timing of events
and hence, candidate B’s is given by
\[ \pi_B (K_{t+1}, g^A_{t+1}, g^B_{t+1}, e_t) = 1 - \pi_A (K_{t+1}, g^A_{t+1}, g^B_{t+1}, e_t) \]

Both two candidates maximize their own vote share such that the optimal announced policies are given by
\[ g_{t+1}^A \in \arg \max \pi_A (g^A_{t+1}, g^B_{t+1}), \]
\[ g_{t+1}^B \in \arg \max \pi_B (g^A_{t+1}, g^B_{t+1}). \]

Given the concavity of \( V \), the first order condition for the candidates’ maximization problem is given by
\[
\phi_{t+1}^k (0) \int_0^{e_t} \partial V^k (z) \frac{dz}{g_{t+1}} + (1 - \mu_{t+1}^0) \phi_{t+1}^l (0) \frac{\partial V^l}{\partial g_{t+1}} = 0 \tag{27}
\]
where \( \phi_{t+1}^m (0) = \phi_{t+1}^m (s_2) \). Equation (27) has an integral with respect to the ability index because the derivative of \( V^k (K_{t+1}, g_{t+1}, \cdot) \) varies according to \( e_t \). \( \phi_{t+1}^m (0) \) captures the fraction of \( m \)-type voters with no political bias i.e. those who are independent in the ideological sense. The independents are thought of as “swing voters” because their voting pattern must be unaffected by the political bias but extremely sensitive to the change of the announced policy. Summarizing these results yields the following lemma.

**Lemma 3** Given capital stock \( K_t \), the probabilistic voting game yields a unique Nash equilibrium, \( \eta_{t+1} \in (\eta_{t+1}, \bar{\eta}_{t+1}) \) which is characterized by the first order condition (27).

In the meantime, equation (27) is equivalent to the first order condition for a welfare maximization problem. The social welfare function for the problem is given by
\[
\zeta (g_{t+1}) := \phi_{t+1}^k (0) \int_0^{e_t} V^k (z) dz + \phi_{t+1}^l (0) (1 - \mu_{t+1}^0) V^l \tag{28}
\]

In fact, the Nash equilibrium outcome \( \eta_{t+1} \) can be the maximizer of social welfare function \( \zeta (\cdot) \). \( \phi_{t+1}^k (0) \) and \( \mu_{t+1}^0 \) are the weights for the welfare of capitalists whereas \( \phi_{t+1}^l (0) \) and \( (1 - \mu_{t+1}^0) \) are the weights for the welfare of workers. The notion of the weights for the welfare function helps to interpret the results of numerical exercises in later sections. Lastly, we establish the following definition.

**Definition 2** A political equilibrium is a sequence of prices \( \{w_t, r_{t+1}\}_{t=0}^{\infty} \), consumption and savings \( \{c^k_t, c^l_t, c^o_t, c^l_o, s_t\}_{t=0}^{\infty} \), capital stocks \( \{K_{t+1}\}_{t=0}^{\infty} \), distribution of agents \( \{\mu_{t+1}^o (= \mu_{t+1}^o)\}_{t=0}^{\infty} \), constant tax rates \( \tau^k, \tau^l \), sequence of productive public good \( \{g_{t+1}\}_{t=0}^{\infty} \) and fixed costs \( \{\gamma_t\}_{t=0}^{\infty} \) given
the initial capital stocks $K_0$, the initial public good $g_0$, and initial fraction of old capitalists $\mu_0$, such that:

(i) individuals rationally choose agent types, and maximize utility;
(ii) firms maximize profits;
(iii) all markets are cleared;
(iv) government budget is balanced;
(v) electoral competition is in Nash equilibrium where both candidates announce an identical policy.

2.7 Steady state

This subsection derives the steady-state properties that form spatial difference in the long-run growth. In the political equilibrium, the output is given by

$$Y_{t+1} = AL_t^{1-\alpha} \frac{\mu_t^{\eta_t} - N \rho}{1 + \beta} K_t,$$

and the rate of growth, $\nu_t$ is derived as

$$1 + \nu_t := \frac{Y_{t+1}}{Y_t} = \left(\frac{\eta_t + L_t}{\eta_t L_t + 1}\right)^{\alpha - 1} \left(1 - \delta + \frac{N \mu_t^{\eta_t} \beta [(1 - \tau)^{(1 - \mu)]}}{(1 - \alpha) L_t^{\alpha - \alpha - 1} - \rho)}\right),$$

Once the bias distribution $\phi^m_{t+1}$ for $m \in \{k, l\}$ converges to the stationary distribution, the rate of growth becomes constant as

$$1 + \nu := 1 - \delta + \frac{N \mu^* \beta [(1 - \tau)^{(1 - \mu)]}}{(1 - \alpha) L^*^{\alpha - \alpha - 1} - \rho)}$$

where superscript $^*$ denotes for a constant value. Political constraint (27) characterizing $\eta^*$ is now given by

$$\phi^k(0) \int \frac{\partial V^k(z)}{\partial g_{t+1}} dz + (1 - \mu^*) \phi^l(0) \frac{\partial V^l}{\partial g_{t+1}} = 0$$

The distribution of agent type, $\mu^*$ is contingent on ex ante decision. Using lemma 1 with equation (10), the equilibrium distribution is characterized by

$$U^k(K_t, \mu^*) = U^l(K_t)$$

Finally, the equilibrium labor is given by

$$L^* = l^* + N (1 - \mu^*) = \frac{\mu^* \left(1 - \frac{\mu^*}{2}\right)}{(1 - \mu^*) \theta}$$

Even though individuals face the idiosyncratic shock on their political bias in the political equilibrium, the growth rate becomes constant in the steady state because the distributions of political bias converge to unique stationary distributions. Thus this model turns out to be a stochastic $AK$-type growth model nesting Barro (1990). Summarizing these results in the steady state yields the following lemma.

**Lemma 4** Once $\phi^m_{t+1}$ converges to the stationary distribution, $c_{t+1}^{y,k}, c_{t+1}^{y,l}, c_{t+1}^{o,k}, c_{t+1}^{o,l}, s_t, K_{t+1}$ and $g_{t+1}$
grow at an endogenous constant rate given by equation (29). This steady-state equilibrium is characterized by equation (30), (31) as well as (32), and referred to as the balanced growth path.

Equation (29) shows the relationship between growth rate and region-specific parameters. However, we cannot analytically investigate whether and how the parameters increase or decrease growth rate because equation (29) includes endogenous variables, $\eta^*, \mu^*, L^*$ that are all determined by the system of non-linear equations, (30), (31), (32). In short, we require a quantitative discipline to shed light on the equilibrium behavior conditional on the region-specific channels. I therefore calibrate the model in section 3 and perform numerical comparative statics in section 4 to obtain some intuitions for the effects of the parameters on growth.

3 Calibration

3.1 Strategy

In the subsequent sections, I conduct numerical exercises with cross-state data in the U.S., focusing on the steady-state equilibrium. Earlier mentioned, the motivation essentially follows from the non-analytical form of the equilibrium, but there is another important justification for the exercises. Recall that growth accounting with conventional Solow-Swan-type model simply decomposes regional difference in growth into market fundamentals. This study releases our sight from the tied-hand framework. It seeks to reveal complex politico-economic interactions shaped by region-specific driving forces, and link them with the variations in the market fundamentals. Up to the last section, I have shown region-specific channels affecting economic growth, but yet considered the contribution of the individual channel to growth. Now, through the calibration exercises, I identify the important channels that have significant impact on growth. It shall be stressed that I assess the impact of parameters in a relative manner with the quantitative analysis. I do not aim to evaluate the reproducibility of the model for some observations like business cycle literature or conduct policy-oriented welfare analysis like life-cycle model literature. Rather, I attempt to obtain “qualitative” findings on the regional disparity in growth using the numerical results, given the theoretical structure. In fact, we lack essential information (e.g. cross-state effective tax rates, state-level government expenditure in the whole of infrastructure) to ensure that the fundamental environment in the model economy are fit enough with the actual economy. However, this does not matter because I do not evaluate the absolute values of numerical results, but focus on the relative difference in them from the qualitative perspective.

In what follows, I clarify the calibration process in order. First, section 3.2 details the calibration for the political bias and its estimation method as well. It provides a foundation to interpret the entire calibrated economy. Next, section 3.3 sets the model and agent-life period. Finally section 3.3 shows the process for the other parameters.
3.2 Political preference

The dynamics in the model economy relies on the exogenous transition of the individual preference in regard to ideology i.e. political bias. We therefore need to discipline our choice of the preference parameters such that they mimic the individual-level transition in ideological preference in the regional society. This study therefore estimates the political bias parameters with an American public opinion survey data, rather than reproducing from economic data the political parameters conditional on the given model structure. The data is the nation-wide cross-section survey of American National Election Study (ANES) from 1972 to 2002. The samples employed in this paper cover 26 states with two-year interval. The original data is disaggregated to state-by-state samples with four-year frequency. Narrowing the frequency enhances the numbers of state-level observations such that it can include the samples of more types of individuals every period. The data includes the respondents’ party identification (PID) with three-point measure (democratic, independent, republican) and other socio-economic attributes (sex, age, race and occupation). PID is a popular concept in political science, defined as “an attachment to a party that helps the citizen locate him/herself and others on the political landscape” (Campbell et al., 1960). It can be thought of as a proxy of ideology, and hence political bias that the individual has. This study approximates the dynamics of the political bias using the observations of party identification.

State space $S$ is assumed to be an ideological space in section 2.6. Now instead, suppose $S$ is a PID space where $s_1$ represents for democrat, $s_2$ for independent and $s_3$ for republican. Given the re-interpretation of the state space, I estimate Markov chain (24) using a two-step method proposed by Okabe (2014). This method allows one to estimate the individual-level dynamics i.e. transitional matrix $\lambda$, from the sequence of aggregate-level data i.e. repeated cross section. It consists of multinomial logistic regression (the first step) and maximum entropy method (the second step). The first step estimates the sequence of distribution, $\{\Phi_t\}_{t=0}^7$, and the second step estimates $\lambda$. The types of individuals are identified by the respondents’ occupation. In particular, it is assumed that those with occupation category, “professional and managerial” represents capitalists whereas those with occupation category “skilled, semi-skilled and service workers” represents workers. Once we estimate $\lambda$, we can compute the stationary distribution with equation (26). Appendix E lists the step-by-step process for the estimation.

3.3 Period configuration

The model period is set to four years. Individuals are assumed to be active at age 18 in the economy, become old in four periods (i.e. at age 34) and gets inactive after further four periods (i.e. at age 46). The duration of the agents being active is set to 32 years such that it is equal to

---

11 We may be able to pin down the values of the bias parameters in the Nash equilibrium of voting game such that a moment matches with the first order condition. However, this strategy would not be desirable because it fails to identify the underlying dynamics to shape the moment.

12 In the observations, occupation are categorized into six: 1. Professional and managerial, 2. Clerical and sales workers, 3. Skilled, semi-skilled and service workers, 4. Laborers, except farm, 5. Farmers, farm managers, farm laborers and foremen; forestry and fishermen, 6. Homemakers
Figure 3: Eight cohorts at time $t$

The sampling periods of the political data. It allows the calibrated model to emulate the estimated transition of political bias for the same duration as the entire life of agents.

Figure 3 shows that there exist eight cohorts, given a period $t$. Old agents are supposed to cast votes at the period when they just get aged such that the electoral outcomes are shaped by period-by-period transition of the political bias. Now the individual-level consumption of $i$-aged, $m$-type agent, $c_{t}^{i,m}$ shall be interpreted as the average of those of the four cohorts. Likewise, the other individual-level variables e.g. savings, $s_{t}$ as well as the distribution of agent types, $\mu^{i}_{t}$ are also re-interpreted as the averaged values.

An interpretation of the model is then that agents are involved in the political economy when they become eligible to participate in the democratic election (i.e. age 18). Their political decisions are however, consolidated into a one-shot vote in the period when they just become old (i.e. age 34). The lifetime of agents are scaled to 46 years that is far from the actual life expectancy whereas the transition of PID goes with the real time scale. The seemingly unrealistic assumption on lifetime does not impede the quantitative analysis. Recall that the objective of the calibration exercise is to identify crucial region-specific channels to affect on “growth”. We are not required, for our purpose, to consider the level effects conditional on the lifetime setting.
Table 1: Calibrated region-common parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Reference/process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.36</td>
<td>Cooley and Prescott (1995)</td>
</tr>
<tr>
<td>$1 - (1 - \delta)^{1/4}$</td>
<td>depreciation rate</td>
<td>0.048</td>
<td>Cooley and Prescott (1995)</td>
</tr>
<tr>
<td>$A$</td>
<td>TFP</td>
<td>3.11</td>
<td>set to match with reference values of capital-output ratio</td>
</tr>
<tr>
<td>$\beta^{1/16}$</td>
<td>discount factor</td>
<td>1.011</td>
<td>Hurd (1989)</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>labor tax</td>
<td>0.2241 (for Group A), 0.2000 (for Group B)</td>
<td>Gomme and Rupert (2007)</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>capital tax</td>
<td>0.2921 (for Group A), 0.2500 (for Group B)</td>
<td>Gomme and Rupert (2007)</td>
</tr>
</tbody>
</table>

3.4 Benchmark

Table 1 shows the calibrated values of region-common parameters. Capital share of output, $\alpha$ and depreciation rate, $\delta$ are taken from Cooley and Prescott (1995), but the latter is adjusted to four-year basis. TFP parameter, $A$ in the production function is set such that capital-public-good ratio $\eta^*$ takes reasonable values satisfying constraint (23), given cross-state observations of capital-output ratio. The capital-output ratios are calculated using data of Bureau of Economic Analysis (BEA) and estimates by Yamarik (2013). The values of output are simply taken from gross state product (GSP) by BEA. The values of capital stocks are initially taken from Yamarik (2013) and then re-scaled such that the national-level capital stock by Yamarik (2013) is equal to the national-level fixed capital assets by BEA. This re-scaling offers reasonable values of the gross interest rate in the steady state, $r^* := \alpha (K/Y)^{-1} - \delta \approx 6.2 - 9.2\%$ (annual base). Discount factor, $\beta$ is taken from an empirical study by Hurd (1989) along the line of OLG model literature e.g. Heer and Mauner (2009). The literature emphasizes the discount factor for OLG models should be measured with data because the economic theory does not impose any restriction on the size of the discount factor unlike business cycle models. Labor and capital tax, $\tau^k$, $\tau^l$ are taken from Gomme and Rupert (2007). Since there are no studies that estimate state-level effective taxes in the U.S., this study treats those values as the averages of the state-level rates. The calibrated tax rates vary with grouping of the states: Group A takes the exactly same as Gomme and Rupert (2007); Group B takes slightly lower than the averages. The small adjustment for the latter case shall be conducted to keep consistency with the observations and the other parameters. See table 2 for the grouping for the tax rates.

Table 2 shows the calibrated region-specific parameters. As earlier mentioned, the political bias parameters $\phi^m(0)$ for $m = \{k, l\}$ are not taken from literature, but estimated with ANES data. The remaining parameters, $\rho$ and $\theta$ are pinned down together with endogenous distribution $\mu^*$ using equation (29), (30) and (31), given the observations of growth rate, $1 + \nu$ and capital-output ratio, $K/Y = (A\eta^*\alpha^{-1})^{-1}$. Finally, population size $N$ is pinned down by equation (32), given $\mu^*$,

---

13 The original measures by Yamarik (2013) yield very low capital-output ratios (0.9 - 1.2 on annual basis), and hence very high steady-state interest rates that cause a failure in matching with the GSP growth rates.
<table>
<thead>
<tr>
<th>name of state</th>
<th>pecuniary cost parameter $\rho$</th>
<th>fraction of neutral capitalists $\phi^*(0)$</th>
<th>fraction of neutral workers $\phi^*(0)$</th>
<th>time efficiency parameter $\theta$</th>
<th>size of population $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alabama</td>
<td>AL 0.149</td>
<td>0.290</td>
<td>0.319</td>
<td>0.050</td>
<td>2.295</td>
</tr>
<tr>
<td>Arkansas</td>
<td>AR 0.077</td>
<td>0.307</td>
<td>0.344</td>
<td>0.162</td>
<td>2.101</td>
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<tr>
<td>California</td>
<td>CA 0.134</td>
<td>0.147</td>
<td>0.175</td>
<td>0.065</td>
<td>2.284</td>
</tr>
<tr>
<td>Connecticut</td>
<td>CT 0.019</td>
<td>0.365</td>
<td>0.349</td>
<td>0.281</td>
<td>1.881</td>
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<tr>
<td>Florida</td>
<td>FL 0.179</td>
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<td>0.182</td>
<td>0.035</td>
<td>2.335</td>
</tr>
<tr>
<td>Georgia</td>
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<td>0.242</td>
<td>0.281</td>
<td>0.090</td>
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</tr>
<tr>
<td>Illinois</td>
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<td>0.315</td>
<td>1.847</td>
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<tr>
<td>Indiana</td>
<td>IN 0.103</td>
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<td>0.292</td>
<td>0.133</td>
<td>2.180</td>
</tr>
<tr>
<td>Iowa</td>
<td>IA 0.150</td>
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<td>0.316</td>
<td>0.049</td>
<td>2.298</td>
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<tr>
<td>Maryland</td>
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<td>0.284</td>
<td>0.003</td>
<td>2.245</td>
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<td>Massachusetts</td>
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<td>0.218</td>
<td>0.295</td>
<td>0.042</td>
<td>2.225</td>
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<td>Michigan</td>
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<td>0.263</td>
<td>0.135</td>
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<tr>
<td>Minnesota</td>
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<td>New Jersey</td>
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<td>North Carolina</td>
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<td>0.318</td>
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<td>Ohio</td>
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<td>Oregon</td>
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<td>0.183</td>
<td>0.357</td>
<td>1.787</td>
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<td>Tennessee</td>
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<td>0.038</td>
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<td>Virginia</td>
<td>VA 0.188</td>
<td>0.277</td>
<td>0.215</td>
<td>0.039</td>
<td>2.303</td>
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<td>Washington</td>
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<td>Wisconsin</td>
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<td>0.260</td>
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<tr>
<td><strong>Group B</strong></td>
<td></td>
<td></td>
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<tr>
<td>Colorado</td>
<td>CO 0.090</td>
<td>0.301</td>
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<td>0.045</td>
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<td>0.187</td>
<td>0.176</td>
<td>2.022</td>
</tr>
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<td>Texas</td>
<td>TX 0.048</td>
<td>0.254</td>
<td>0.200</td>
<td>0.183</td>
<td>1.995</td>
</tr>
</tbody>
</table>

$\theta$ as well as normalized aggregated labor i.e. $L^* = 1$.

4 Effects of exogenous factors on government policy, distribution of heterogeneous agents and growth

This section shows how market institution parameter $\rho$, political preference $\phi^m(0)$ and innate productivity parameter $\theta$ affect the political equilibrium. Since our interest is the behavior of the equilibrium in the long-run, we focus on the steady state. The endogenous variables, $\nu, \eta^*, \mu^*, L^*$ are determined by the system of equations (29) - (32). Given all the other parameters fixed, I examine how the steady-state equilibrium outcomes change according to the value of a parameter. I conduct this exercise for all the states, and find that the directions of the effects are identical. Therefore, what follows discusses the general findings, illustrating the results for state of Alabama.

Figure 4 shows the results. It consists of nine panels: (A1)-(A4) show the effects of change in
Figure 4: Effects of change in parameters on political equilibrium

\(\rho; (B1)-(B4)\) the effects of change in \(\phi^k(0)\), given \(\phi^l(0)\); and \((C1)-(C4)\) the effects of change in \(\theta\). The observed effects are all monotonic with respect to the parameter.

### 4.1 Market institution

From panel (A1)-(A4), the effects of change in \(\rho\) are summarized as follows.

\[
\frac{\partial \eta^*}{\partial \rho} > 0, \quad \frac{\partial \mu^*}{\partial \rho} < 0, \quad \frac{\partial L^*(\mu^*(\rho))}{\partial \rho} < 0, \quad \frac{\partial \nu(\eta^*(\rho), \mu^*(\rho), L^*(\rho), \rho)}{\partial \rho} < 0
\]

Consider economic intuition for the decreasing growth rate in panel (A4). First, recall that \(\rho\) is pecuniary cost for being capitalists. An increment in \(\rho\) therefore decreases savings, thus having a direct impact of decreasing the growth rate. (See LHS of equation (29) where a negative \(\rho\) term explicitly emerges.) Second, observe that the growth rate also depends on the other endogenous variables, \(\eta^*(\rho), \mu^*(\rho), L^*(\mu^*(\rho))\). The change in those endogenous variables also influences on the growth. Since the aggregated labor \(L^*(\cdot)\) is an increasing function of \(\mu^*\) (see equation (32)), the sign of the partial derivative is identical to that of \(\mu^*\). We then focus on the change in capital-public-good ratio \(\eta^*(\rho)\) and agent-type distribution \(\mu^*(\rho)\). Those two variables are determined by
non-linear simultaneous equations (30), (31) where the former is the first order condition for the voting game; the latter is the cut-off condition for the agent-type choice. The numerical calculation results in monotonic decrease in regard to both variables as shown in panel (A1) and (A2), but the notion of economic intuition is complicated. Consider the case where \( \rho \) is marginally increased. In the agent-type choice, on one hand, an increment in the pecuniary cost for being capitalists reduces the net amount of savings, and hence ex post lifetime utility of “capitalists”. On the other hand, given the electoral outcome fixed, it decreases the capital tax revenue, and hence the income transfer to workers, thus decreasing the ex post lifetime utility of “workers”. Since the lifetime utility of both agents are lowered, it is ambiguous whether the proportion of capitalists, \( \mu^* \) is eventually increased or decreased. However, the consequence generates a forward effect anyhow on the electoral competition that subsequently happens. In the election, given the current capital stock, the change in \( \eta^* \) depends on the agent-type distribution. Recalling that \( \mu^* \) is a weight for capitalists in social welfare function (28), increase in \( \mu^* \) lowers \( \eta^* \) whereas decrease in \( \mu^* \) raises \( \eta^* \). Since individuals have perfect foresight for the subsequent events, the effect of the electoral outcome propagates backward to the agent-type choice. Eventually, all those back-and-forth effects results in increase in \( \eta^* \) and decrease in \( \mu^* \) such that it shall be consistent with the graphs of panel (A1) and (A2). Increase in \( \eta^* \) is equivalent to decrease in public good expenditure whereas decrease in \( \mu^* \) lowers the savings, and hence decrease capital stock. Both those effects reduces the growth rate. \( L^* \) turns out to decrease because \( \mu^* \) decreases. Decrease in \( L^* \) raises the marginal product of labor (see equation (12)), and hence savings. This raises the growth rate, but this positive effect is canceled out by the other effects. The growth rate is, in turn monotonically decreased by an increment in \( \rho \).

4.2 Political preference

From panel (B1)-(B4), the effects of political preference are given by

\[
\frac{\partial \eta^*}{\partial \phi^{sk}(0)} < 0, \quad \frac{\partial \mu^*}{\partial \phi^{sk}(0)} > 0, \quad \frac{\partial L^*(\mu^*(\phi^{sk}(0)))}{\partial \phi^{sk}(0)} > 0,
\]

\[
\frac{\partial \nu}{\partial \phi^{sk}(0)} \left( \eta^*(\phi^{sk}(0)), \mu^*(\phi^{sk}(0)), L^*(\phi^{sk}(0)) \right) > 0.
\]

Likewise to the last subsection, consider the intuition for panel (B4). Since the political bias parameter is not explicitly included in function \( \nu(\eta^*, \mu^*, L^*) \), the change in \( \phi^{sk}(0) \) does not directly affect growth. (See LHS of equation (29) where there is no \( \phi^{sk}(0) \) terms.) Consider the case where \( \phi^{sk}(0) \) is marginally increased, given \( \phi^{el}(0) \). In the electoral competition, given the current capital, an increment in \( \phi^{sk}(0) \) lowers \( \eta^* \), and hence raises public good expenditure because \( \phi^{sk}(0) \) is a weight for capitalists’ welfare in the voting game. (See equation (28).) This results in increase in wage and rent of capital. In the agent-type decision, the enhanced prices raise ex post lifetime utility of capitalists and workers, but the effects on \( \mu^* \) is again ambiguous. However, panel (B2) indicates that more individuals turn out to choose to be capitalists because the ex post welfare of capitalists are much better off than that of workers. Thus increase in \( \phi^{sk}(0) \) enhances both capital
stock and the public good expenditure. The capital stock is raised by increase in $\mu^*$. The public good expenditure is enhanced by increase both in $\phi^k(0)$ and $\mu^*$. $L^*$ is enhanced due to increase in $\mu^*$, thus lowering savings. However this negative effect is very limited. The growth rate is, in turn raised due to the dominant positive effects.

4.3 Innate productivity

From panel (C1)-(C4), the effects of innate productivity are given by

$$\frac{\partial \eta^*}{\partial \theta} > 0, \quad \frac{\partial \mu^*}{\partial \theta} < 0, \quad \frac{\partial L^*}{\partial \theta} \geq 0, \quad \frac{\partial \nu(\eta^*(\theta), \mu^*(\theta), L^*(\theta))}{\partial \theta} < 0$$

Finally, we study the case for innate productivity. Since the innate parameter is not an explicit parameter of function $\nu(\eta^*, \mu^*, L^*)$, we focus on the effects of change in the endogenous variables. Consider the case where $\theta$ is marginally increased. In the agent-type decision, on one hand, an increment in $\theta$ raises the ex post lifetime utility of workers. On the other hand, given the electoral outcome fixed, the increase in $\theta$ raises the labor tax revenue, thus decreasing $\eta^*$. Again, the effects on $\mu^*$ is vague although it affects on the electoral outcome. Similar to the case for the exercise on $\rho$, the back-and-forth effects result in increase in $\eta^*$ and decrease in $\mu^*$. The change in $L^*$ is not monotonic because the change in $\theta$ directly affects the aggregate-level of labor. Given all those effects, the growth rate in turn monotonically decreases as the productivity increases. This implies that the non-monotonic effect of change in $L^*$ is washed out.
5 Spatial variation

This section examines the effect of each of the region-specific parameters on growth in the steady state.

Figure 5 shows the disparity of the parameters among states. Panel (2) implies a weak seeming-correlation between \( \rho \) and \( \theta \). The less institutional cost for being capitalists a state has, the more efficient labor supply by workers it tends to have. In contrast, panel (1) indicates that there are various combinations of the political bias and the institutional costs, and hence so are in the combination of the bias and the labor efficiency. The variations in \( \theta \) and \( \rho \) are relatively large.

Table 3 shows the variations in the steady-state outcomes. Recall that the model is calibrated such that it matches with the observation of the growth rate and capital-output ratio. Those two observations in table 3 are on four-year basis. The growth rate approximately varies 6–10% whereas agent distribution \( \mu^* \) varies 0.52–0.67. The ranges of the other endogenous variables slightly differ according to the state groups. The capital-output ratio is approximately 0.60 – 0.75 for Group A whereas 0.79 – 0.82 for Group B. The capital-public good ratio takes approximately 2.6 – 3.8 for Group A whereas 4.0 – 4.3 for Group B. The public-good-tax-revenue ratio approximately varies 77 – 82% for Group A whereas 85 – 87% for Group B. These group-specific ranges of the variation
Table 3: Variation in political equilibria

<table>
<thead>
<tr>
<th>state</th>
<th>growth rate $\nu$ (⋆)</th>
<th>capital-output ratio (⋆⋆)</th>
<th>distribution $\mu^*$</th>
<th>capital-public-good ratio $\eta^*$</th>
<th>public-good-tax-revenue ratio $g_t/T_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AL</td>
<td>8.00%</td>
<td>0.698</td>
<td>0.588</td>
<td>3.360</td>
<td>78.0%</td>
</tr>
<tr>
<td>AR</td>
<td>8.16%</td>
<td>0.741</td>
<td>0.564</td>
<td>3.687</td>
<td>77.9%</td>
</tr>
<tr>
<td>CA</td>
<td>7.44%</td>
<td>0.724</td>
<td>0.577</td>
<td>3.557</td>
<td>76.9%</td>
</tr>
<tr>
<td>CT</td>
<td>10.48%</td>
<td>0.718</td>
<td>0.594</td>
<td>3.509</td>
<td>81.5%</td>
</tr>
<tr>
<td>FL</td>
<td>7.41%</td>
<td>0.676</td>
<td>0.585</td>
<td>3.193</td>
<td>78.6%</td>
</tr>
<tr>
<td>GA</td>
<td>9.17%</td>
<td>0.718</td>
<td>0.591</td>
<td>3.509</td>
<td>78.2%</td>
</tr>
<tr>
<td>IL</td>
<td>7.29%</td>
<td>0.767</td>
<td>0.557</td>
<td>3.891</td>
<td>78.7%</td>
</tr>
<tr>
<td>IN</td>
<td>7.20%</td>
<td>0.738</td>
<td>0.551</td>
<td>3.665</td>
<td>77.2%</td>
</tr>
<tr>
<td>IA</td>
<td>7.89%</td>
<td>0.700</td>
<td>0.588</td>
<td>3.370</td>
<td>77.9%</td>
</tr>
<tr>
<td>MD</td>
<td>8.74%</td>
<td>0.604</td>
<td>0.666</td>
<td>2.676</td>
<td>82.2%</td>
</tr>
<tr>
<td>MA</td>
<td>10.05%</td>
<td>0.705</td>
<td>0.637</td>
<td>3.411</td>
<td>78.3%</td>
</tr>
<tr>
<td>MI</td>
<td>6.09%</td>
<td>0.712</td>
<td>0.520</td>
<td>3.462</td>
<td>78.1%</td>
</tr>
<tr>
<td>MN</td>
<td>9.27%</td>
<td>0.748</td>
<td>0.583</td>
<td>3.741</td>
<td>79.3%</td>
</tr>
<tr>
<td>MS</td>
<td>7.56%</td>
<td>0.693</td>
<td>0.570</td>
<td>3.321</td>
<td>78.3%</td>
</tr>
<tr>
<td>NJ</td>
<td>9.18%</td>
<td>0.721</td>
<td>0.575</td>
<td>3.529</td>
<td>79.2%</td>
</tr>
<tr>
<td>NC</td>
<td>8.52%</td>
<td>0.653</td>
<td>0.618</td>
<td>3.025</td>
<td>79.9%</td>
</tr>
<tr>
<td>OH</td>
<td>6.88%</td>
<td>0.728</td>
<td>0.542</td>
<td>3.585</td>
<td>77.5%</td>
</tr>
<tr>
<td>OR</td>
<td>7.24%</td>
<td>0.753</td>
<td>0.555</td>
<td>3.777</td>
<td>79.8%</td>
</tr>
<tr>
<td>TN</td>
<td>8.37%</td>
<td>0.683</td>
<td>0.601</td>
<td>3.245</td>
<td>78.7%</td>
</tr>
<tr>
<td>VA</td>
<td>9.80%</td>
<td>0.624</td>
<td>0.596</td>
<td>2.817</td>
<td>82.3%</td>
</tr>
<tr>
<td>WA</td>
<td>8.20%</td>
<td>0.672</td>
<td>0.632</td>
<td>3.164</td>
<td>78.7%</td>
</tr>
<tr>
<td>WI</td>
<td>8.11%</td>
<td>0.710</td>
<td>0.589</td>
<td>3.446</td>
<td>77.6%</td>
</tr>
<tr>
<td>Group B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO</td>
<td>9.23%</td>
<td>0.785</td>
<td>0.688</td>
<td>4.031</td>
<td>84.6%</td>
</tr>
<tr>
<td>NY</td>
<td>7.32%</td>
<td>0.793</td>
<td>0.619</td>
<td>4.095</td>
<td>87.2%</td>
</tr>
<tr>
<td>PA</td>
<td>7.62%</td>
<td>0.795</td>
<td>0.619</td>
<td>4.118</td>
<td>85.3%</td>
</tr>
<tr>
<td>TX</td>
<td>8.37%</td>
<td>0.816</td>
<td>0.640</td>
<td>4.288</td>
<td>84.9%</td>
</tr>
</tbody>
</table>

(⋆) The calibrated growth rate are equal to the averages of four-year GSP growth rates for 1972-2003.
(⋆⋆) The calibrated ratios are equal to the average of the re-scaled capital stock at the beginning of the year divided by sum of GSP for subsequent four years for 1972-2003.

result from different tax rates. Nevertheless, ratio $g_t/T_t$ representing the government expenditure composition does not much differ within the group.

Next we study which parameter has the most significant effect on growth. In particular, we compute the total differential of growth rate with respect to region-specific parameters. Applying chain rule for equation (29), it is derived by

$$
\frac{dv}{db} = \frac{\partial v}{\partial \eta^*} \frac{\partial \eta^*}{\partial b} + \frac{\partial v}{\partial \mu^*} \frac{\partial \mu^*}{\partial b} + \frac{\partial v}{\partial L^*} \frac{\partial L^*}{\partial b} + \frac{\partial v}{\partial b}
$$

where $b \in B : \{ \rho, \phi^{*k}(0), \theta \}$. For any of the states, the signs of the computed differential are given by

$$
\frac{dv}{d\rho} < 0, \quad \frac{dv}{d\phi^{*k}(0)} > 0, \quad \frac{dv}{d\theta} < 0
$$

25
These consequences coincides the results of comparative statics in section 4. We then calculate the ratio of the differential on percentage given by

\[ R_b = \frac{\left| \frac{d\nu}{db} \right|}{\left| \frac{d\nu}{d\rho} \right| + \left| \frac{d\nu}{d\phi(0)} \right| + \left| \frac{d\nu}{d\theta} \right|} \times 100 \, (\%) \]  

(34)

where \( R_{\rho} + R_{\phi(0)} + R_{\theta} = 100 \, (\%) \). The absolute value is taken for each of the derivatives to conform the signs.

Figure 6 - 8 are color-coded American maps showing the cross-state outcomes. First, focus on identifying the most influential channel on growth. The effect of \( \rho \) is the largest in most of the states (\( R_{\rho} \approx 40 - 60\% \)). The exceptions are Florida, Washington, North Carolina and Maryland where \( \theta \) has the largest impact (\( R_{\theta} \approx 40 - 80\% \)). The effects of political preference are moderate (\( R_{\phi(0)} \approx 20 - 30\% \)) in the entire states. In sum, the market institution channel has the most considerable power that forms the cross-state disparity in growth, but in some of states, innate productivity of workers has the most significant effect on growth. The impact of political preference is relatively moderate.

Second, we can observe cross-state variation within the individual figure. Roughly speaking, on figure 6, the effect of \( \rho \) is larger in the southeast part (e.g. Alabama, Florida, North Carolina, Tennessee) whereas it is smaller in the northeast part (e.g. Illinois, Michigan, New York, Pennsylvania). In addition, taking into consideration the calibrated value of \( \rho \) in table 2, states with higher costs for generating capitalists (e.g. Alabama, California, Florida, Washington) exhibit the larger impact viz. that the value of the total derivative is larger as \( \rho \) takes the higher value. This implies that, in those states, the market institution considerably prevents the economic growth.

Figure 7 shows the large disparity in the cross-state effect with respect to the political preference. The impact of the effect is large in California, Florida and Mississippi whereas not much in the other states. Given the calibration in table 2 it is interesting that the effect is small in states with remarkably more \( k \)-type swing voters i.e. higher \( \phi^k(0)/\phi^l(0) \) (e.g. New York, Pennsylvania, Texas). This implies the difference in political preference does not critically affects growth.

Similarly, figure 8 shows the large disparity in the effect by \( \theta \). The effect is large especially in North Carolina, Maryland and Washington. However, the calibrated value of \( \theta \) for those states takes small value (see table 2). This implies that in those states, there is limited room to enhance economic growth by raising the labor productivity of workers somehow.
Notes:
(1) Color codes indicate the level of total differentiation $\frac{du}{d\rho}$.
(2) Percentages below state names are ratio $R_\rho$.

Figure 6: Impact of market institution parameter $\rho$ on growth
Notes:

(1) Color codes indicate the level of total differentiation \( \frac{d\nu}{d\phi^k(0)} \).

(2) Percentages below state names are ratio \( R_{\phi^k(0)} \).

Figure 7: Impact of political institution parameter \( \phi^k(0) \) on growth
Notes:
(1) Color codes indicate the level of total differentiation $\frac{d\nu}{d\theta}$. The color code for Maryland is adjusted to that for the second largest value, (i.e. 2.5 for Washington) because the absolute value of the total differential for Maryland is extremely large (> 7.7).
(2) Percentages below state names are ratio $R_\theta$.

Figure 8: Impact of innate productivity parameter $\theta$ on growth
6 Conclusion

This paper proposes a new political-economy growth model to shed light on cross-region disparity in economic growth. Given no population growth and identical level of technology among regions, the proposed model introduces some institutional channels such that they shape the growth of capital, infrastructure expenditure and hence output in the local economy. The endogenous distribution of the agents plays an important role to bridge the region-specific institutions and economic growth in a non-linear fashion. The calibration exercise with cross-state U.S. data shows that the market institution causes the largest impact on the growth in most of the selected American states whereas political preference exhibits the moderate impact.

References


Campbell, Albert Angus, Philip Ernest Converse, Warren E. Miller, and Donald Elkin-tont Stokes. 1960. The American Voter.: Wiley & Sons , Inc..


A Proof for lemma 1

Using equation (5) and (6), the first order derivatives of $c_{t}^{y,k}$ and $c_{t+1}^{o,k}$ with respect to $e_{t}$ are derived as

\[
\frac{\partial c_{t}^{y,k}}{\partial e_{t}} = - \frac{(1 - \tau^{l})w_{t}}{1 + \beta} < 0
\]

\[
\frac{\partial c_{t+1}^{o,k}}{\partial e_{t}} = - \frac{\beta(1 - \tau^{k})(1 + \tau_{t+1})(1 - \tau^{l})w_{t}}{1 + \beta} < 0
\]

Therefore,

\[
\frac{\partial U^{k}}{\partial e_{t}} = \frac{1}{c_{t}^{y,k}} \frac{\partial c_{t}^{y,k}}{\partial e_{t}} + \frac{\partial c_{t+1}^{o,k}}{\partial e_{t}} < 0
\]

Thus $U^{k}(K_t, \cdot)$ is strictly decreasing on $E$. In the meantime, given $K_t$, $U^{l}(K_t)$ takes a constant. Letting $U^{k}$ denote a whole set of $U^{k}(K_t, \cdot)$, the graphs of $U^{k}(K_t, \cdot)$ and $U^{l}(K_t)$ obviously have a unique intersection in $(E \times U^{k})$ space. (Q.E.D)
B  \textit{Ex ante} decision with smooth distribution on ability space

Suppose \( e_t \) is an i.i.d. random variable, and the distribution is given by a smooth density function \( h(\cdot) \) on \([0, 1]\). What follows focuses on some derivations that differ from the case of uniform distribution. First, lemma 1 obviously holds also in this case, the fraction of agents who choose to be capitalists is given by

\[
\mu^y_t = \int_0^{e_t^*} h(z) dz
\]

where \( z \) is a dummy variable for \( e_t \). This re-shapes the aggregate-level variables: the aggregated labor supply by capitalists is given by

\[
l_t = N \int_0^{e_t^*} h(z) dz;
\]

the aggregated consumption is by

\[
C_t = \int_0^{e_t^*} c^y_t h(z) dz + (1 - \mu^y_t)c^d_t + \int_0^{e_t^*} c^o_t h(z') dz' + (1 - \mu^o_t)c^d_t
\]

\[
= (1 - \tau^l)L_tw_t - (S_t + N\mu^y_t\gamma_t) + (1 - \tau^k)(1 + r_t)S_{t-1} + N(1 - \mu^o_t)c_{g,t};
\]

the aggregated savings is by

\[
S_t = \int_0^{e_t^*} s_t h(z) dz
\]

where \( s_t \) is given by equation (4). Note that the market clearing condition (18) holds in this case, too. In the electoral competition, the expected vote share is given by

\[
\pi_A(K_{t+1}, g^A_{t+1}, g^B_{t+1}, e_t) = N \int_0^{e_t^*} h(z) \phi^k_{t+1} \left[ V^k(K_{t+1}, g^A_{t+1}, e_t) - V^k(K_{t+1}, g^B_{t+1}, e_t) \right] dz
\]

\[
+ N(1 - \mu^o_t)\Phi^l_{t+1} \left[ V^l(K_{t+1}, g^A_{t+1}) - V^l(K_{t+1}, g^B_{t+1}) \right]
\]

The first order condition is then given by

\[
\phi^k_{t+1}(0) \int_0^{e_t^*} \frac{\partial V^k(z)}{\partial g_{t+1}} h(z) dz + (1 - \mu^k_{t+1})\phi^l_{t+1}(0) \frac{\partial V^l}{\partial g_{t+1}} = 0
\]

With these characterization, the political equilibrium exists and lemma 4 is ensured to hold.

C  Proof for lemma 2

The first order derivatives of \( V^m \) with respect to \( g_{t+1} \) are given by

\[
\frac{\partial V^m}{\partial g_{t+1}} = \beta \frac{1}{c^o_{t+1}} \frac{\partial c^o_{t+1}}{\partial g_{t+1}}
\]

where

\[
\frac{\partial c^o_{t+1}}{\partial g_{t+1}} = \beta \frac{(1 - \tau^k) \left[(1 - \tau^l)(1 - e_t) w_t - \gamma_t \right]}{1 + \beta} \frac{\partial (1 + r_{t+1})}{\partial g_{t+1}} > 0
\]
\[
\frac{\partial c_{o,l}^{t+1}}{\partial g_{t+1}} = \frac{1}{N(1 - \mu_{o,t}^{t+1})} \left[ \frac{\partial T_{t+1}}{\partial g_{t+1}} - 1 \right]
\]

Therefore, \( V^k \) is monotonically increasing whereas \( V^l \) not. The second order derivatives are given by
\[
\frac{\partial^2 V^m}{\partial g_{t+1}^2} = -\beta \frac{1}{(c_{m,t+1}^{t+1})^2} \left( \frac{\partial c_{o,m}^{t+1}}{\partial g_{t+1}} \right) + \beta \frac{1}{c_{m,t+1}^{t+1}} \frac{\partial^2 c_{o,m}^{t+1}}{\partial g_{t+1}^2}
\]

where
\[
\frac{\partial^2 c_{o,k}^{t+1}}{\partial g_{t+1}^2} = \frac{\beta (1 - \tau_k) \left[ (1 - \tau_l) (1 - e_t) w_t - \gamma_l \right]}{1 + \beta} \frac{\partial^2 (1 + r_t)}{\partial g_{t+1}^2} < 0
\]
\[
\frac{\partial c_{o,l}^{t+1}}{\partial g_{t+1}} = \frac{1}{N(1 - \mu_{o,t}^{t+1})} \left[ \frac{\partial^2 T_{t+1}}{\partial g_{t+1}^2} \right]
\]
\[
= \frac{1}{N(1 - \mu_{o,t}^{t+1})} \left[ \tau_l L_{t+1} \frac{\partial^2 w_{t+1}}{\partial g_{t+1}^2} + \tau_k S_t \frac{\partial^2 (1 + r_{t+1})}{\partial g_{t+1}^2} \right] < 0
\]

Therefore,
\[
\frac{\partial^2 V^m}{\partial g_{t+1}^2} < 0 \quad \forall m \in \{k, l\}
\]

Given the concavity and the monotonic increasing property, \( V^k \) is maximized at a corner point given by
\[ g_{t+1}^{*k} = T_{t+1} \]

Likewise, given the concavity, \( V^l \) is maximized at a unique point given by
\[ g_{t+1}^{*l} \in G_{t+1} \quad \text{such that} \quad \left. \frac{\partial V^l}{\partial g_{t+1}} \right|_{g_{t+1}=g_{t+1}^{*l}} = 0 \] (Q.E.D.)

### D Proof for proposition 1

From lemma \(2\) \( V^m(\cdot) \quad \forall m \in \{k, l\} \) is monotonically increasing on \([0, g_{t+1}^{*l}]\). Therefore, given \( K_{t+1} \), the conflicting preference arises on \((g_{t+1}^{*l}, T_{t+1})\). Corner point \( g_{t+1}^{*l} \) is given as a solution of an equation given by
\[
\frac{\partial V^l}{\partial g_{t+1}} = 0
\]
\[ \Leftrightarrow \frac{\partial T_{t+1}}{\partial g_{t+1}} - 1 = 0 \]
\[ \Leftrightarrow \Lambda \frac{\partial Y_t}{\partial g_{t+1}} - 1 = 0 \]
\[ \Leftrightarrow \eta_{t+1} = \left[ \frac{1}{(1 - \alpha) \Lambda L_{t+1}^{1-\alpha}} \right]^{\frac{1}{\alpha}} \]
Likewise, $g_t^{k+1} = T_{t+1}$ is a solution of an equation given by

$$T_{t+1} = g_t$$

$$⇔ \Lambda Y_t + \tau^k N \mu_{t+1}^y \rho K_{t+1} - g_{t+1} = 0$$

$$⇔ \Lambda AL_{t+1}^{1-\alpha} \eta_{t+1}^\alpha + \tau^k \mu_{t+1}^y \rho K_{t+1} - 1 = 0$$

Summarizing these results are equivalent to the proposition. (Q.E.D.)

### E Estimation of political bias

This part outlines how to estimate Markov chain (24) step by step. For more details, see Okabe (2014).

#### E.1 Multinomial logistic regression

Letting $T$ be the total number of sampling periods, the first step computes $\{\Phi\}_{t=0}^{T-1}$ using estimators of multinomial logistic (MNL) regression. The original data set is repeated cross-section with 26 states for 1972 - 2002. First, we disaggregate the data into state-by-state samples and further separate them into eight periods (i.e. $T = 8$). The data is now divided into (26 states) × (8 periods) sample groups, $M$. Table E1 summarizes the sample sizes of $M$. Next, we conduct the MNL regressions with each of the group samples. Given a sample group $m \in M$, Let $d_i$ denote individual $i$’s response, and $c_i$ ($K + 1$)-vector of individual $i$’s attributes. The MNL model gives the response probability that individual $i$ chooses PID $s_j \in S = \{s_1, s_2, s_3\}$

$$\text{Prob} (d_i = s_j | c_i) = \pi_{ij}$$

where $0 < \pi_{ij} < 1$ and $\sum_j \pi_{ij} = 1$. In particular, the response probability is parameterized as

$$\left\{ \begin{array}{l}
\pi_{i1} = \frac{1}{1 + \sum_{r=2}^J \exp (c'_i \beta_r)} , \\
\pi_{ij} = \frac{\exp (c'_i \beta_j)}{1 + \sum_{r=2}^J \exp (c'_i \beta_r)} , \\
\end{array} \right. \quad j = 2, 3$$

where $\beta_j$ is a ($K + 1$)-vector for $j = 2, 3$, and $c'_i \beta_j = \beta_{j0} + \beta_{j1} c_{i1} + \cdots + \beta_{jK} c_{iK}$. Given the sample of $I$-paired observations $(d_i, c_i)$, the log-likelihood function is then given by

$$\ln L(\beta_2, \beta_3; d, c) = \sum_{i=1}^I \sum_{j=1}^3 d_{ij} \log \pi_{ij}$$

where $d = \{d_1, \cdots, d_I\}'$ and $c = \{c_1, \cdots, c_I\}'$. Maximizing the log-likelihood function gives the estimators $\hat{\beta}_j$ for $j = 2, 3$. Suppose individuals with attributes $c_h$ are categorized as agent-group
Table E1: ANES cross-section data

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Total  3,519  3,751  2,563  3,918  3,650  3,753  2,669  2,816  26,639
The estimated probability that those individuals choose alternative \( s_j \) is then given by

\[
\hat{\pi}_{hj} = \frac{\exp \left( c'_h \hat{\beta}_j \right)}{1 + \sum_{r=2}^{J} \exp \left( c'_h \hat{\beta}_r \right)} \quad \text{for } j = 1, 2, 3
\]

(E.1)

Replacing \( \hat{\pi}_{hj} \) with \( \phi_t(s_j) \) in equation (25), we can obtain the estimated distribution \( \hat{\Phi}_t \). The predicted distribution has the confidence interval which is ensured by the asymptotic normality assumption of the MNL model.

### E.2 Maximum entropy method

Given the sequence of PID distribution, \( \{\hat{\Phi}_t\}_{t=0}^{T-1} \) obtained in the first step, the next step estimates the transitional matrix \( \lambda \) using maximum entropy method. Entropy is a measure for the level of uncertainty in a probability distribution parametrized on a set of events, proposed by Shannon (1948). Using the entropy concept, Jaynes (1957a,b) propose the principle of maximum entropy. The key idea is to estimate the unknown parameters such that they maximize the entropy subject to the imposed constraints on the observations and other available information. The second step estimation employs generalized entropy method (GME) that allows researchers to take into consideration the measurement error. Suppose the observation constraints are given by

\[
b = A p + e
\]

where

\[
b := \begin{bmatrix} \hat{\Phi}_1, \cdots, \hat{\Phi}_T \end{bmatrix}',
\]

\[
A := \begin{bmatrix} A(0) \\ \vdots \\ A(T-1) \end{bmatrix}, \quad A(t) := \begin{bmatrix} \hat{\Phi}_t & 0 & \cdots & 0 \\ 0 & \hat{\Phi}_t & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\Phi}_t \end{bmatrix},
\]

\[
p := [\lambda(1), \lambda(2), \lambda(3)]', \quad \lambda(j) := [\lambda_{j1}, \lambda_{j2}, \lambda_{j3}]
\]

\[
e := [e_{11}, e_{12}, e_{13}, \cdots, e_{T1}, e_{T2}, e_{T3}]'
\]

where \( b \) is a 3\( T \)-vector of the observations, \( A \) a \( (3T \times 3^2) \) matrix of the observations, \( e \) a \( 3T \)-vector of unobserved noise. The noise term shall be included in the constraint because sequence \( \{\hat{\Phi}_t\}_{t=0}^{T-1} \) are estimated with the PID observations that are “subjective” responses based on personal belief concerning ideology. Next suppose a compact support for each \( \lambda_{xy} \) that allows one to treat it as a random variable. \( p \) matrix is then re-parameterized as

\[
p = Z q = \begin{bmatrix} Z(1) & 0 & 0 \\ 0 & Z(2) & 0 \\ 0 & 0 & Z(3) \end{bmatrix} \begin{bmatrix} q(1) \\ q(2) \\ q(3) \end{bmatrix}, \quad Z(j) := \begin{bmatrix} z_{j1} & z_{j1} & 0 & 0 & 0 & 0 \\ 0 & 0 & z_{j2} & z_{j2} & 0 & 0 \\ 0 & 0 & 0 & 0 & z_{j3} & z_{j3} \end{bmatrix}
\]

36
\[ q(j) := \left[ q_{j1}, q_{j2}, q_{j3}, q_{j1}', q_{j2}', q_{j3}' \right]' \]

where \( Z \) is a \((3^2 \times 2 \cdot 3^2)\) sparse matrix representing the supports, \( q \gg 0 \) is a \((2 \cdot 3^2)\)-vector of probabilities. Likewise, suppose each element of noise vector \( e \) has a compact support. The re-parametrization is then given by

\[
\begin{align*}
\mathbf{e} &= \mathbf{Vw} = \\
&= \begin{bmatrix} \mathbf{V}(0) & 0 & \cdots & 0 \\
0 & \mathbf{V}(1) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \mathbf{V}(T-1) \end{bmatrix} \begin{bmatrix} \mathbf{w}(0) \\
\mathbf{w}(1) \\
\vdots \\
\mathbf{w}(T-1) \end{bmatrix},
\end{align*}
\]

\[
\mathbf{V}(t) := \begin{bmatrix} \tau_{t1} & \tau_{t1} & 0 & 0 & 0 & 0 \\
0 & 0 & \tau_{t2} & \tau_{t2} & 0 & 0 \\
0 & 0 & 0 & 0 & \tau_{t3} & \tau_{t3} \end{bmatrix}, \quad \mathbf{w}(t) := [w_{t1}, w_{t1}, w_{t2}, w_{t2}, w_{t3}, w_{t3}]'
\]

where \( \mathbf{V} \) is a \((3T \times 2T \cdot 3)\) sparse matrix representing the supports and \( \mathbf{w} \gg 0 \) a \((2T \cdot 3)\) vector of probabilities. The GME estimators are given by solving the optimization problem given by

\[
\max_{\mathbf{q}, \mathbf{w}} \Gamma(\mathbf{q}, \mathbf{w}) = -\mathbf{q}' \ln(\mathbf{q}) - \mathbf{w}' \ln(\mathbf{w})
\]

subject to

\[
\mathbf{b} = AZ\mathbf{q} + \mathbf{Vw}
\]

\[
1_J = (I_J \otimes 1_J')\mathbf{q}
\]

\[
1_T = (I_T \otimes 1_T')\mathbf{w}
\]

\[
\mathbf{q} \geq 0
\]

\[
\mathbf{w} \geq 0
\]

where \( \Gamma \) is the entropy.

In addition, this paper employs extra constraints such that the transitional matrix is given by

\[
\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & 0 \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
0 & \lambda_{32} & \lambda_{33} \end{bmatrix}
\]

This follows an hypothesis by Mebane and Wand (1997) that American individual PID gradually changes in the short-run. The hypothesis can be verified by Wald test. The idea for adding the extra constraints is also illustrated in Okabe (2014).
E.3 Results

Since the estimations include so many regressions, this section shows only the result for Alabama. I conduct the first step estimation with four control variables, i.e. sex, age (10-year interval, race (white, black and others) and occupation (see footnote 12 in section 3.3). Table E2 shows the estimated distributions. The estimated distribution differ only in occupation, given all the other control variables are fixed. Some of the distributions are identical due to the insignificance of the Wald test. Table E3 summarizes the estimated transitional matrices, stationary distributions and the relevant results. Wald test with the null hypothesis $H_0 := \lambda_{13} = \lambda_{31} = 0$ results in failing to reject the null at more than 99% level.

\footnote{The whole results are available upon request.}
Table E2: Estimated distributions for Alabama state

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<th>Skilled workers</th>
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<td>Democrats</td>
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<td>Period 1</td>
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<td>Period 2</td>
<td>0.651</td>
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<td>Period 3</td>
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Note that * indicates that the distribution is identical between two groups. This is the case where the Wald test for the whole regressors is not significant at 10% level.

Table E3: Estimated transitional matrices for Alabama

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<tr>
<td>PID_{t-1}</td>
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<tr>
<td></td>
<td>I</td>
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<tr>
<td></td>
<td>R</td>
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<tr>
<td>( \Phi^* )</td>
<td>0.393</td>
</tr>
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</table>

\[ \Gamma(q) = \begin{bmatrix} 0.694 \end{bmatrix}, \quad \Gamma(q) = \begin{bmatrix} 0.713 \end{bmatrix} \]

where \( \Gamma(q) \) is the normalized entropy, \( v \) the support of the noise.