WHY ARE HEALTH CARE REPORT CARDS SO BAD (GOOD)?

Yijuan Chen
School of Economics
College of Business and Economics
Australian National University
E-mail: yijuan.chen@anu.edu.au

JEL codes: I18, D82, C31

Working Paper No: 511
ISBN: 0 86831 511 7

December 2009
Why Are Health Care Report Cards So Bad (Good)?*

Yijuan Chen†

First Version 2008-11-22; This Version 2009-11-18

Abstract

This paper provides a signaling-game theoretical foundation for empirically testing the effects of quality report cards in the U.S. health care industry. It shows that, when health care providers face an identical distribution of patient illness severities, a trade-off between multidimensional measures in the existing report cards renders them a mechanism that reveals the providers’ qualities without causing them to select patients. However, non-identical patient type distributions between providers, attributed to the referring physician, may force the high-quality provider to shun patients in order to signal himself. Despite this imperfection, the existing report cards cause the minimum provider selection compared with alternative report mechanisms.

Since the report cards not only may cause providers to select patients, but also cause patients to select providers, the single difference-in-differences estimates used in previous studies are not sufficient to indicate providers’ selection behavior, and cannot capture the report cards’ long-run welfare effect with short-run data. In an updated empirical framework, a treatment effect will be estimated once every period.

Keywords: Report Cards, Signaling Game, Difference-in-differences, Experts

JEL Classification: I18, D82, C31

*I am very grateful to Jan Eeckhout and Mark Pauly for their generous guidance and support. This paper also benefits from discussions with and comments from Andrew Postlewaite, Elena Krasnokutskaya, David Dillenberger, Roberto Pinheiro and seminar participants at the University of Pennsylvania, Australian National University, Australian Centre for Economic Research on Health, Cheung Kong Graduate School of Business, and the 7th International Industrial Organization Conference. Scott McCracken provided excellent research assistance. All the remaining errors are mine.

†School of Economics, Australian National University, H W Arndt Building, Canberra, ACT 0200, Australia
E-mail: yijuan.chen@anu.edu.au
1 Introduction

This paper provides a theoretical foundation for empirically testing the effects of quality report cards in the U.S. health care industry, with particular focus on the well-known report cards for the coronary artery bypass graft (CABG) surgery. The report cards, planned to give outsiders better information about the quality of health care providers, have drawn doubts and criticisms since their inception. Arguably the most authoritative verdict comes from Dranove et al. (2003). They point out that, despite the risk adjustment procedures used in producing the report cards, "providers are likely to have better information on patients’ conditions than even the most clinically detailed database" and may use such private information to improve their results by selecting patients. Using a single difference-in-differences estimate for each treatment effect, they show that report cards caused health care providers to avoid sicker patients and overall decreased social welfare. In other words, the report cards, as a solution toward adverse selection, result in moral hazard.

The existing report cards, however, are worth deeper examination for the following reasons. First, interpretations of empirical results in prior research are largely based on intuitions and/or conventional wisdom, lacking support of a solid theoretical foundation. Second, following prior research results, one may start searching for a new report mechanism. But if the existing report cards need only a small fix, then it may cost much less than a new mechanism, which may call for time-consuming institutional changes. Knowing how to fix the existing report cards, if fixing is necessary at all, entails better understanding of the cards. In particular, a largely neglected fact is that the existing cards show multidimensional measures of the providers’ performance1. How do these measures collectively affect the providers’ decisions? How do the patients parse the report cards, and how do they interact with the providers? Moreover, how does a provider’s decision vary with his true quality? A purely empirical approach cannot answer these questions in an integral way, and a theoretical signaling-game model is needed.

My studies of the above positive questions lead to some striking normative answers. First, even if the providers possess private patient information, when they are facing an identical distribution of patient types, the existing report cards are actually the optimal mechanism in the sense that they fully reveal the providers’ types without causing providers to select patients. The reason lies in the trade-off between two measures, volume and outcome, in the existing report cards. To improve outcome, measured by the mortality rate, a provider has to avoid sicker patients, resulting in a smaller patient volume. Due to this trade-off, a

1See Appendix 2 for a sample of the CABG report cards.
low-quality provider has no way to imitate a high-quality one. Consequently, no providers select patients and the report cards reveal true quality. This result on one hand provides a benchmark for studying the effects of the existing report cards, and on the other hand calls a stop to searching for a better mechanism under the assumption of identical patient distributions between providers.

I then investigate a more realistic situation where providers face non-identical distributions of patient types, attributed to the existence of a referring physician. When sicker patients are matched with the high-quality provider, sufficiently sick patients can dampen the high-quality provider’s outcome, forcing him to shun sicker patients in order to signal his quality. The consequent selection behavior in a separating equilibrium is characterized by three ranges of the high-quality provider’s type: In the bottom range, the degree of selection behavior increases with the provider’s type, in the middle range, the degree decreases with his type, while in the top range, when the provider’s quality is sufficiently high, there is no selection behavior. I further show that, despite their imperfection, the existing report cards cause the minimum selection behavior compared with alternative report mechanisms. This is because, given the existing cards have revealed all available information about a provider’s performance, other report mechanisms necessarily temper the information from the existing report cards, only making it easier for the low-quality provider to imitate the high-quality one, and forcing the high-quality provider to shun more sicker patients to separate himself.

The theoretical results shed new light on the empirical study. The model implies that the report cards give rise to two selection behaviors, which take place in sequence: The providers select patients before the report cards are issued, and the patients select providers after. Each selection behavior has an impact on the treatment effects of the report cards, such as the incidence effect, the quantity effect, the matching effect, and the welfare effect. Consequently, all treatment effects of the report cards vary across periods, entailing a difference-in-differences estimate for each treatment effect in each period. In contrast, by pooling all the data after the policy and comparing them with those before, the single difference-in-differences estimate used in previous literature only captures the average of the two periods’ effects. Hence the results in the previous literature need a more careful interpretation. In particular while prior studies have used a negative incidence effect, namely the decrease of the average illness severity of surgical patients, to indicate the existence of providers’ selection behavior, my theoretical model shows that even if the providers are not selecting patients, the patients’ selection behavior alone can lead to a negative incidence effect. Moreover, I argue that a once-every-period difference-in-differences estimation would allow researchers to capture the report cards’ long-run welfare effect with short-run data.
The rest of the paper is organized as follows: I briefly review the background of the health care report cards and the previous literature in Section 2. The theoretical model is presented in Section 3, and the empirical implications are discussed in Section 4. Section 5 concludes.

2 Background and Previous Research


Previous research: Since the inception of the CABG report cards, a large literature, most of which based on survey or empirical approaches, has been focused on studying their impact on the health care industry. Nonetheless, as Epstein (2006) points out, in the empirical literature "prior research...has failed to distinguish the effect of public reporting from other possible confounders associated with an underlying predisposition to performance improvement". One exception is Dranove et al.(2003), who in addition point out that "the failure of previous studies to consider the entire population at severity for CABG, rather than those who received it, is a potentially severe limitation". Using longitudinal cardiac-patient data from Medicare claims from 1987 to 1994, and hospital data from American Hospital Association, they estimate treatment effects of the report cards using a difference-in-differences approach. Based on the estimated negative incidence effects, they conclude that the report cards resulted in providers avoiding sicker patients. In addition, based on the estimated positive quantity effects, they suggest the report cards led the providers to shift the distribution of patient illness severity toward healthier patients. Furthermore, they conclude that the report cards lead to decreased social welfare based on estimated increased post-surgical expenses and readmission rates. Using a similar approach with Florida as the control state, Epstein (2004) shows that mortality dropped sharply in New York and New Jersey around the time of the first report.

\(^2\)See Epstein (2004) for a detailed introduction to the history of the health care report cards in the U.S.  
\(^3\)For a comprehensive review, see Epstein (2006).
card publication. In addition Werner (2004) shows that racial and ethnic differences in CABG use rose significantly in New York after the state’s CABG report card was released, which she attributes to physicians’ beliefs about different clinic uncertainties in different racial and ethnic groups.

Despite the impact on surgery providers, literature that summarizes survey information shows that the report cards had small effects on the referring pattern of referring cardiologists. Based on a 1995 Pennsylvania survey, Schneider and Epstein (1996) show that 82% of surveyed cardiologists were aware of the report cards in 1995, but fewer than 10% discussed the guide with more than 10% of their patients needing CABG surgery. Hannan et al.(1997) show that, in New York, 85% of surveyed cardiologists received the 1995 report card, but only 22% routinely discussed the report card with patients. Nonetheless the report cards did affect the difficulty had in placing patients. Schneider and Epstein (1996) show that, 59% of cardiologists reported increased difficulty since 1992 in placing their high-severity CABG patients, and 63% of cardiac surgeons reported being less willing to operate on those patients, offering circumstantial evidence for existence of providers’ selection behavior.

At the patient level, Omoigui et al. (1996) show that after the release of report cards in New York, the number of patients transferred to Ohio’s Cleveland Clinic has increased by 31%, and that in general the illness severity of these transferred patients was higher than those transferred to the Cleveland Clinic from other states, offering further circumstantial evidence for providers’ selection caused by report cards. In addition Gibbs et al.(1996) show that most participants considered friends and relatives as highly credible and preferred these sources to published information, indicating that word-of-mouth is another channel of information diffusion.

Conclusions drawn from most of the empirical literature stemmed from intuitions and conventional wisdom, lacking support of a comprehensive and equilibrium-based theoretical framework. Among the exceptions is Epstein (2004), who studies a single surgeon’s decision problem under the report-card program. Nonetheless the model lacks necessary components of a signaling game. In particular, it neglects the possibility that a low-quality surgeon may pool with the high-quality counterpart by imitating his performance. In addition, the necessary belief system in a signaling game is absent, as the model does not specify how patients interpret the reports card and base their belief about the surgeon type on it. Fong (2007) proposes an alternative scoring rule in a setting with one provider, whose type, drawn from a binary variable, is unknown to the outsiders. The scoring rule is based on a one dimensional signal, namely the success rate. In equilibrium, however, the "good" type provider typically engages in selection behavior. Since Fong essentially assumes an identical distribution of patient types
between both providers, the scoring rule she proposes is actually inferior to the existing report
cards due to the trade-off between the measures in the existing ones. Another related paper
is Lu et al. (2003). Using a Hotelling-class model, they study the effect of performance-based
contracting on the providers. Though patient types are heterogeneous in their model, the
types (locations) of providers are common knowledge, and so the model cannot be applied to
study the effects of report cards.

3 Theoretical Model

3.1 Set-up

Time is discrete with 2 periods, indexed by \( t \in \{1, 2\} \).

Each period there are a continuum of patients with measure 1, who will be active for one
period. A patient, indexed by \( j \), has an illness severity type \( s_j \) drawn from an identical and
independent distribution on \( \mathbb{R} \) according to an increasing cumulative distribution function
\( F(.) \) with the density function \( f(.) \).

There are 3 health care institutions, indexed by \( i \in \{A, B, C\} \), of whom \( A \) and \( B \) are
surgery providers and \( C \) is a referring physician. A surgery provider \( i \) is characterized by
his quality type \( k_i \in \mathbb{R} \). One provider’s quality type is \( k_h \), the other’s being \( k_l < k_h \). The
providers’ types are known to all the physicians\(^4\) but unknown to the patients, who hold a
prior belief that each provider is equally likely to be type \( k_l \) or \( k_h \).\(^5\)

Denote by \( m_{it} \) the measure of patients that provider \( i \) performs the surgery on in period \( t \).
If \( s_j = s, m_{it} = m \), and \( k_i = k \), the minimum probability of failure of the surgery on patient
\( j \) by provider \( i \) is given by a function \( q(s, m, k) \), which satisfies the following assumption:

Assumption 1:

\[
\begin{align*}
(1) \quad & \frac{\partial q}{\partial s} \geq 0, \text{ with } \frac{\partial q}{\partial s} = 0 \text{ if and only if } q(s, m, k) = 1; \\
(2) \quad & \frac{\partial q}{\partial k} \leq 0, \text{ with } \frac{\partial q}{\partial k} = 0 \text{ if and only if } q(s, m, k) = 0; \\
(3) \quad & \frac{\partial q}{\partial m} \geq 0, \text{ with } \frac{\partial q}{\partial m} = 0 \text{ if and only if } q(s, m, k) = 1. \\
(4) \quad & \lim_{s \to -\infty} q(s, 1, k) = 1, \lim_{s \to +\infty} q(s, 0, k_h) = 1 \text{ and } \lim_{s \to -\infty} q(s, 0, k_l) \in (0, 1). 
\end{align*}
\]

\(^4\)This specification is consistent with the previous survey literature, as discussed in Section 2, which show
that \( C \)'s decision rule is not affected by report cards, indicating that \( C \) already has knowledge of the surgery
providers’ types.

\(^5\)In contrast to simply assuming categorically a high type and a low type, my assumption allows continuity
in the providers’ types, in the sense that the difference between \( k_h \) and \( k_l \) now reflects how good a provider
is relative to another. Such a continuity captures the reality that although most providers do differ in their
quality, they may do so between "fair" and "very good", or "very good" and "excellent", instead of only
between symbolically "good" and "bad".
In Assumption 1, (i) and (ii) imply that the probability of failure is increasing with a patient’s severity type and decreasing with the provider’s quality. In (iii), $\frac{\partial q}{\partial m} \geq 0$ reflects the capacity constraint faced by each provider, which comes from factors such as clinic facility, nursing, and logistic that affect a surgery’s probability of failure.\footnote{An open-heart surgery team requires, among other things, skilled technicians, diagnostic imaging, and sophisticated laboratory support.” Raffel & Barsukiewicz, “The U.S. Health System, Origins and Functions”, 5th Edition, page 126} \lim_{s \to -\infty} q(s, 1, k) = 1 reflects that, as a matter of reality, any provider, no matter how skillful, will be overwhelmed when providing surgery on the whole population. The assumption $\lim_{s \to -\infty} q(s, 0, k_h) = 1$ implies that when the severity type is sufficiently high, even the most skillful provider will fail for sure. $\lim_{s \to -\infty} q(s, 0, k_l) \in (0, 1)$ reflects the minimum requirement for a provider to obtain a license and practice surgery. Moreover, $q$ being the minimum probability of failure implies that a provider is able to do worse, with the worst outcome being a certain failure.\footnote{Chen (2008) shows that, as using "character evidence" is prohibited in the U.S. legal system, in the extreme case implicit collusion between providers will result in no litigation from the patient side.}

Each period a patient needs to visit $C$ before being referred to a surgery provider. A patient can decide which provider he would like to be referred to, and $C$ will make the referral according to the patient’s choice. Alternatively the patient can leave the referral decision to $C$. If a patient is referred to a provider, his severity level is known by the provider while remains unverifiable to himself. The provider can choose between performing the surgery or providing an alternative treatment. A provider $i$’s payoff is $\pi_i = m_{i1} + m_{i2}$. I assume all the patients are Medicare beneficiaries and so monetary charges are abstracted from patients’ payoffs\footnote{Raffel M. W. & Barsukiewicz C. K., "The U.S. Health System, Origins and Functions", 5th Edition, page 32-33}. The payoff of patient $j$ is 1 if the surgery succeeds and 0 if it fails. For simplicity I also assume a patient’s payoff to be 0 if she undergoes the alternative treatment. Such an assumption allows a tractable analysis and also captures the worst possible scenario should the provider refuse performing surgery. Results from a more general assumption about patient payoff are discussed in Subsection 3.3, indicating the robustness of the current assumption.

Two facts pertain to the referring physician. First, in contrast to a surgery provider, the referring physician is a generalist, lacking sufficient expertise, experience, and technical support to accurately determine a patient’s illness severity. To capture this fact, I assume that $C$’s knowledge about a patient’s illness severity is characterized by the median illness severity level $s_{1/2}$\footnote{That is, $F(s_{1/2}) = \frac{1}{2}$.}, such that $C$ can only observe whether a patient’s illness severity is above or below it\footnote{While using $s_{1/2}$ allows a tractable analysis, I leave discussion about a more general specification of $C$’s knowledge to Subsection 3.3.}. Second, two common payment schemes for the referring physician are fixed salary and
"fee for service". The former scheme clearly gives the referring physician a constant payoff. In the current setting, the latter scheme implies a constant payoff for the referring physician as well, since the measure of patients seeking $C$ for referral is constant at 1 in each period. Hence I assume that $C$’s payoff is a constant.

The assumptions on $C$’s knowledge and payoff imply two alternative referral patterns of $C$, to be analyzed in sequence in the next subsection:

**Referral Pattern 1:** Refer sicker patients (those with $s_j \geq s_{1/2}$) to the type $k_h$ provider and healthier patients (those with $s_j < s_{1/2}$) to the type $k_l$ provider.

**Referral Pattern 2:** Refer healthier patients to the type $k_h$ provider and sicker patients to the type $k_l$ provider.

At the end of period 1, a report card of provider $i$ will be published to the public, showing $(m_{i1}, d_{i1})$, where $d_{i1}$ is the mortality rate of the surgeries performed by $i$ in period 1.

Denote by $P$ a patient’s mixed action set $\{(p_A, p_B, p_C) \mid \sum_i p_i = 1 \text{ and } p_i \geq 0 \text{ for } i \in \{A, B, C\}\}$. An element of the action set, $(p_A, p_B, p_C)$, means that for $i \in \{A, B\}$ with probability $p_i$ the patient will ask $C$ to refer him to provider $i$ and with probability $p_C$ the patient will have $C$ make the referral decision. Denote a strategy of patient $j$ in period $t$ by $\phi_{jt}$, where $\phi_{j1} \in P$ and $\phi_{j2}$ is a mapping from $j$’s information set in period 2 to $P$.

An action taken by provider $i$ in period $t$ is summarized by a set of patient types, denoted by $S_{it}$. Only patients with $s_j \in S_{it}$ will receive the surgery. Denote the set of $S_{it}$ by $S$. A strategy of provider $i$ is $\sigma_i = (\sigma_{i1}, \sigma_{i2})$, where $\sigma_{i1} : \{k_l, k_h\} \to S$ and $\sigma_{i2} : \{k_l, k_h\} \times [0, 1] \to S$.

Denote by $M_{it}$ the measure of patients that are referred to $i$ in period $t$, and $F_{it}$ the distribution of severity types in the patients referred to $i$ in $t$. When clear in the context, below I use $h$ in the subscript to denote the type $k_h$ provider and $l$ to denote the type $k_l$ provider.

### 3.2 Analysis

The solution concept is a symmetric perfect Bayesian equilibrium (PBE), consisting of a strategy profile $(\sigma_A^*, \sigma_B^*, \phi_{jt}^*)$ and a belief system, such that (i) given the other players’ strategies specified in the profile each player’s specified strategy is sequentially rational, (ii) the belief system is consistent with the strategy profile, and (iii) $\sigma_A^* = \sigma_B^* = \sigma^*, \phi_{jt}^* = \phi_t^*$ for every patient $j$.

I outline some preliminary results below before studying the equilibrium outcomes:

---

First, in period 2 it must be that each provider performs the surgery on all incoming patients.

Second, two report cards are regarded as different if they differ in either one measure or both measures. An equilibrium is a separating one if in period 1 two providers generate different report cards. The concept of perfect Bayesian equilibrium then requires the patients in period 2 correctly infer the providers’ types in a separating equilibrium. Consequently, in a separating equilibrium, if in period 2 the payoff of one type of provider is less than that of another, then in period 1 the former one must perform the surgery on all the incoming patients. Throughout the analysis I will focus on the separating equilibrium, and I discuss the pooling equilibrium in the next subsection.

Third, under the prior belief, if patient $j$ chooses to make the referral decision on his own, he faces the probability distribution of matching between his severity type and a provider $i$’s quality type as in the table below

<table>
<thead>
<tr>
<th>Prob</th>
<th>$k_i = k_h$</th>
<th>$k_i = k_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_i &lt; 1/2$</td>
<td>1/4</td>
<td>1/4</td>
</tr>
<tr>
<td>$s_i \geq 1/2$</td>
<td>1/4</td>
<td>1/4</td>
</tr>
</tbody>
</table>

while if he lets $C$ make the referral decision and $C$ follows, say, Referral Pattern 1, the matching probability table he faces is

<table>
<thead>
<tr>
<th>Prob</th>
<th>$k_i = k_h$</th>
<th>$k_i = k_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_i &lt; 1/2$</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$s_i \geq 1/2$</td>
<td>1/2</td>
<td>0</td>
</tr>
</tbody>
</table>

Denote by $E_{it}$ both a patient $j$’s expected payoff from being referred to provider $i$ at $t$ when he makes the decision on his own and his expected payoff from letting $C$ make the referral decision when $i = C$. In period 1, for $i \in \{A, B\}$,

$$E_{i1} = \frac{1}{2} \int_{\sigma_{i1}(k_i)} 1 - q(s, m_{i1}, k_i) dF(s) + \frac{1}{2} \int_{\sigma_{i1}(k_h)} 1 - q(s, m_{h1}, k_h) dF(s)$$

and

$$E_{C1} = \int_{\sigma_{i1}(k_i) \cap (-\infty, s_{1/2})} 1 - q(s, m_{i1}, k_i) dF(s) + \int_{\sigma_{i1}(k_h) \cap (s_{1/2}, +\infty)} 1 - q(s, m_{h1}, k_h) dF(s),$$

with $m_{i1} = M_{i1} \cdot \int_{\sigma_{i1}(k_i)} dF_{i1}(s)$, and $m_{h1} = M_{i1} \cdot \int_{\sigma_{i1}(k_h)} dF_{i1}(s)$.

Fourth, in equilibrium it must be that $E_{A2} = E_{B2}$ in period 2. Suppose otherwise, say
\( E_{A2} > E_{B2} \), then all patients in period 2 will choose provider A, resulting in \( M_{A2} = 1 \) and so \( E_{A2} = 0 \) by Assumption 1, contradicting \( E_{A2} > E_{B2} \). This result challenges the conventional wisdom that revelation of provider types will result in sicker patients choosing the high-quality provider and healthier patients choosing the low-quality one. A healthier patient after all is still a patient, whose objective is to seek the best possible treatment. Therefore the healthier patient also wants to choose the high-quality provider as long as he provides a better outcome, and this will not cease until a large patient volume drags the high-quality provider’s outcome to the same level as the low-quality one’s.

Last, the period 2 patients’ belief upon seeing identical report cards, i.e. \( (m_{h1}, d_{h1}) = (m_{l1}, d_{l1}) \), remains the same as the prior belief, since two identical cards cannot help the patients distinguish one provider from the other.

Given these preliminary results, I first study a benchmark case where there is no referring physician \( C \) and so the patients have to make decisions on their own, then I study the full model with \( C \), analyzing the referral patterns one by one. The results of the benchmark case are informative in their own right, and will also be frequently referred to in the subsequent analysis.

### 3.2.1 Without \( C \)

Without the referring physician \( C \), in period 1 the patients will choose providers on their own and so \( F_{it}(.) = F(.) \). With slight abuse of notation, I restrict the patients’ actions to \( P \) such that \( p_{C} = 0 \). Assumption 1 then implies that in equilibrium it must be that \( \phi^{*}_{1} = (\frac{1}{2}, \frac{1}{2}, 0) \).

In words, when patients have no other information about the providers’ types, naturally they will randomize between the providers with equal probabilities. But when all the other patients randomize with equal probabilities, each provider will recieve \( \frac{1}{2} \) measure of patients. Then a patient will be indifferent between the providers and so it is indeed his best response to also randomize with equal probabilities.

Lemma 1 below shows that, if the providers’ types are revealed in period 2, there is a unique measure of patients seeking each provider in period 2, and \textit{ceteris paribus}, that measure increases with the provider’s own quality and decreases with the opponent’s quality.

**Lemma 1** If the providers’ types \((k_{i}, k_{-i})\) are revealed in period 2, then there exists a unique measure of patients \( \hat{M}_{2}(k_{i}, k_{-i}) \) that will choose provider \( i \) in period 2. Moreover, \( \frac{\partial \hat{M}_{2}}{\partial k_{i}} > 0 \) and \( \frac{\partial \hat{M}_{2}}{\partial k_{-i}} < 0 \).

**Proof.** In Appendix. ■
An immediate corollary of Lemma 1 is that, should the period-2 patients know the providers’ types, then the type $k_h$ provider will receive more than $\frac{1}{2}$ measure of patients in that period. For ease of notation, denote $\bar{M}_{i2}(k_h, k_i)$ by $\bar{M}_h$, and denote $1 - \bar{M}_h$ by $\bar{M}_l$.

Denote by $\bar{d}_{i1}$ the minimum mortality rate that provider $i$ can achieve. The lemma below characterizes $\bar{d}_{i1}$ as a function of $m_{i1}$, the measure of $i$’s period-1 surgical patients.

**Lemma 2** For $m_{i1} \in (0, M_{i1}]$, $\bar{d}_{i1}(m_{i1}) = \left[ \int_{-\infty}^{\infty} \frac{m_{i1}}{M_{i1}} q(s, m_{i1}, k_i) dF(s) \right] \frac{M_{i1}}{m_{i1}}$, with $\frac{\partial \bar{d}_{i1}}{\partial m_{i1}} > 0$.

**Proof.** In Appendix. □

Lemma 2 shows the trade-off facing a provider $i$ between the minimum mortality rate $\bar{d}_{i1}$ and the measure of surgical patients $m_{i1}$: As can be seen in Figure 2-1, to improve (lower) the minimum mortality rate, the provider has to shun more sicker patients, which reduces the measure of patients he treats.

![Figure 2-1: Trade-off between $m_{i1}$ and $\bar{d}_{i1}$](image)

Given $(M_{i1}, k_i)$ we can define the set of provider $i$’s possible report card results to be $Z(M_{i1}, k_i) = \{(m_{i1}, d_{i1}) | m_{i1} \in (0, M_{i1}], d_{i1} \in [\bar{d}_{i1}(m_{i1}), 1]\}$ and we call the set $Z(M_{i1}, k_i) = \{(m_{i1}, \bar{d}_{i1}(m_{i1})) | m_{i1} \in (0, M_{i1}]\}$ the frontier of $Z(M_{i1}, k_i)$.

At period 1, patients’ equal randomization leads to $M_{i1} = \frac{1}{2}$ for each provider, which implies

$$\bar{d}_{i1}(\frac{1}{2}) = \int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_i) dF(s)$$

and consequently $\bar{d}_{h1}(\frac{1}{2}) < \bar{d}_{i1}(\frac{1}{2})$, as shown in Figure 2-1. Therefore, if the type $k_h$ provider performs the surgery on all the incoming patients, then his report card will show $(\frac{1}{2}, \bar{d}_{h1}(\frac{1}{2}))$,
which cannot be imitated by the type $k_l$ provider. Hence in a perfect Bayesian equilibrium, upon seeing $(\frac{1}{2}, d_{k1}(\frac{1}{2}))$ the patients in period 2 must form a belief that the provider’s type is certainly $k_h$. Therefore in period 1 performing surgeries on all the incoming patients gives the type $k_h$ provider the highest possible payoff $\frac{1}{2} + \hat{M}_{k_h}$ and so dominates all other actions. On the other hand, since the type $k_l$ provider by no means can imitate the type $k_h$ provider, it is the type $k_l$ provider’s best response to perform surgeries on all the incoming patients in period 1 too. The consistent belief system can be easily constructed accordingly. The proposition below summarizes these results:

**Proposition 1** If there is no referring physician $C$, then there exists a unique separating equilibrium such that $\phi_1^* = (\frac{1}{2}, \frac{1}{2}, 0)$ and each provider performs the surgery on all incoming patients. At period 2 the patients correctly infer the providers’ types from the report cards and randomize between the providers by choosing the type $k_h$ provider with probability $\hat{M}_{k_h}$ and the type $k_l$ provider with probability $\hat{M}_{k_l}$.

Proposition 1 implies that, when patients and providers are matched randomly, thanks to the trade-off between the volume measure, $m_{i1}$, and the outcome measure, $d_{i1}$, the existing report cards are actually a mechanism that fully reveals the providers’ types without causing them to select patients. This result shows the necessity of understanding the specific features of the existing report cards and their influence on the participants in the programs. Unfortunately, the feature of multidimensional measures in the existing report cards has been largely neglected in the previous studies, with some directly calling them "mortality report cards". Some normative studies have also been conducted under the assumption that only mortality rates are released, while in the mean time implicitly assuming identical distribution of patient types between providers, and as a result the proposed mechanism is inferior to the existing report cards. Paying attention to the actual features of the existing report cards thus helps to avoid conducting research under unnecessarily unrealistic assumptions. Moreover, it is worth noting that, thanks to the absence of the referring physician $C$ here, the result in Proposition 1 is robust to the specification of $C$’s objective and behavior. Therefore Proposition 1 provides a benchmark for further study on the report cards.

We now turn to the full model with the referring physician $C$. In subsection 3.2.2 we analyze the situation where $C$ chooses Referral Pattern 1, and in subsection 3.2.3 we analyze Referral Pattern 2.

### 3.2.2 With $C$, Referral Pattern 1

Suppose $C$ will refer patients with $s_j \geq s_{1/2}$ to the type $k_h$ provider and the rest to the other.
We first look at period 2. Suppose in period 2 the patients correctly infer the providers’ types from the report cards. Then in equilibrium not all the patients will ask $C$ to make the referral decision, since otherwise each provider will receive patients with measure $1/2$, while $q(s, \frac{1}{2}, k_h) < q(s, \frac{1}{2}, k_l)$ implies a patient will strictly prefer choosing the type $k_h$ provider. However, on the other hand, a patient can induce more information about his severity type from turning to $C$: If he asks $C$ to make the referral decision and is referred to, say, the type $k_l$ provider, then he will know that his severity type is below $s_{1/2}$. At this point the patient may want to reconsider the referral decision\textsuperscript{12}. Therefore in this subsection I take into account this extra information when patients correctly infer the provider types from report cards, and allow a patient to base his action on both the providers’ types and the category of his severity types characterized by $s_{1/2}$.

If a patient knows both the provider types and the category of his severity types, he can do at least as well as letting $C$ make the decision. Now there are three possibilities in a separating equilibrium regarding the distribution of patients’ severity types facing the providers in period 2:

(i) All patients with $s_j \geq s_{1/2}$ and some with $s_j < s_{1/2}$ choose the type $k_h$ provider
(ii) All patients with $s_j < s_{1/2}$ and some with $s_j \geq s_{1/2}$ choose the type $k_h$ provider
(iii) Each provider is facing patients with all possible $s_j$’s.

To analyze these possibilities, we see that, on one hand, for a patient with $s_j \geq s_{1/2}$ to be indifferent between a type $k_h$ provider and a type $k_l$ provider, it must be that for patients

\textsuperscript{12}To see how this extra information affects the patients’ actions, note that when the patients know the providers’ types, then, with the extra information about severity types, the period-2 patients’ equilibrium actions derived in the previous subsection will no longer be part of an equilibrium even if self-referral gives a patient a higher expected payoff than letting $C$ make the decision. That is,

$$1 - \int_{-\infty}^{s_{1/2}} q(s, \hat{M}_h, k_h)dF(s) > 1 - \left[ \int_{-\infty}^{s_{1/2}} q(s, \hat{M}_l, k_l)dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \hat{M}_h, k_h)dF(s) \right], \quad (\diamond)$$

where the left hand side is the payoff from seeing a type $k_h$ provider, which, as discussed before, is equal to the payoff from seeing a type $k_l$ provider, and the right hand side is the payoff from leaving the referral decision to $C$. This is because $(\diamond)$ implies

$$1 - \int_{-\infty}^{s_{1/2}} q(s, \hat{M}_h, k_h)dF(s) > 1 - \int_{-\infty}^{s_{1/2}} q(s, \hat{M}_l, k_l)dF(s),$$

but now if a patient induces information from $C$ and knows that his severity type is below $s_{1/2}$, he will strictly prefer the type $k_h$ provider to the type $k_l$ provider, and so he would like to "renegotiate" with $C$ and choose the type $k_h$ provider for sure rather than randomizing between them or following $C$’s decision.
with a measure denoted by $\tilde{M}_h \in (\frac{1}{2}, 1]$ that choose the type $k_h$ provider:

$$Y(\tilde{M}_h) \equiv \int_{s_{1/2}}^{+\infty} q(s, \tilde{M}_h, k_h)df(s) - \int_{s_{1/2}}^{+\infty} q(s, 1 - \tilde{M}_h, k_l)df(s) = 0.$$ 

Since $\lim_{\tilde{M}_h \to 1/2} Y(\tilde{M}_h) < 0$, $Y(1) > 0$, and $\frac{\partial Y}{\partial \tilde{M}_h} > 0$, there exists a unique $\tilde{M}_h^*$ such that $Y(\tilde{M}_h^*) = 0$. Note that $\tilde{M}_h^*$ is a function of $(k_h, k_l)$, and when necessary I will use the notation $\tilde{M}_h^*(k_h, k_l)$. Moreover, $\frac{\partial \tilde{M}_h^*}{\partial k_h} > 0$. On the other hand, for a patient with $s_j < s_{1/2}$ to be indifferent between a type $k_h$ provider and a type $k_l$ provider, it must be that, for patients with a measure denoted by $\tilde{M}_h \in (\frac{1}{2}, 1]$ that choose the type $k_h$ provider:

$$G(\tilde{M}_h) \equiv \int_{s_{1/2}}^{+\infty} q(s, \tilde{M}_h, k_h) - q(s, 1 - \tilde{M}_h, k_l)df(s) = 0.$$ 

Analogously we can show there exists a unique $\tilde{M}_h^*$ such that $G(\tilde{M}_h^*) = 0$. Also $\tilde{M}_h^*$ is a function of $(k_h, k_l)$ and when necessary I will use the notation $\tilde{M}_h^*(k_h, k_l)$. In addition, $\frac{\partial \tilde{M}_h^*}{\partial k_h} > 0$.

Now the aforementioned three possibilities correspond to the comparison between $\tilde{M}_h^*$ and $\tilde{M}_h^*$:

(i) $\tilde{M}_h^* > \tilde{M}_h^*$, as shown in Figure 2-2

![Figure 2-2: $\tilde{M}_h^* > \tilde{M}_h^*$](image)

Then in a separating equilibrium in period 2 it must be that all patients with severity types $s_j \geq s_{1/2}$ and a fraction $\frac{\tilde{M}_h^* - 1}{2} = 2\tilde{M}_h^* - 1$ of the patients with severity types $s_j < s_{1/2}$ choose the type $k_h$ provider, with the remaining patients choosing the type $k_l$ provider. In total in period 2 the type $k_h$ provider receives patients with measure $\tilde{M}_h^*$ and the type $k_l$ provider receives $1 - \tilde{M}_h^*$ patients. The reason for such an outcome is that, as $\tilde{M}_h^* > \tilde{M}_h^*$, in the specified outcome the patients with severity types $s_j \geq s_{1/2}$ strictly prefer the type $k_h$ provider to the type $k_l$ provider\(^{13}\), whereas the patients with severity types $s_j < s_{1/2}$ are indifferent between

\(^{13}\)On the other hand the patients with $s_j \geq s_{1/2}$ are indifferent between choosing the type $k_h$ surgeon on his own and letting $C$ make the referral decision.
the two providers, and so no patients have the incentive to deviate. It is then easy, though
tedious, to verify that in any other outcomes at least one category of patients will have the
incentive to deviate.

(ii) $\tilde{M}_h^* < \tilde{M}_h^*$, as shown in Figure 2-3

Then in a separating equilibrium in period 2 all patients with severity types $s_j < s_{1/2}$ and a
fraction $2\tilde{M}_h^* - 1$ of the patients with severity types $s_j \geq s_{1/2}$ choose the type $k_h$ provider,
with the remaining patients choosing the type $k_l$ provider. In total in period 2 the type $k_h$
provider receives patients with measure $\tilde{M}_h^*$ and the type $k_l$ provider receives $1 - \tilde{M}_h^*$ patients.

(iii) $\tilde{M}_h^* = \tilde{M}_h^*$

Then in a separating equilibrium in period 2 the type $k_h$ provider receives $\tilde{M}_h^*$ patients.
A fraction $\theta$ of patients with $s_j \geq s_{1/2}$ and a fraction $\gamma$ of patients with $s_j < s_{1/2}$ choose the
type $k_h$ provider, with $\frac{1}{2}\theta + \frac{1}{2}\gamma = \tilde{M}_h^*$.\footnote{Note that $\tilde{M}_h^* = \tilde{M}_h^*$ implies that $Y(\tilde{M}_h^*) + G(\tilde{M}_h^*) = \int_{-\infty}^{+\infty} q(s, \tilde{M}_h^*, k_h) dF(s) - \int_{-\infty}^{+\infty} (s - \tilde{M}_h^*, k_l) dF(s) = L(\tilde{M}_h^*) = 0$, that is, $\tilde{M}_h^* = \tilde{M}_h^* = \tilde{M}_h$, but on the other hand $L(\tilde{M}_h^*) = 0$ does not necessarily mean $Y(\tilde{M}_h^*) = 0$ and $G(\tilde{M}_h^*) = 0$.}

The fact that $\frac{\partial \tilde{M}_h^*}{\partial k_h} > 0$ and $\frac{\partial \tilde{M}_h^*}{\partial k_h} > 0$ implies that the type $k_h$ provider’s payoff increases
with $k_h$ if the type is known by the patients. The lemma below summarizes the results
obtained so far.

Lemma 3 If in period 2 the patients know both the providers’ types and the category of their
severity types characterized by $s_{1/2}$, then in equilibrium the type $k_h$ provider will receive patients
with measure $M_h^*(k_h, k_l) \equiv \min\{\tilde{M}_h^*, \tilde{M}_h^*\}$, and the type $k_l$ provider will receive patients with
measure $1 - M_h^*(k_h, k_l)$.

We now turn to period 1. There are two possible outcomes in the first period. The patients
may make the referral decisions on their own, or leave the decision to the referring physician.
The lemma below characterizes these possibilities.
Lemma 4 There exists a separating equilibrium such that in period 1 the patients make referral decisions on their own if and only if
\[
\int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_l) - q(s, \frac{1}{2}, k_h) dF(s) < \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_l) - q(s, \frac{1}{2}, k_h) dF(s).
\] (1)

Proof. In Appendix. ■

The intuition of Lemma 4 is as follows: Compared with letting C make the decision, there is a benefit and a cost of choosing a provider on one’s own. The benefit comes from choosing a type \(k_h\) provider when one’s severity type is below \(s_{1/2}\), while the cost comes from choosing a type \(k_l\) provider when one’s severity type is above \(s_{1/2}\). For a patient to prefer self-referral to letting C make the decision, it must be that the cost of self-referral, represented by the left hand side of the inequality in the lemma, is outweighed by the benefit, represented by the right hand side of the inequality.

The equilibrium outcome in the separating equilibrium with self referral in period 1 is similar to that when there is no referring physician C. Equal randomization of the patients allows the type \(k_h\) provider to separate himself from the type \(k_l\) provider without shunning any patients, and consequently the type \(k_l\) provider will not shun patients either. At period 2, the report cards fully reveal the providers’ types and the patients choose providers in the way implied by Lemma 2 and its preceding discussion.

When the inequality in Lemma 4 is reversed, the patients leave the referring decision to C. We now turn to this scenario.

If period-1 patients let C make the referral decision, then \(M_{i_1} = \frac{1}{2}\) for each provider i, but the providers face different distributions of severity types\(^\text{15}\).

Denote by \(\bar{d}_{h1}\) the minimum mortality rate that a type \(k_h\) provider can achieve. The first part of the lemma below characterizes \(\bar{d}_{h1}\) as a function of the measure of the type \(k_h\) provider’s period-1 surgical patients. The second part of the lemma provides an analogous characterization for \(\bar{d}_{l1}\), the minimum mortality rate that a type \(k_l\) provider can achieve.

\(^{15}\)Specifically, the type \(k_h\) provider receives only patients with \(s_j \geq s_{1/2}\), so

\[
F_{h1}(s) = \begin{cases} 
2F(s) - 1 & \text{if } s_j \geq s_{1/2} \\
0 & \text{if } s_j < s_{1/2}
\end{cases}
\]
whereas the type \(k_l\) provider receives only patients with \(s_j < s_{1/2}\), so

\[
F_{l1}(s) = \begin{cases} 
1 & \text{if } s_j \geq s_{1/2} \\
2F(s) & \text{if } s_j < s_{1/2}
\end{cases}
\]
Lemma 5  (i) For \( m_{h1} \in (0, M_{h1}] \), \( \bar{d}_{h1}(m_{h1}) = \int_{s_{1/2}}^{F^{-1}(m_{h1})} q(s, m_{h1}, k_h)dF(s) \cdot \frac{M_{h1}}{m_{h1}} \), with \( \frac{\partial \bar{d}_{h1}}{\partial m_{h1}} > 0 \) and \( \frac{\partial \bar{d}_{h1}}{\partial k_h} < 0 \). (ii) For \( m_{l1} \in (0, M_{l1}] \), \( \bar{d}_{l1}(m_{l1}) = \int_{-\infty}^{F^{-1}(m_{l1})} q(s, m_{l1}, k_l)dF(s) \cdot \frac{M_{l1}}{m_{l1}} \), with \( \frac{\partial \bar{d}_{l1}}{\partial m_{l1}} > 0 \) and \( \frac{\partial \bar{d}_{l1}}{\partial k_l} < 0 \).

Proof. In Appendix. ■

Following Lemma 5, with \( M_{l1} = \frac{1}{2}, \) for \( m \in (0, \frac{1}{2}] \) we have

\[
\bar{d}_{h1}(m) = \frac{1}{m} \cdot \int_{s_{1/2}}^{F^{-1}(m+\frac{1}{2})} q(s, m, k_h)dF(s)
\]

and

\[
\bar{d}_{l1}(m) = \frac{1}{m} \cdot \int_{-\infty}^{F^{-1}(m)} q(s, m, k_l)dF(s)
\]

Denote by \( \bar{k}_h \) the type of \( k_h \) that solves \( \bar{d}_{h1}(\frac{1}{2}) = \bar{d}_{l1}(\frac{1}{2}) \). Also denote by \( \underline{k}_h \) the type \( k_h \) that solves \( \lim_{m \to 0} \bar{d}_{h1}(m) = \lim_{m \to 0} \bar{d}_{l1}(m) \). The fact \( \frac{\partial \bar{d}_{h1}}{\partial k_h} < 0 \) implies that \( k_l < \underline{k}_h < \bar{k}_h \).

\( \underline{k}_h \) and \( \bar{k}_h \) divide \( k_h \) into three ranges. As shown in Table 2-1, for \( k_h > \bar{k}_h \), the frontier of the type \( k_h \) provider’s report card results lies entirely outside the type \( k_l \) provider’s set of report card results, implying that if the type \( k_h \) provider stays on the frontier, then the type \( k_l \) provider cannot imitate his report card result. For \( k_h < \underline{k}_h \), the set of the type \( k_h \) provider’s report card results becomes a proper subset of the type \( k_l \) provider’s set of report card results, meaning that the type \( k_l \) can mimic any report card result from the type \( k_h \) provider. For \( k_h \in (\underline{k}_h, \bar{k}_h) \), there is a generic subset of the type \( k_h \) provider’s report card results that does not belong to the type \( k_l \) provider’s set of results, implying that the type \( k_l \) provider can imitate the type \( k_h \) provider, but only to a certain extent. The remaining non-generic cases are characterized by \( k_h = \underline{k}_h \) and \( k_h = \bar{k}_h \), as limits of the generic cases.

16There are \( \bar{d}_{h1}(\frac{1}{2}) = 2 \int_{s_{1/2}}^{1} q(s, \frac{1}{2}, k_h)dF(s) \) and \( \bar{d}_{l1}(\frac{1}{2}) = 2 \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_l)dF(s) \). The existence and uniqueness of \( \bar{k}_h \) result from the Intermediate Value Theorem and monotonicity of \( \bar{d}_{h1}(\frac{1}{2}) - \bar{d}_{l1}(\frac{1}{2}) \). The same result applies to \( \bar{k}_h \) below.

17By L’Hospital’s rule, \( \lim_{m \to 0} \bar{d}_{h1}(m) = \lim_{m \to 0} q(F^{-1}(m + \frac{1}{2}), m, k_h) + \int_{s_{1/2}}^{F^{-1}(m+\frac{1}{2})} q(s, m, k_h)dF(s) = q(s_{1/2}, 0, k_h) \) and \( \lim_{m \to 0} \bar{d}_{l1}(m) = \lim_{m \to 0} q(F^{-1}(m), m, k_l) + \int_{-\infty}^{F^{-1}(m)} q(s, m, k_l)dF(s) = \lim_{s \to -\infty} q(s, 0, k_l) \)
Table 2-1: Categories of $k_h$

<table>
<thead>
<tr>
<th>$k_h &gt; k_{h1}$</th>
<th>$\lim_{m \to 0} d_{h1}(m)$ vs. $\lim_{m \to 0} d_{l1}(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_h = k_{h1}$</td>
<td>&lt;</td>
</tr>
<tr>
<td>$k_h \in (k_{h1}, k_{h2})$</td>
<td>&gt;</td>
</tr>
<tr>
<td>$k_h = k_{h2}$</td>
<td>=</td>
</tr>
<tr>
<td>$k_h \in (k_l, k_{h2})$</td>
<td>&gt;</td>
</tr>
</tbody>
</table>

For tractability in the case with $k_h \in (k_{h1}, k_{h2})$, I impose the following assumption:

**Assumption 2:** For $k_h \leq \bar{k}_h$, if $m' < m''$, then $\tilde{d}_{l1}(m'') - \tilde{d}_{l1}(m') > \tilde{d}_{l1}(m'') - \tilde{d}_{l1}(m')$.

Assumption 2 is essentially a single crossing condition, stating that for $k_h \leq \bar{k}_h$, the increase of the same measure of patients will result in a higher increase of the minimum mortality rate for the type $k_h$ provider than for the type $k_l$ provider. The validity of this assumption can be better seen from $k_h = k_l$, where the assumption essentially means that a provider will face a higher increase of the minimum mortality rate when treating high-severity patients than when treating low-severity patients. Assumption 2 then implies this result holds for higher $k_h$'s as well.

Under Assumption 2, the relation between $\tilde{d}_{h1}$ and $\tilde{d}_{l1}$ implied by Table 2-1 can be fully shown in Figure 2-4. For $k_h \in (k_{h1}, k_{h2})$, denote by $\bar{m}(k_h, k_l)$ the solution of $\tilde{d}_{h1}(\bar{m}) = \tilde{d}_{l1}(\bar{m})$. The fact $\frac{\partial \tilde{d}_{h1}}{\partial k_h} < 0$ implies that $\frac{\partial \bar{m}}{\partial k_h} > 0$.

![Figure 2-4: Report Card Frontiers of $h$ and $l$](image-url)
We can now analyze the providers’ period 1 equilibrium actions according to the three ranges of \( k_h \). Without loss of generality and for ease of the subsequent analysis, I further assume that, if a provider engages in selecting patients, the minimum measure of patients that he can avoid is an infinitesimal constant \( \varepsilon \).

The proposition below summarizes the analysis, with the results graphically shown in Figure 2-5.

**Proposition 2** If in period 1 the patients let \( C \) make the referral decision, there is a separating equilibrium where for every \((k_l, k_h)\) there is a unique \( k_b \in (k_h, \bar{k}_h) \) such that

\[
 m^*_h = \begin{cases} 
 1 - M^*_h(k_h, k_l) & \text{if } k_h \in (k_l, k_h] \\
 \bar{m}(k_h, k_l) - \varepsilon & \text{if } k_h \in (k_b, \bar{k}_h] \\
 \frac{1}{2} & \text{if } k_h > \bar{k}_h 
\end{cases}
\]

**Proof.** In Appendix. ■

![Figure 2-5: \( m^*_h \) in the separating equilibrium with \( C \) making the period 1 referral decision.](image)

The intuition of Proposition 2 is as follows: When patients let \( C \) make the referral decision, the type \( k_h \) provider faces sicker patients than the type \( k_l \) provider does, forcing the former to shun patients in order to signal himself. The degree of the type \( k_h \) provider’s selection behavior is characterized by \( k_b \) and \( \bar{k}_h \). Intuitively, when \( k_h = k_l \), there is no gain from signaling, and naturally there is no selection behavior in equilibrium. When \( k_h \) increases, the gain from separating from the type \( k_l \) provider increases as well, therefore the type \( k_h \) provider is willing to shun more patients in period 1 in exchange for more patients in period 2. Hence the degree of patient selection initially increases with \( k_h \). But when \( k_h > \bar{k}_h \), there comes the possibility that the \( k_h \) provider can generate a signal that cannot be imitated by the type \( k_l \).
provider. However, initially the cost of generating such a signal is larger than generating a
signal that the type $k_l$ provider does not want to mimic, and so for $k_h < k_l$, the degree of
selection behavior keeps increasing with $k_h$. On the other hand, the cost of generating a signal
that the type $k_l$ provider cannot mimic keeps decreasing with $k_h$. The turning point occurs
at $k_h = k_l$, where the costs of the two kinds of signals are equal. When $k_h$ is larger than $k_l$,
the type $k_h$ provider switches to generating the signal that his counterpart cannot mimic, and
consequently the degree of his selection behavior decreases with his type. In the end, when $k_h$
is sufficiently high, the type $k_h$ provider can signal himself without turning down any patients.

3.2.3 With $C$, Referral Pattern 2

Suppose $C$ will refer patients with $s_j < s_{1/2}$ to the type $k_h$ provider and the rest to the other.

Analogous to the analysis in the previous subsection, in period 2 if the patients correctly
infer the providers’ types from the report cards, then a patient can induce more information
about his severity type from turning to $C$. Allowing a patient to base his action on the
providers’ types as well as the category of his severity types characterized by $s_{1/2}$, we can
readily see that Lemma 3 also holds under Referral Pattern 2.

The following lemma is the counterpart of Lemma 4, characterizing the situation where
in period 1 patients make referral decisions on their own. Its intuition also parallels that of
Lemma 4: Compared with leaving the referral decision to $C$, there is a benefit and a cost from
making a self-referral. Thus for a patient to prefer self-referral to letting $C$ make the decision,
it must be that the benefit of self-referral outweighs the cost.

**Lemma 6** There exists a separating equilibrium such that in period 1 the patients make re-
ferral decisions on their own if and only if

$$\int_{s_{1/2}}^{s_{1/2}} q(s, \frac{1}{2}, k_l) - q(s, \frac{1}{2}, k_h) dF(s) > \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_l) - q(s, \frac{1}{2}, k_h) dF(s).$$

(2)

**Proof.** In Appendix. □

Analogous to the discussion in the previous subsection, the equilibrium outcome in the
separating equilibrium with self-referral in period 1 is similar to the benchmark case where
there is no referring physician $C$, with the report cards fully revealing the providers’ types
without causing any of them to select patients.

When the inequality in Lemma 6 is reversed, in period 1 the patients leave the referring
decision to $C$. Because the type $k_h$ provider is receiving healthier patients while the type $k_l$
provider receiving sicker patients, compared with the benchmark case, now the report card frontiers of the providers are further apart. The result is then analogous to the benchmark case: Since, given any measure of surgical patients, the minimum mortality rate of the type $k_l$ provider will be higher than that of the type $k_h$ provider, the type $k_l$ provider cannot imitate the type $k_h$ provider whatsoever. Consequently, the report cards will reveal the providers’ types without causing providers to select patients.

3.2.4 Alternative Report Mechanisms

The following table summarizes the results about the providers’ equilibrium selection behavior in the previous two subsections:\(^{18}\):

<table>
<thead>
<tr>
<th>Inequality (1) holds</th>
<th>Referral Pattern 1</th>
<th>Referral Pattern 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providers do not select patients</td>
<td>Providers do not select patients</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inequality (2) holds</th>
<th>Referral Pattern 1</th>
<th>Referral Pattern 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provider with $k_h \in (k_l, k_h)$ selects patients at $t=1$</td>
<td>Providers do not select patients</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2: Summary of providers’ equilibrium selection behavior

The existence of provider selection under Referral Pattern 1 and inequality (2) naturally makes one ponder whether there are better report mechanisms. In the current setting, however, the answer is no. In general, a report mechanism is a function $R : [0,1] \times [0,1] \rightarrow \Omega[0,1] \times \Omega[0,1]$ where $\Omega[0,1]$ is the $\sigma-$algebra on $[0,1]$. Given $R, r_i \in \Omega[0,1] \times \Omega[0,1]$ is the report card of $i$ published to the public at the end of period 1. For example, in the existing report mechanism, $R(m_{i1}, d_{i1}) = (m_{i1}, d_{i1})$ with $r_i = (m_{i1}, d_{i1})$. In a PBE induced by a report mechanism $R$, I use $\hat{x}$ to denote the equilibrium value of a variable $x$. Also in a PBE induced by $R$, if $r_i = r_{-i}$ then $M_{ij}(r_i, r_{-i}) = \frac{1}{2}$, as two identical signals yield no new information. A report mechanism $R$ is revealing if $\hat{r}_l \neq \hat{r}_h$ in an induced PBE, and a revealing report mechanism $R$ is said to cause provider selection if in equilibrium there is a provider $i$ such that $\hat{m}_{i1} < \hat{M}_{i1}$. Similar to the discussion before, if a report mechanism $R$ is revealing, then it must be that in equilibrium $\hat{m}_{i1} = \frac{1}{2}$.

The following proposition shows that, despite their imperfection in terms of the resulted provider selection, the existing report cards cause the minimum provider selection compared

\[^{18}\] Without loss of generality I skip the 0 probability case where 
\[
\int_{s_{1/2}}^{\infty} q(s, \frac{1}{2}, k_i) - q(s, \frac{1}{2}, k_h) dF(s) = \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_i) - q(s, \frac{1}{2}, k_h) dF(s)
\]
with other revealing report mechanisms.

**Proposition 3** Among all the revealing report mechanisms, the existing report mechanism \( R(m_1, d_1) = (m_1, d_1) \) causes the minimum provider selection.

**Proof.** In Appendix. ■

The intuition of Proposition 3 is as follows: In the present setting the existing report cards have shown the maximum information available, and any other revealing mechanisms necessarily temper information based on the existing cards, only to make it easier for the type \( k_l \) provider to pool with the type \( k_h \) provider, forcing the the latter to shun even more patients to signal himself. The result therefore suggests that future study aimed at a better report mechanism should be focused on eliciting information beyond the present setting, such as inducing better information about \( s_j \) and so improving the accuracy of the risk adjustment procedure.

### 3.3 Discussion

**C’s Referring Pattern** More generally we can characterize \( C \)’s referring pattern by a cutoff level \( s_c \), which can be larger or smaller than \( s_{1/2} \). Referral Pattern 1 now means that, above \( s_c \), \( C \) refers a patient to the type \( k_h \) provider, and otherwise refers her to the type \( k_l \) provider. Referral Pattern 2 means \( C \) refers patients with illness severity lower than \( s_c \) to the type \( k_h \) provider, and the rest to the type \( k_l \) provider.

First look at Referral Pattern 1. For simplicity let’s focus on the case where patients let \( C \) make the decision in period 1. If \( s_c > s_{1/2} \) then the previous analysis carries through. The type \( k_h \) provider’s report card frontier is similarly characterized by three ranges of \( k_h \), with \( k_h \) unchanged while \( k_h \) is lower than before due to a lower measure of incoming patients. However, if \( s_c < s_{1/2} \), then in any scenario the report cards will reveal the providers’ types without causing patient selection. This is because the type \( k_h \) provider is now receiving more incoming patients than the type \( k_l \) provider, and so by accepting all the patients he will generate a report card with \( m_{h1} = 1 - F(s_c) \) that can not be imitated by the type \( k_l \) provider. Nonetheless, I should point out that throughout the analysis on separating equilibrium I have put no restriction on patients’ off-equilibrium-path belief, as long as the belief system supports the equilibrium strategies. It would not be unrealistic to assume that the patient’s belief system is characterized by "indifference curves" such that a provider with a report card result on a lower indifference curve (lower mortality rate and higher volume) will be regarded
as the type $k_h$ provider. Given any such belief systems, the type $k_h$ provider’s incentive to signal himself by selecting patients is restored.

For Referral Pattern 2, it is easy to see that the analysis under $s_{1/2}$ carries through, because the report card frontiers are further apart compared with the benchmark case, making it impossible for the type $k_l$ provider to imitate the the type $k_h$ provider.

**Payoff from the Alternative Treatment**  For simplicity I have assumed 0 payoff for patients undergoing the alternative treatment, which captures the worst possible scenario should providers engage in selection. The set-up could be more realistic. For example, assume a patient’s payoff from receiving the alternative treatment is $\alpha \in (0, 1)$, then the patients’ period 1 expected payoffs are, for $i \in \{A, B\}$,

$$E_{i1} = \frac{1}{2} \int_{\sigma_{i1}(k_i)} 1 - q(s, m_{i1}, k_i)dF(s) + \frac{1}{2} \int_{\sigma_{i1}(k_h)} 1 - q(s, m_{h1}, k_h)dF(s)$$

$$+ \alpha \left[ 1 - \frac{1}{2} \left( \int_{\mathbb{R} \setminus \sigma_{i1}(k_i)} dF(s) + \int_{\mathbb{R} \setminus \sigma_{i1}(k_h)} dF(s) \right) \right]$$

and

$$E_{C1} = \int_{\sigma_{i1}(k_i) \cap (-\infty, s_{1/2})} 1 - q(s, m_{i1}, k_i)dF(s) + \int_{\sigma_{i1}(k_h) \cap (s_{1/2}, +\infty)} 1 - q(s, m_{h1}, k_h)dF(s)$$

$$+ \alpha \left[ 1 - \int_{\mathbb{R} \setminus \sigma_{i1}(k_i) \cup \sigma_{i1}(k_h)} dF(s) \right].$$

$\alpha \in (0, 1)$ implies the alternative treatment yields a positive payoff lower than a successful surgery. Such a generalization captures several facts. First, a standard alternative treatment, percutaneous transluminal coronary angioplasty (PTCA), is known to suffer from recurrence of symptoms. The reported failure rates due to restenosis (recurrent narrowing) in the first 6-12 months following the PTCA procedure are 30-60%. (Rupprecht, et al. 2005; Kulick, 2005). Second, an alternative treatment may mean transferring patients to other providers that are not subject to the report cards, including those in the neighbor states (Omoigui et al. 1996) and within-state government hospitals, including county and city hospitals\(^{19}\). Such a generalization, however, reduces the tractability of the model, and I believe it will not offer more insights to the analysis. This is because, first, the patients’ decisions will be similar to those in the 0 payoff setting: In period 2, a patient will still choose the provider that gives her the highest expected payoff, and in period 1 the patients will equally randomize between

\(^{19}\)“In large urban areas, these tend to be safety net hospitals (serving the poor and uninsured).” Raffel & Barsukiewicz, "The U.S. Health System, Origins and Functions", 5th Edition, page 128
the providers when there are no $C$, or when $C$ is in action choose between self-referral and $C$ in a way analogous to the 0 payoff setting. Given such similarity in patients’ actions and response to report cards, the results derived from the original setting should remain robust.

**Pooling Equilibrium**  
So far the analysis has been focused on separating equilibrium, where the providers’ types are revealed by the report cards. However, pooling equilibria also exist, though they involve an unappealing off-equilibrium-path patient belief. In a pooling equilibrium, the providers yield the same report card results, keeping the period 2 patients’ beliefs the same as the prior, and thus each provider receives measure $\frac{1}{2}$ patients in period 2. To construct a pooling equilibrium, one only needs to specify an equilibrium report card result with a sufficiently large $m_{i1}$, and then have the patients hold the belief that any provider unilaterally deviating from that result will be regarded as of type $k_l$. However, it follows that to support *any* pooling equilibrium, the period 2 patients must hold the belief that a provider is of type $k_l$ if his report card result is slightly different$^{20}$ from the equilibrium outcome. This is because, if the period 2 patients hold the opposite belief, i.e. a provider will be of type $k_h$ should he yield a slightly different report card result, then discontinuity between the period 2 payoff $1/2$ from pooling and $M_h^*(k_h, k_l) > 1/2$ from separating means the type $k_h$ provider will always slightly deviate in period 1, which is a contradiction. But such a belief system implies that the patients prefer not to distinguish the high quality provider from the low quality one, which is implausible.

**More than Two Periods**  
While the two-stage game helps us gain insights of the effects of the report cards in a simple setting, in reality there may be more than two periods. When there are multiple periods, they can be broken into two categories that are represented by the two stages in my model: the signaling periods and the signaled periods. In the latter periods, since the providers’ types are revealed, the strategic interaction between providers and patients in each period will coincide with the second period in the two-stage game. The earlier periods, however, may differ from the original model, because adding more periods implies that the type $k_l$ provider has more incentive to imitate the high-quality provider. An immediate consequence is that $k_b$ will be lower than before since the type $k_h$ provider’s cost of generating a signal that the type $k_l$ provider does not want to imitate is now higher. Moreover, for $k_h < k_b$, a higher payoff from pooling with the type $k_h$ provider due to adding more periods means that the type $k_l$ provider may imitate any report card results from the type $k_h$ provider, resulting in pooling in early periods. Nonetheless, aiming at a one-signaling-period equilibrium, we can realistically introduce a discount factor, which discounts future

---

$^{20}$More precisely, "slightly different" means the report card result is $\varepsilon$ away from the the equilibrium outcome.
Learning-by-Doing

Existence of learning-by-doing among providers has also been documented in empirical studies, such as Ramanarayanan (2006). Since I assume provider types to be numerical rather than categorical, the model has the potential to be further extended to incorporate learning-by-doing. A starting point can be to assume the increment of provider quality $\Delta k_i$ to be an increasing function of $m_{i1}$, and so the provider’s period 2 quality increases with the measure of patients he treats in period 1. Consequently, upon seeing the report cards, period 2 patients now will figure out not only $k_i$ but also $k_i + \Delta k_i(m_{i1})$. Then we may conjecture that learning-by-doing will attenuate the providers’ selection behavior, since a provider is now facing a second layer of trade-off: Though avoiding patients can help to signal oneself or pool with the other, it also lowers $\Delta k_i$. In extreme cases, it may be that $k_i + \Delta k_i(m_{i1}) > k_h + \Delta k_h(m_{h1})$, that is, the type $k_l$ provider grows up to a provider with higher quality. The concern of lowered $\Delta k_i$ therefore should curb a provider’s incentive to avoid patients.

Word-of-Mouth

Other than resorting to published information, patients may rely on word-of-mouth from friends and relatives to gain information about the providers (Gibbs et al., 1996). Though such information is inevitably noisy, if it is positively correlated with a provider’s quality, we may conjecture that existence of word-of-mouth alleviates the providers’ incentive to select patients, since it gives the providers with another channel to signal their types. In particular, if one assumes that patients suffering from failed surgeries (death) do not engage in spreading provider information, then the larger the amount of successful surgeries, the larger the good word-of-mouth. A caution, however, is that the trade-off between $m_{i1}$ and $\bar{d}_{i1}$ implies that the measure of successful surgeries, $m_{i1}(1 - \bar{d}_{i1})$, may not achieve its maximum at $m_{i1} = M_{i1}$, and so requires more careful specification and analysis.

Previous studies also show that, compared with a small town with a small number of providers, it is harder for patients to gain information about providers through word-of-mouth in a big city with a large number of providers, since the chance that friends and relatives know a randomly picked provider is low (Satterthwaite, 1979; Pauly and Satterthwaite, 1981). Following this line, I conjecture that providers’ selection behavior will be more severe in urban areas than in suburban and rural areas. In other words, selection will be most severe in the areas where report cards are needed the most. Future extension in this direction bears both theoretical and empirical interest.

Trade-off between Outcome and Volume

The analysis of the theoretical model hinges on the trade-off between the outcome and volume measures. Though one may argue that, by shifting
the distribution of illness severity toward healthy patients, the providers may improve outcome without cutting volume, the notion is self-contradictory, because it implies the providers will not be short of patient sources, and as a result they should not be concerned by the report cards, and there will be no selection behavior at the outset. Moreover, the existence of referring physicians as "gatekeepers" renders such a strategy infeasible in reality.

4 Empirical Implications

In empirical studies, treatment effects of the report-card policy refer to the changes of variables caused by implementing the policy. For example, treatment effects studied by Dranove et al. (2003) include (i) Incidence Effect, measured by the change of the mean illness severity of the patients receiving the surgery, (ii) Quantity Effect, referring to the average change of the measure of patients undergoing the surgery, (iii) Matching Effect, showing the average change in the variation of illness severity in the surgical patients, and (iv) Welfare Effect, indicating the change of social welfare. Thanks to its advantage in controlling the possible confounding factors, the difference-in-differences estimation has been a common approach to estimate these effects. The theoretical model, however, implies that this estimation strategy should be implemented more carefully.

As the theoretical model shows, the report-card policy may cause two selection behaviors. First, before the report cards are issued, the policy causes providers to select patients if doing so helps them signal or hide their types. Second, after the publication of the report cards, revelation of providers’ types allows patients to select providers. Each selection behavior, if present, will affect the variables of empirical concern. To see this more clearly, let’s take the incidence effect as an example. At period 1, in cases where providers are not selecting patients, the incidence effect at that period is 0. But in the case where provider selection occurs, avoiding sicker patients implies the mean illness severity of the surgical patients will decrease, i.e. a negative incidence effect. At period 2, however, the incidence effect is ambiguous. If $\bar{M}_h^* > \bar{M}_h^*$, then in addition to treating all the patients with $s_j \geq s_{1/2}$ the type $k_h$ provider also treats a fraction of patients with $s_j < s_{1/2}$, which lowers the mean illness severity of his patients, and thus leads to a negative incidence effect. If $\bar{M}_h^* < \bar{M}_h^*$ then the type $k_h$ provider treats all the patients with $s_j < s_{1/2}$ plus a fraction of patients with $s_j \geq s_{1/2}$, leading to a lower mean illness severity of his patients, but the type $k_l$ provider now only treats patients with $s_j \geq s_{1/2}$, resulting in a higher mean illness severity for him. Consequently the incidence effect at period 2 is unclear. Analogously, one can show that other treatment effects also vary across the two periods due to the two selection behaviors.
The variation of each treatment effect across periods implies that a difference-in-differences estimate should be used for each treatment effect in each period. However, the commonly used single difference-in-differences estimate only captures the average of the two periods’ effects. The following simple model illustrates this. Let time period \( t = 0, 1, 2 \), where 0 stands for the period before the report-card program is effective, with 1 for the first period and 2 for the second. Let the superscript \( T \) denote the treatment group and \( NT \) the control (non-treatment) group. Denote \( y \) a variable whose change from \( t = 0 \) to \( t = 1, 2 \) in the treatment group encompasses a treatment effect of interest. Suppose there are \( n \) members in the treatment group and \( m \) members in the control group. For simplicity, ignore covariates. Suppose

\[
\begin{align*}
y_{j0}^T &= \alpha_0 + \varepsilon_{j0}, \quad y_{j0}^{NT} = \alpha_0 + \varepsilon_{j0} \\
y_{j1}^T &= \alpha_1 + \beta_1 + \varepsilon_{j1}, \quad y_{j1}^{NT} = \alpha_1 + \varepsilon_{j1} \\
y_{j2}^T &= \alpha_1 + \beta_2 + \varepsilon_{j2}, \quad y_{j2}^{NT} = \alpha_1 + \varepsilon_{j2}
\end{align*}
\]

where \( \varepsilon_{jt} \) and \( \varepsilon_{j't} \) are i.i.d. with \( E[\varepsilon_{jt}] = E[\varepsilon_{j't}] = 0 \). Then there are

\[
\begin{align*}
E[y_{j0}^T] &= \alpha_0, \quad E[y_{j0}^{NT}] = \alpha_0 \\
E[y_{j1}^T] &= \alpha_1 + \beta_1, \quad E[y_{j1}^{NT}] = E[y_{j2}^{NT}] = \alpha_1 \\
E[y_{j2}^T] &= \alpha_1 + \beta_2
\end{align*}
\]

and identification of the period 1 effect \( \beta_1 \) and the period 2 effect \( \beta_2 \) stems from

\[
\begin{align*}
\beta_1 &= [E[y_{j1}^T] - E[y_{j0}^T]] - [E[y_{j1}^{NT}] - E[y_{j0}^{NT}]] \\
\beta_2 &= [E[y_{j2}^T] - E[y_{j0}^T]] - [E[y_{j2}^{NT}] - E[y_{j0}^{NT}]].
\end{align*}
\]

On the other hand the single difference-in-differences estimate \( \hat{\beta} \) implies

\[
\hat{\beta} = \frac{\sum y_{j1}^T + \sum y_{j2}^T}{2n} - \frac{1}{n} \sum y_{j0}^T - \frac{\sum y_{j1}^{NT} + \sum y_{j2}^{NT}}{2m} - \frac{1}{m} \sum y_{j0}^{NT}
\]

\[
= \frac{1}{2} \left( \frac{\sum y_{j1}^T}{n} + \frac{\sum y_{j2}^T}{n} - \frac{1}{n} \sum y_{j0}^T \right) - \frac{\sum y_{j1}^{NT} + \sum y_{j2}^{NT}}{2m} - \frac{1}{m} \sum y_{j0}^{NT}
\]

\[
\rightarrow \frac{1}{2} \left\{ [(E[y_{j1}^T] + E[y_{j2}^T]) - 2E[y_{j0}^T]] - 2 [E[y_{j1}^{NT}] - E[y_{j0}^{NT}]] \right\}
\]

\[
= \frac{1}{2} \left( \beta_1 + \beta_2 \right). \quad (3)
\]

Since the single difference-in-differences estimation can only capture the average of the
periodic effects, results in the previous literature using this estimation strategy are worth revisiting and should be interpreted more carefully. For example, Dranove et al. (2003) use a negative incidence effect to indicate the existence of providers’ selection behavior, but as the above discussion shows, a negative incidence effect from a single difference-in-differences estimation can also be found in the situation where the providers are not selecting patients ($\beta_1 = 0$) but only patients are selecting providers ($\beta_2 < 0$). Put in another way, to indicate the existence of provider selection, one needs to show a negative period-1 incidence effect, which can be estimated by using difference-in-differences once every period.

Another crucial, yet subtle, point is that the effective date of the report-card policy is not the date when the report cards are published (i.e. the beginning of period 2), rather it is the date when the report-card producer starts collecting data (i.e. the beginning of period 1). For example, in the case of New York, the first report cards were published in 1991, showing providers’ performance from 1989 to 1990. Hence, to study the report cards’ effects, the researcher should set the effective date to 1989. Setting the effective date to 1991 further compromises the results from a single difference-in-differences estimation, as it essentially compares period-2 data with a mix of period-1 data and the data before that. Unfortunately this has not been paid enough attention to in the previous literature.

Another implication of (3) is that the single difference-in-differences estimate cannot capture the long-run welfare effect with short-run data, while estimating the welfare effect once every period can do so. In the theoretical model, two stages suffice for the analysis, but in reality there may be more than one period after provider types are revealed. To count the actual welfare effect, one needs to sum up the (discounted) effects in all periods. Consequently, even though a negative period 1 effect may outweigh a positive period 2 effect, leading to a negative single difference-in-differences estimation result, the sum of all periods’ effects may still be positive, rendering the single difference-in-differences result inconclusive. A simple example is as follows: Suppose in total there are 4 periods, with the first period’s welfare effect being $-3$, and the others being $2$. Thus the actual welfare effect is $3$. Suppose one only has data of the first two periods, then the single difference-in-differences estimation will lead to $-0.5$, the average of the first two periods’ effects. In contrast, separately estimating the first two periods’ effects allows us to capture $-3$ and $2$, and consequently capture the long-run welfare effect.
5 Conclusion

Providing a theoretical foundation to the empirical framework helps to shed new light on the studies of health care report cards. When critics argue that the health care providers may use private patient information to "game" the system, a game-theoretical model is the best candidate to help us understand why they game and how. Based on a two-stage signaling game, I show that, when patients and providers are matched randomly, the trade-off between the measures in the existing report cards render them the optimal mechanism that reveals provider types without causing them to select patients. However, asymmetric distributions of patient types between providers, attributed to the referring physician, may force the high-quality provider to shun patients in order to separate himself from the low-quality one. The report cards not only give rise to the possibility of providers selecting patients, but also help the patients select providers. To identify the providers’ selection behavior, we need a difference-in-differences estimation once every period.

Toward a deeper understanding of the report cards’ effects, the two-stage signaling game that I employ has the flexibility to be extended to incorporate more realistic features, including word-of-mouth among patients, and learning-by-doing of providers. These remain future directions.

The existence of selection behavior documented in previous literature may daunt people planning to introduce the report-card program to other fields in the health care industry and more broadly to other industries, such as law and education, where goods and services are also provided by skilled experts. But as shown in this paper, the providers’ selection behavior is not caused by the report cards alone, rather it is caused by the combination of the report cards and the particular features of the health care industry. This implies the report-card program has the potential to be successfully introduced to other industries, especially those where producers and consumers are randomly matched. Nonetheless, each industry is distinct in its own features, so specific conclusions should be drawn only upon a sufficient understanding of the industry where the report-card program is applied to.
Appendix 1

Proof of Lemma 1:

Suppose the providers’ types \((k_i, k_{-i})\) are revealed at \(t = 2\), then the facts \(F_{it}(\cdot) = F(\cdot)\) and that each provider performs surgeries on all incoming patients in period 2 imply \(E_{i2} = 1 - \int_{-\infty}^{+\infty} q(s, M_{i2}, k_i) dF(s)\) and \(E_{-i2} = 1 - \int_{-\infty}^{+\infty} q(s, 1 - M_{i2}, k_{-i}) dF(s)\). Following the discussion prior to the lemma, it must be \(E_{i2} = E_{-i2}\) in equilibrium, which implies

\[
L(M_{i2}) \equiv \int_{-\infty}^{+\infty} q(s, M_{i2}, k_i) - q(s, 1 - M_{i2}, k_{-i}) dF(s) = 0.
\]

Since \(L(1) \equiv \int_{-\infty}^{+\infty} [1 - q(s, 0, k_{-i})] dF(s) > 0\), \(L(0) \equiv \int_{-\infty}^{+\infty} [q(s, 0, k_i) - 1] dF(s) < 0\), and \(L'(M_{i2}) = \int_{-\infty}^{+\infty} \frac{\partial}{\partial M_{i2}} q(s, M_{i2}, k_i) + \frac{\partial}{\partial (1 - M_{i2})} q(s, 1 - M_{i2}, k_{-i}) dF(s) > 0\), by the Intermediate Value Theorem, for every \((k_i, k_{-i})\) there exists a unique \(\hat{M}_{i2}(k_i, k_{-i})\) such that \(E_{i2} = E_{j, -i2}\).

By the Implicit Function Theorem,

\[
\int_{-\infty}^{+\infty} \left[ \frac{\partial}{\partial k_i} q(s, \hat{M}_{i2}, k_i) + \frac{\partial}{\partial \hat{M}_{i2}} q(s, \hat{M}_{i2}, k_i) \frac{\partial \hat{M}_{i2}}{\partial k_i} + \frac{\partial}{\partial (1 - \hat{M}_{i2})} q(s, 1 - \hat{M}_{i2}, k_{-i}) \frac{\partial \hat{M}_{i2}}{\partial k_i} \right] dF(s)
\]

\[
= \int_{-\infty}^{+\infty} \frac{\partial}{\partial k_i} q(s, \hat{M}_{i2}, k_i) dF(s) + \frac{\partial \hat{M}_{i2}}{\partial k_i} \int_{-\infty}^{+\infty} \left[ \frac{\partial}{\partial \hat{M}_{i2}} q(s, \hat{M}_{i2}, k_i) + \frac{\partial}{\partial (1 - \hat{M}_{i2})} q(s, 1 - \hat{M}_{i2}, k_{-i}) \frac{\partial \hat{M}_{i2}}{\partial k_{-i}} \right] dF(s)
\]

\[
= 0,
\]

which implies \(\frac{\partial \hat{M}_{i2}}{\partial k_i} > 0\) since \(\frac{\partial}{\partial k_i} q(s, \hat{M}_{i2}, k_i) < 0\) and \(\frac{\partial}{\partial \hat{M}_{i2}} q(s, \hat{M}_{i2}, k_i) > 0\).

Similarly,

\[
\int_{-\infty}^{+\infty} \left[ \frac{\partial}{\partial \hat{M}_{i2}} q(s, \hat{M}_{i2}, k_i) \frac{\partial \hat{M}_{i2}}{\partial k_i} - \frac{\partial}{\partial k_{-i}} q(s, 1 - \hat{M}_{i2}, k_{-i}) \frac{\partial \hat{M}_{i2}}{\partial k_i} \right] dF(s)
\]

\[
= \frac{\partial \hat{M}_{i2}}{\partial k_{-i}} \int_{-\infty}^{+\infty} \left[ \frac{\partial}{\partial \hat{M}_{i2}} q(s, \hat{M}_{i2}, k_i) + \frac{\partial}{\partial (1 - \hat{M}_{i2})} q(s, 1 - \hat{M}_{i2}, k_{-i}) \frac{\partial \hat{M}_{i2}}{\partial k_{-i}} \right] dF(s)
\]

\[
- \int_{-\infty}^{+\infty} \frac{\partial}{\partial k_{-i}} q(s, 1 - \hat{M}_{i2}, k_{-i}) dF(s)
\]

\[
= 0,
\]

which implies \(\frac{\partial \hat{M}_{i2}}{\partial k_{-i}} < 0\). Q.E.D.
**Proof of Lemma 2:**

Given $M_i$, for every $m_{i1} \leq M_{i1}$ there exists a unique severity level $s_i$ such that $m_{i1} = M_{i1} \cdot F(s_i)$. For each $m_{i1} \leq M_{i1}$, the minimum mortality rate $\bar{d}_{i1}$ a provider $i$ can achieve comes from avoiding treating patients with $s_j > s_i$, implying

$$\bar{d}_{i1} = \frac{\int_{-\infty}^{s_i} q(s, m_{i1}, k_{i1})dF(s)}{F(s_i)} = \frac{\int_{-\infty}^{s_i} q(s, M_{i1}F(s_{i1}), k_{i1})dF(s)}{F(s_i)}$$

We have

$$\frac{\partial m_{i1}}{\partial s_i} = f(s_i) > 0$$

and

$$\frac{\partial \bar{d}_{i1}}{\partial s_i} = \left[ \frac{\int_{-\infty}^{s_i} q(s, M_{i1}F(s_{i1}), k_{i1}) \cdot f(s_i)}{[F(s_i)]^2} \cdot f(s_i) - \frac{\int_{-\infty}^{s_i} q(s, M_{i1}F(s_{i1}), k_{i1})dF(s) \cdot f(s_i)}{[F(s_i)]^2} \right] > 0,$$

where the first inequality in the $\frac{\partial \bar{d}_{i1}}{\partial s_i}$ part comes from $q(s, M_{i1} \cdot F_{i1}(s_i), k_{i1}) < q(s_i, M_{i1} \cdot F_{i1}(s_i), k_{i1})$ for all $s < s_i$, as implied by Assumption 1-(i).

Since we can write $s_i$ as $s_i = F^{-1}(\frac{m_{i1}}{M_{i1}})$, for $m_{i1} \in (0, M_{i1}]$ we can write $\bar{d}_{i1}$ as

$$\bar{d}_{i1}(m_{i1}) = \left[ \int_{-\infty}^{F^{-1}(\frac{m_{i1}}{M_{i1}})} q(s, m_{i1}, k_{i1})dF(s) \right] \cdot \frac{M_{i1}}{m_{i1}}.$$

Consequently $\frac{\partial \bar{d}_{i1}}{\partial m_{i1}} = \frac{\partial \bar{d}_{i1}}{\partial s_i} \cdot \frac{\partial s_i}{\partial m_{i1}} > 0$. Q.E.D.

**Proof of Lemma 4:**

In the previous subsection we already know that in period 1 when all the other patients randomize between the providers with equal probabilities a patient will also be indifferent between choosing each provider on his own. Also we show that if all patients in period 1 equally randomize between the providers, then the type $k_h$ provider will perform surgeries on all the incoming patients and thus reveals his type in period 2. Consequently the type $k_l$ provider will also perform surgeries on all the incoming patients. To complete the proof, we only need to show that when all the other patients equally randomize between the providers,
a patient will prefer this equal randomization to letting $C$ make the referral decision. That is,

$$\frac{1}{2}[\int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_1) dF(s) + \int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s)]$$

$$< \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_1) dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s),$$

which implies $\int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_1) dF(s) + \int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s) < 2[\int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_1) dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s)]$

which is equivalent to $\int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_1) dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s) < \int_{s_{1/2}}^{s_{1/2}} q(s, \frac{1}{2}, k_1) dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s)$, which after rearrangement leads to the inequality condition stated in the lemma. Q.E.D.

**Proof of Lemma 5**

For the type $k_h$ provider, given $M_{h1}$, for every $m_{h1} \leq M_{h1}$ there exists a unique $s_h \geq s_{1/2}$ such that

$$m_{h1} = M_{h1} \cdot F_{h1}(s_h).$$

For each $m_{h1} \leq M_{h1}$, the minimum mortality rate $\bar{d}_{h1}$ he can achieve comes from avoiding patients with $s_j > s_h$, implying

$$\bar{d}_{h1} = \frac{\int_{s_{1/2}}^{s_h} q(s, m_{h1}, k_h) dF_{h1}(s)}{F_{h1}(s_h)} = \frac{\int_{s_{1/2}}^{s_h} q(s, M_{h1}, F_{h1}(s_h), k_h) dF_{h1}(s)}{F_{h1}(s_h)}.$$

Similar to those in the proof of Lemma 2, we have $\frac{\partial m_{h1}}{\partial s_h} > 0$ and $\frac{\partial \bar{d}_{h1}}{\partial s_h} > 0$. Moreover, for $s_h \geq s_{1/2}$ we can write $s_h$ as $s_h = F_{h1}^{-1}(M_{h1})$, and so for $m_{h1} \in (0, M_{h1}]$ we can write $\bar{d}_{h1}$ as

$$\bar{d}_{h1}(m_{h1}) = \int_{s_{1/2}}^{F_{h1}^{-1}(M_{h1})} q(s, m_{h1}, k_h) dF_{h1}(s) \cdot \frac{M_{h1}}{m_{h1}}.$$

It follows that $\frac{\partial \bar{d}_{h1}}{\partial m_{h1}} = \frac{\partial m_{h1}}{\partial s_h} \cdot \frac{\partial s_h}{\partial m_{h1}} > 0$ and $\frac{\partial \bar{d}_{h1}}{\partial k_h} < 0$.

Similarly, for the type $k_l$ provider, given $M_{l1}$, for every $m_{l1} \leq M_{l1}$ there exists a unique $s_l \leq s_{1/2}$ such that

$$m_{l1} = M_{l1} \cdot F_{l1}(s_l).$$

For each $m_{l1} \leq M_{l1}$, the minimum mortality rate $\bar{d}_{l1}$ he can achieve comes from avoiding
treating patients with $s_j > s_l$, implying

$$\tilde{d}_{l1} = \frac{\int_{-\infty}^{s_l} q(s, m_{l1}, k_l) dF_{l1}(s)}{F_{l1}(s_l)} = \frac{\int_{-\infty}^{s_l} q(s, M_{l1} \cdot F_{l1}(s_l), k_l) dF_{l1}(s)}{F_{l1}(s_l)}.$$

We have $\frac{\partial m_{l1}}{\partial s_l} > 0$ and $\frac{\partial \tilde{d}_{l1}}{\partial s_l} > 0$. Moreover, for $s_l < s_{1/2}$ we can write $s_l = F_{l1}^{-1}(\frac{m_{l1}}{M_{l1}})$, and so for $m_{l1} \in (0, M_{l1}]$ we can write $\tilde{d}_{l1}$ as

$$\tilde{d}_{l1}(m_{l1}) = \int_{-\infty}^{F_{l1}^{-1}(\frac{m_{l1}}{M_{l1}})} q(s, m_{l1}, k_l) dF_{l1}(s) \cdot \frac{m_{l1}}{M_{l1}}.$$

It follows that $\frac{\partial \tilde{d}_{l1}}{\partial m_{l1}} = \frac{\partial \tilde{d}_{l1}}{\partial s_l} \cdot \frac{\partial s_l}{\partial m_{l1}} > 0$ and $\frac{\partial \tilde{d}_{l1}}{\partial k_l} < 0$. Q.E.D.

**Proof of Lemma 6:**

The proof parallels the proof of Lemma 4. To complete the proof, we only need to show that when all the other patients equally randomize between the providers, a patient will prefer this equal randomization to letting $C$ make the referral decision. That is,

$$\frac{1}{2} [\int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_l) dF(s) + \int_{-\infty}^{+\infty} q(s, \frac{1}{2}, k_h) dF(s)] < \int_{-\infty}^{s_{1/2}} q(s, \frac{1}{2}, k_h) dF(s) + \int_{s_{1/2}}^{+\infty} q(s, \frac{1}{2}, k_l) dF(s),$$

which after rearrangement leads to the inequality condition stated in the lemma. Q.E.D.

**Proof of Proposition 2:**

It is easy to see if $k_h > \bar{k}_h$ then there exists a separating equilibrium such that $m_{h1}^* = \frac{1}{2}$, because $k_h > \bar{k}_h$ implies $\tilde{d}_{h1}(\frac{1}{2}) < \tilde{d}_{l1}(\frac{1}{2})$, so the type $k_h$ provider can separate himself from the type $k_l$ provider by accepting all patients. Similarly, if $k_h = \bar{k}_h$, then $\tilde{d}_{h1}(m) < \tilde{d}_{l1}(m)$ for all $m < \frac{1}{2}$, so the type $k_h$ provider can separate himself from the type $k_l$ provider by accepting virtually all patients while shunning $\varepsilon$ measure of patients.

Now turn to the case where $k_h < \bar{k}_h$, that is, $\tilde{d}_{h1}(\frac{1}{2}) > \tilde{d}_{l1}(\frac{1}{2})$. First, if $k_h < \bar{k}_h$, then a necessary condition for a separating equilibrium is that $m_{l1} \neq m_{l1}$, since otherwise the fact that $1 - M_{h2}^* < \frac{1}{2}$ implies that the type $k_l$ provider will have the incentive to mimic the type $k_h$ provider’s signal. In other words, in a separating equilibrium it must be that the type $k_h$ provider generates a signal that the type $k_l$ provider does not want to mimic. Due to this,
there is a separating equilibrium with

\[ m_{h1} + \frac{1}{2} = \frac{1}{2} + 1 - M^*_h(k_h, k_l), \]

which implies

\[ m_{h1} = 1 - M^*_h(k_h, k_l). \]

In such a separating equilibrium the type \( k_h \) provider generates a signal that the type \( k_l \) provider is indifferent to imitate. The type \( k_h \) provider receives a payoff \( m_{h1} + M^*_h(k_h, k_l) = 1 > \frac{1}{2} + 1 - M^*_h(k_h, k_l) \), where the right hand side of the inequality is the payoff if type \( k_h \) provider accepts all the patients in period 1 and is regarded as a type \( k_l \) provider in period 2, and so he has no incentive to deviate given the patients’ consistent beliefs, which can be constructed accordingly.\(^{21}\)

For \( k_h \in (k_h, \bar{k}_h) \), to separate himself from the type \( k_l \) provider, the type \( k_h \) provider can choose \( m_{h1} = \bar{m}(k_h, k_l) - \epsilon \), a signal that cannot be imitated by the type \( k_l \) provider, or \( m_{h1} = 1 - M^*_h(k_h, k_l) \), a signal that the type \( k_l \) provider has no incentive to mimic. Since for \( k_h = k_h \), \( \bar{m}(k_h, k_l) = 0 < 1 - M^*_h(k_h, k_l) + \epsilon \); for \( k_h = \bar{k}_h \), \( \bar{m}(\bar{k}_h, k_l) = \frac{1}{2} > 1 - M^*_h(\bar{k}_h, k_l) + \epsilon \), and \( \bar{m} \) and \( M^*_h \) are increasing in \( k_h \), there exists a unique \( k_b \in (k_h, \bar{k}_h) \) such that \( \bar{m}(k_b, k_l) = 1 - M^*_h(k_b, k_l) + \epsilon \). For \( k_h < k_b \), \( 1 - M^*_h(k_h, k_l) > \bar{m}(k_b, k_l) - \epsilon \), so the type \( k_h \) provider prefers the signal that the type \( k_l \) provider has no incentive to mimic. For \( k_h \in (k_h, \bar{k}_h) \), \( \bar{m}(k_h, k_l) - \epsilon > 1 - M^*_h(k_h, k_l) \), so the type \( k_h \) provider prefers the signal that the type \( k_l \) provider cannot mimic.

To support the equilibrium strategies, the patients’ off-equilibrium-path belief can be constructed accordingly. We focus on the two cases \( k_h \in (k_b, \bar{k}_h) \) and \( k_h \in (k_1, k_b) \), as the other two cases are straightforward. Note that we only need to be concerned with two categories of the patients’ off-equilibrium-path belief, one with a provider \( i \)’s report card being \( (m_{i1}, d_{i1}) = (\frac{1}{2}, \bar{d}_{i1}(\frac{1}{2})) \) and the other with a provider \( i \)’s report card being \( (m_{i1}, d_{i1}) = (m^*_1, \bar{d}_{i1}(m^*_1)) \). First, for \( k_h \in (k_b, \bar{k}_h) \), for the type \( k_h \) provider not to deviate, the patients can hold the belief that if \( (m_{i1}, d_{i1}) = (\frac{1}{2}, \bar{d}_{i1}(\frac{1}{2})) \) but \( m_{-i1} > \bar{m}(k_h, k_l) - \epsilon \) then \( k_i = k_l \) and \( k_{-i} = k_l \). For the type \( k_l \) provider not to deviate, the patients can hold the belief that if \( (m_{i1}, d_{i1}) = (\bar{m}(k_h, k_l) - \epsilon, \bar{d}_{i1}(\bar{m}(k_h, k_l) - \epsilon)) \) but \( (m_{-i1}, d_{-i1}) \neq (\frac{1}{2}, \bar{d}_{i1}(\frac{1}{2})) \) then \( k_i = k_h \) and \( k_{-i} = k_l \). Second, for \( k_h \in (k_1, k_b) \), for the type \( k_h \) provider not to deviate, the patients can hold the belief that if \( (m_{i1}, d_{i1}) = (\frac{1}{2}, \bar{d}_{i1}(\frac{1}{2})) \) but \( (m_{-i1}, d_{-i1}) \neq (1 - M^*_h(k_h, k_l), \bar{d}_{i1}(1 - M^*_h(k_h, k_l))) \)

\(^{21}\)Moreover, analogously we can show actually every \( m_{h1} \in [\frac{1}{2} - 2M^*_h(k_h, k_l), 1 - M^*_h(k_h, k_l)] \) can be a part of a report card result that the type \( k_l \) provider does not want to imitate, and so it can also be supported as a part of a separating equilibrium. The one with \( m_{h1} = 1 - M^*_h(k_h, k_l) \) has the maximum payoff for the providers and so implies the minimum selection behavior.
then $k_i = k_h$ and $k_{-i} = k_l$. For the type $k_l$ provider not to deviate, the patients can hold the belief that (i) if $(m_{i1}, d_{i1}) = (1 - M_h^*(k_h, k_l), \tilde{d}_{h1}(1 - M_h^*(k_h, k_l)))$ but $(m_{-i1}, d_{-i1}) \neq (m_{i1}, d_{i1})$ and $(m_{-i1}, d_{-i1}) \neq (\frac{1}{2}, \tilde{d}_{i1}(\frac{1}{2}))$, then $k_i = k_h$, $k_{-i} = k_l$, and (ii) if $(m_{i1}, d_{i1}) = (m_{-i1}, d_{-i1}) = (1 - M_h^*(k_h, k_l), \tilde{d}_{h1}(1 - M_h^*(k_h, k_l)))$, then $\Pr(k_i = k_h) = \frac{1}{2}$. Q.E.D.

Proof of Proposition 3:

The results summarized in Table 2-2 imply that we only need to focus on the scenario where $C$ chooses Referral Pattern 1 and the patients let $C$ make the referral decision in period 1.

For $k_h > \bar{k}_h$, we already know that the existing report mechanism causes no provider selection.

For $k_h \leq \bar{k}_h$, we only need to focus on the conditions under which the existing report mechanism $R(m_{i1}, d_{i1}) = (m_{i1}, d_{i1})$ causes provider selection. Suppose there is a revealing mechanism such that in equilibrium $\hat{m}_{h1} > m_{h1}^*$. There are two cases:

(i) For $k_h \in (k_h, \bar{k}_h]$, $m_{h1}^* = \bar{m}(k_h, k_l) - \varepsilon$, so

$$\hat{m}_{h1} \geq \bar{m}(k_h, k_l) > 1 - M_h^*(k_h, k_l),$$

which implies

$$\hat{m}_{h1} + \frac{1}{2} > \frac{1}{2} + [1 - M_h^*(k_h, k_l)].$$

Then the type $k_l$ provider can be better off by deviating to the actions that lead to $r_l = \hat{r}_h$, which he is capable to do now thanks to the above result $\hat{m}_{h1} > 1 - M_h^*(k_h, k_l)$, contradicting the report mechanism being a revealing one.

(ii) For $k_h \in (k_l, k_h]$, $m_{h1}^* = 1 - M_h^*(k_h, k_l)$, then $\hat{m}_{h1} > m_{h1}^*$ implies that

$$\hat{m}_{h1} + \frac{1}{2} > \frac{1}{2} + [1 - M_h^*(k_h, k_l)].$$

Then the type $k_l$ provider can also be better off by deviating to the actions that lead to $r_l = \hat{r}_h$, contradicting the report mechanism being a revealing one. Q.E.D.
Appendix 2

Sample of New York CABG report card, hospital level, 1994

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Cases</th>
<th>Deaths</th>
<th>OMR</th>
<th>EMR</th>
<th>RAMR</th>
<th>95% CI for RAMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albany Medical Center</td>
<td>1167</td>
<td>18</td>
<td>1.54</td>
<td>1.93</td>
<td>1.98</td>
<td>(1.18, 3.14)</td>
</tr>
<tr>
<td>Arnot-Ogden</td>
<td>236</td>
<td>4</td>
<td>1.69</td>
<td>2.42</td>
<td>1.74</td>
<td>(0.47, 4.45)</td>
</tr>
<tr>
<td>Bellevue</td>
<td>93</td>
<td>6</td>
<td>6.45</td>
<td>2.28</td>
<td>7.05</td>
<td>(2.57, 15.34)</td>
</tr>
<tr>
<td>Beth Israel</td>
<td>270</td>
<td>4</td>
<td>1.48</td>
<td>2.98</td>
<td>1.24</td>
<td>(0.33, 3.17)</td>
</tr>
<tr>
<td>Buffalo General</td>
<td>1173</td>
<td>25</td>
<td>2.13</td>
<td>1.95</td>
<td>2.71</td>
<td>(1.75, 4.00)</td>
</tr>
</tbody>
</table>

Sample of Pennsylvania CABG report card, hospital level and surgeon level, 1994-95

FIGURE 3B: Actual to Expected Mortality, by Cardiac Surgeon, 1994-1995
References


