Asset Value, Interest Rates and Oil Price Volatility

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JEL codes: E37, F47, Q43

Working Paper No: 536
ISBN: 0 86831 536 2

January 2011
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Abstract

Simulations from a standard two-region model where producers respond to changes in interest rates are better able to match observed data than an identical model without supply-side responses. This indicates that incorporating the supply-side behaviour of oil producers is quantitatively important when endogenously modeling oil prices. These results have two implications. First, adding supply-side responses can change the oil price/output relationship, which is a continuing topic of research interest. Second, if production is unable to adjust to interest rate changes, an important explanatory factor of oil price volatility may be missing.

JEL Classification: E37, F47, Q43

Keywords: Oil price, volatility, two regions, dynamic model, interest rates

\textsuperscript{1} I have benefitted greatly from the comments and suggestions of Rod Tyers, Warwick McKibbin, Mardi Dungey, Tim Kam, Junsang Lee, Pedro Gomis-Porqueras, Jaime Alonso-Carrera, Bruce Preston, Don Harding, Jan Libich, Shaun Vahey, Shane Evans, Aki Asano, Justin Wang, and Yiyong Cai. I also received very helpful comments in seminar presentations at The Australian National University, The University of Tasmania, La Trobe University, as well as the 2010 Australian Conference of Economists in Sydney, and the 2010 conference of the Society for the Advancement of Economic Theory in Singapore.
1. Introduction

Following the work of Barsky and Kilian (2004), there have been a number of dynamic general equilibrium models in which oil prices are determined endogenously. Most of these papers focus on oil’s importance as an input to production, abstracting from its supply. In these models, changes in oil prices are driven by demand shocks, and the direct supply-side responses of oil producers to interest rates are not captured.

This paper studies the importance of such supply-side responses to interest rates when endogenously modeling oil prices. Research suggests that a relationship between oil prices and interest rates does exist and is also important. Mabro (1998) and Barsky and Kilian (2004) have argued that over the medium-run, interest rates will impact producer extraction and investment decisions. Both Akram (2008) and Frankel (2006) find evidence of a negative relationship between interest rates and the level of oil prices. Arora (2010) argues that low real interest rates may have played a part in the rising oil prices observed through 2008 by influencing producer extraction decisions.

The results of this paper show that the supply-side responses of oil producers to changes in interest rates are quantitatively important, especially for modeling oil price volatility. This matters for three reasons. First, the simulated effect that oil price changes have on output may be altered when considering supply-side responses. Some papers that endogenously model oil prices are looking at this relationship. Second, if production is unable to adjust to interest rate changes, an important explanatory factor of oil price volatility may be missing. Finally, incorporating supply-side decisions also helps to explain one factor driving extraction in a model with endogenous oil prices.

Theoretically, changes in interest rates will alter oil prices through producer extraction decisions if oil in the ground has value (Hotelling, 1931). Oil in the ground can have value because it is scarce, and this might differ from the value it has as a final good or as an input to production above ground. From a producer’s perspective, this additional value adds a facet to their extraction decision. Both the revenues from extracting and selling a barrel of oil, and the rate of return on oil in the ground need to be considered.

The model developed in this paper incorporates producer responses to changes in interest rates by making

\[^2\text{Nakov and Pescatori (2007) and Nakov and Pescatori (2009) are exceptions.}\]

\[^3\text{For example Hirakata and Sudo (2009).}\]
the stock of oil finite. This gives oil in the ground asset value. The model has two regions, complete asset markets, and demand shocks where oil is an input to production. The conditional expected rate of return on Arrow securities is used as a proxy for the real interest rate.

Some authors believe the exhaustibility of oil is irrelevant (see Mabro (1998)). However, it is a simple and intuitive way to incorporate the asset value of oil into a macroeconomic model. Additionally, as argued by Salehi-Isfahani (1995), the key issue is not whether the oil stock is actually finite, but if the possibility that oil will run out gives it a scarcity rent. Another potential issue with this framework is that the price of oil is taken as an equilibrium outcome in a competitive market. This may or may not be reasonable given the market power of OPEC, especially when global demand for oil is strong.

Simulations from the model with and without asset value are compared against various stylised facts for U.S. data. There are two primary results. First, standard business cycle statistics are similar between the two models. More importantly, the model with asset value is better able to account for the oil price related data. The absolute volatility of changes in the oil price, the correlation between real interest rates and changes in the oil price, the relative volatility of changes in the oil price with respect to output, and the relative volatility of changes in the oil price with respect to oil production all match the data better when oil has asset value than when it does not.

2. Relevant Data

U.S. data for the period JAN1982-MAR2010 on seasonally adjusted real GDP, seasonally adjusted real consumption, the short-term (one month) real interest rate, the log difference in real oil prices, and oil production are used to assess the model. The first two data sets are taken from the Federal Reserve Economic Database (FRED) GDP and components section, the short-term real interest rate is provided by the Federal Reserve Bank of Cleveland research department, and the oil data is taken from the U.S. Energy Information Administration (EIA).4 Following Stock and Watson (1998), all time series are H-P filtered, and with the exception of the real interest rate, are in logs.

The first three rows of Table 1, panel (a) provide standard business cycle statistics. The results for both GDP and consumption are in line with those reported in Stock and Watson (1998). The volatility of GDP

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4 Additional details on data collection and manipulation can be found in Appendix 1.
is lower than in Stock and Watson (1998), but this is likely because the time period under consideration
capsulates the majority of the “Great Moderation”. Consumption has over 75% of the volatility of output
over this time horizon, and the correlation between the two is closer to 85%.

<table>
<thead>
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<th>Variables</th>
<th>Description</th>
<th>SD</th>
<th>Relative SD</th>
<th>Corr w/ Qr</th>
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</thead>
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<tr>
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<td>GDP</td>
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<td>1.00</td>
<td>1.00</td>
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<tr>
<td>C</td>
<td>Consumption</td>
<td>0.0087</td>
<td>0.784</td>
<td>0.846</td>
</tr>
<tr>
<td>r</td>
<td>Real Interest Rate</td>
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<td>1.27</td>
<td>0.271</td>
</tr>
<tr>
<td>V</td>
<td>Log Difference Oil Price</td>
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<td>4.91</td>
<td>0.367</td>
</tr>
<tr>
<td>Y</td>
<td>Oil Production</td>
<td>0.006</td>
<td>0.647</td>
<td>0.054</td>
</tr>
</tbody>
</table>

(a) Business Cycle Statistics for the U.S. JAN82-MAR10

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Relative SD</th>
<th>Corr(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V and r</td>
<td>Log Diff Oil Price; Real Int Rate</td>
<td>3.86</td>
<td>-0.056</td>
</tr>
<tr>
<td>V and Y</td>
<td>Log Diff Oil Price, Oil Production</td>
<td>8.74</td>
<td>-0.286</td>
</tr>
</tbody>
</table>

(b) Selected Statistics for the U.S. JAN82-MAR10

Table 1: Selected U.S. Statistics

The one-month real interest rate estimates are taken from the Federal Reserve Bank of Cleveland as of
August 2010. These differ from the results in Stock and Watson (1998) for the 3-month real Treasury Bill
rate. The reported absolute volatility is an order of magnitude lower, and the contemporaneous correlation is
of the opposite sign. These differences are likely due to the different procedures used in calculating inflation
expectations.

The fourth row of Table 1, panel (a) gives the statistics for the quarterly log difference in oil prices.
Consistent with the literature, the annualised standard deviation of log price differences (returns) is used to
measure oil price volatility. Oil price data is taken from the EIA’s U.S. Refiner Acquisition Costs (RAC).

Returns are almost five times as volatile as output. While a large variation in oil price volatilities is
reported in the literature, the 5.40% shown in Table 1, panel (a) is on the low end of most estimates. There
are two reasons for this. First, real returns are used are instead of nominal. Using the same data without
deflating by the CPI gives a volatility closer to other studies. Second, the data are monthly, consolidated
to quarterly. Using daily or weekly data would likely raise the reported absolute volatility. The fifth row
from Table 1, panel (a) lists the relevant statistics for annual global oil production, taken from the EIA’s
international energy statistics. These are less volatile than both returns and output.

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5See http://www.clevelandfed.org/research/data/inflation_expectations/index.cfm
Panel (b) of Table 1 provides details on the relationship between returns and the real interest rate, and between oil production and returns. Over the time period, returns have been almost four times as volatile as the short-term real interest rate. The variables also have had a slightly negative correlation. The second row of this panel shows that returns are over eight times more volatile than production, and the variables also have a negative correlation.

The statistics reported in Table 1 will be used to assess simulation results from the model outlined in the next section.

3. The Model

The model has one oil producing and one oil consuming region. Within each region there is one representative consumer and one representative firm. Figure 1 presents the structure in detail.

The producing region’s sole output is oil, and the firm’s Cobb-Douglas technology depends on oil in the ground and domestic labour. This producing region generates all income from exports, and imports any final consumption goods. The consuming region imports oil for use in production. Final consumption goods are produced by the firm via a CES technology using oil and domestic labour. These final goods can be exported or consumed locally.

There is one source of uncertainty in the model: stochastic technology on final production in the consuming region. This technology induces a stochastic event, $s_t$, in each period $t$. There are finitely many
possible events, and the history of events up to and including \( t \) is denoted by \( s^t = (s_o, s_1, ..., s_t) \). The initial realisation, \( s_o \), is known. The probability at period 0 of any history \( s^t \) is \( \chi(s^t) \). Although all equilibrium prices and allocations are a function of these histories, the dependence will be suppressed throughout the paper for simplicity, save where it is absolutely necessary.

There are complete asset markets. Each regional household has access to a contingent claims market where an array of Arrow securities, denoted by \( B_{t+1}(s_{t+1}|s^t) \), are traded. These claims pay one unit of final consumption goods at \( t + 1 \) if \( s_{t+1} \) is realised given the history at \( t \) is \( s^t \). The price of that same security is denoted \( P_{b,t}(s_{t+1}|s^t) \).

Each variable representing the oil producing region has a subscript of 1, those representing the oil consuming region have a subscript of 2. If there is no subscript number, the variable is the same in both regions.

### 3.1. Consumers

Representative consumers in both regions choose current consumption \((C_t)\), labour supply \((N^{s}_t)\), and holdings of Arrow securities (one for each possible realisation of \( s_{t+1} \)) to maximise expected utility using a time-separable function of the coefficient of relative risk aversion (CRRA) form:

\[
\max \{C_t, N^{s}_t, (B_{t+1}(s_{t+1}|s^t))_{s_{t+1}}\}_{t=0}^\infty \quad E_t \left\{ \sum_{t=0}^\infty \beta^t \frac{C_{t}^{1-\eta_c}}{1- \eta_c} \right\}
\]

where \( \beta \) is the discount factor and \( \eta_c \) the CRRA. Consumers in either region can spend their wage income (the wage rate, \( w_t \), can differ between regions) and payout from one period claims \((B_t)\) on current consumption or the purchase of additional contingent claims:

\[
C_t + \int_{s_{t+1}} P_{b,t}(s_{t+1}|s^t)B_{t+1}(s_{t+1}|s^t) = w_t N^{s}_t + B_t
\]

### 3.2. Firms

Representative firms in each region have differing technologies. In the oil producing region, the firm chooses current extraction \((E_t)\), next period’s oil stock \((J_{t+1})\), and labour demand \((N^{d}_{1,t})\) each period to maximise profits.

\[
\max_{E_t, J_{t+1}, N^{d}_{1,t}} P_{o,t} Y_{o,t} - w_{1,t} N^{d}_{1,t}
\]
where $P_{o,t}$ is the oil price and $Y_{o,t}$ oil production. The production technology is Cobb-Douglas, with $\alpha_1$ the oil share in production:

$$Y_{o,t} = E_{t}^{\alpha_1} N_{1,t}^{(1-\alpha_1)}$$

(4)

The oil stock is depleted with extraction:

$$J_{t+1} = J_t - E_t$$

(5)

In the oil consuming region, the representative firm chooses oil demand ($Y_{o,t}$) and labour demand ($N_{d,2,t}$) to maximise profits,

$$\max_{Y_{o,t},N_{d,2,t}} Q_t - w_{2,t} N_{d,2,t}^d - P_{o,t} Y_{o,t}$$

(6)

where $Q_t$ is production of the final consumption good, and the price of final goods is set equal to one. This firm’s production technology is of the Constant Elasticity of Substitution (CES) form, with the elasticity of substitution between oil and labour defined as $\omega = \frac{\rho - 1}{\rho}$:

$$Q_t = Z_t (\psi Y_{o,t}^p + (1 - \psi) N_{d,2,t}^p)^\frac{1}{\rho}$$

(7)

where $\psi$ is the CES weight on oil. $Z_t$ is a stochastic total factor productivity shock with persistence $\pi$, which evolves according to:

$$\ln Z_t = \pi \ln Z_{t-1} + \epsilon_t$$

(8)

The innovation $\epsilon_t \sim i.i.d N(0, \sigma^2_\epsilon)$, and $\sigma_\epsilon$ is its standard deviation.

### 3.3. Optimality and Equilibrium

Consolidating the first-order conditions from the consumer and firm in the oil producing region gives the following optimality conditions:

$$P_{b,t} (s_{t+1} | s_t) = \beta_{1} E_{t} \left\{ \frac{C_{1,t}^{\rho_1}}{C_{1,t+1}^{\rho_1}} \right\}$$

(9)

$$\mu_t = \beta_{2} E_{t} \{ \mu_{t+1} \}$$

(10)

$$\mu_t = \frac{\alpha_1 P_{o,t} E_{t}^{(\alpha_1 - 1)}}{C_{1,t}^{\rho_1}}$$

(11)
These are for all $t$, and equation (9) is over all $s_{t+1}$. $\mu_t$ is the Lagrange multiplier on the oil constraint from the consumer’s utility maximisation, and can be interpreted as the value of oil in the ground.

Equation (9) defines the contingent price of Arrow securities as the discounted ratio of expected marginal utility over time. Equation (10) encapsulates the Hotelling (1931) Rule, the expected value of oil in the ground must grow at the inverse of the discount rate. The final equation specifies that the oil producing firm sets marginal cost equal to marginal benefits when choosing extraction.

As with the producing region, the oil consuming region’s firm and consumer first-order conditions are combined into the following optimality conditions:

$$P_{b,t}(s_{t+1}|s^t) = \beta_2 \mathbb{E}_t \left\{ \frac{C_{2,t}^{\omega_2}}{C_{2,t+1}^{\omega_2}} \right\}$$  \hfill (12)

$$P_{o,t} = \psi Y_{o,t} Z_t[\psi Y_{o,t} + (1 - \psi)N_{2,t}](\sigma - 1)$$  \hfill (13)

These are for all $t$, and equation (12) is over all $s_{t+1}$. Equation (12) defines the contingent price of Arrow securities as before. Equation (13) specifies that the oil consuming region imports oil until the marginal cost equals the marginal benefit.

The model is closed with two market clearing conditions:

$$Q_t = C_{1,t} + C_{2,t}$$  \hfill (14)

$$B_{1,t+1}(s_{t+1}|s^t) + B_{2,t+1}(s_{t+1}|s^t) = 0$$  \hfill (15)

Both are over all $t$, and equation (15) is over all $s_{t+1}$. Equation (14) is the final goods market clearing condition, and equation (15) states that Arrow securities are in zero net supply for every possible realisation of $s_{t+1}$.

A competitive equilibrium for this economy is a process of prices $\{P_{b,t}(s_{t+1}|s^t), P_{o,t}, w_t, \mu_t\}_{t=0}^{\infty}$, a process of allocations $\{C_t, N_t, J_{t+1}, E_t, B_{t+1}(s_{t+1}|s^t), Y_{o,t}, Q_t\}_{t=0}^{\infty}$, and an exogenous technology process $\{Z_t\}_{t=0}^{\infty}$ such that (i) representative firms in each region maximise profit; (ii) representative consumers in each region maximise utility; (iii) the final goods market clears; (iv) the securities market clears; and (v) each consumer’s budget constraint is met.
4. Solution Method and Parameter Values

4.1. Solution Method

Technology is represented by a Markov process, so the events $s_t$ summarise the effects of past events and current information. The model is solved using recursive methods, with equilibrium outcomes functions of two state variables: the stock of oil in the producing region and the technology shock in the oil consuming region. The model is solved by finding a stationary equilibrium with a decision rule that is a function of these state variables.

As in Backus et al. (1992), the stationary decision rule is obtained by solving a social planner’s problem to find the Pareto optimal allocations, given a weight ($\tau$) on each region’s consumption. The second welfare theorem can then be used to back out prices and the remainder of allocations from each respective consumer and firm’s problem.\(^6\) Specifically, the planner chooses allocations of consumption and labour in each region, along with oil stock next period, to maximise expected utility:

$$\begin{align*}
\max_{\{C_{1,t}, C_{2,t}, N_{1,t}, N_{2,t}, J_{t+1}\}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( \tau \frac{C_{1,t}^{1-\eta_1}}{1-\eta_1} + (1-\tau) \frac{C_{2,t}^{1-\eta_2}}{1-\eta_2} \right) \right\} \end{align*}$$

(16)

This is subject to a consolidated budget constraint:

$$C_{1,t} + C_{2,t} = Q_t$$

(17)

Le Van et al. (2010) show that under standard assumptions there is a stationary decision rule that is the optimal solution to this problem.\(^7\) The first-order conditions are provided in Appendix 2. The decision rule is approximated using continuous state dynamic programming with the CompEcon Toolbox of Miranda and Fackler (Miranda and Fackler, 2002).\(^8\)

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\(^6\)See Appendix 2 for a proof of the equivalence between the two approaches.

\(^7\)The utility function must be strictly concave, strictly increasing, continuously differentiable, and satisfy $u(0) = 0$ and $u'(0) = \infty$. The production function must be continuously differentiable, strictly increasing, and strictly concave when $J > 0$.

\(^8\)This toolbox approximates the value function at select points in the domain. The value function is approximated using a combination of piecewise linear spline basis functions. Piecewise linear splines are chosen as the basis functions because they provide reasonable approximations for functions that have discontinuities, or which are not smooth. This is important here, as the depletion of oil may result in solutions with significant non-linearities. Additional simulations were also performed using both Chebychev polynomials and piecewise polynomial splines as basis functions. However, neither performed as well as piecewise linear splines.
The grid for the stock of oil ranges from 2 to 100,000 for the benchmark solution. Up to a lower bound of 100, the Bellman equation residual error of 1e-7 is in line or better than that seen with comparable models of non-renewable resources (see Miranda and Fackler (2002) p. 245 and p. 259).\(^9\) The shock values range from -0.577 to 0.577.

### 4.2. Parameter Values

The model does not have a steady state apart from zero oil stock, and cannot be calibrated in the usual way. In order to put some structure on the results, the Planner’s weight on consumption in the oil producing region is varied until the time-series output in the oil consuming region matches the long-run consumption to output ratio in the U.S. This has been roughly between 0.60-0.70 over the previous 40 years (King and Rebelo, 2000).

Following King and Rebelo (2000), the persistence of the technology shock \(\pi\) is set to 0.979, with a standard deviation \(\sigma_v\) of 0.007. These are standard values for U.S. data. The CRRA parameter \(\eta\) is set at 1.5 in both regions as in Attanasio et al. (1999), and the discount factor \(\beta\) has a quarterly value of 0.99. To simplify the oil producer’s problem, the production elasticity of oil \(\alpha_1\) is set to 1.

The CES weight on oil in production \(\psi\) is set to 0.02. This estimate comes from Blanchard and Gali (2007), who use U.S. data. The elasticity of substitution between oil and labour in production of final goods is 0.50. Manne et al. (1979) emphasise the value is below 1 for the United States, and Kemfert and Welsch (2000) estimate \(\omega\) to be near 0.70 for Germany. Both of these parameters are varied to gauge the sensitivity of the results to their values.

Finally, the stock of oil is 100,000 barrels. Any choice for the stock is arbitrary, but this value is also changed in the sensitivity analysis.

### 5. Analytical Relationship

This section shows analytically (i) the factors that affect oil price volatility; and (ii) the relationship between interest rates and oil price volatility in the context of this model. These are best seen by looking individually

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\(^9\) 800 linear spline basis functions are used over the range of approximation for the oil stock (the first state variable). 18 linear spline basis functions are used to approximate the technology shock (the second state variable). The coefficients associated with the basis functions are found by iterating on the Bellman equation.
at the direct relationship between returns and the real interest rate (the asset value channel), and the indirect relationship between the two (the production channel).

5.1. Asset Value Channel

The asset value channel gives a direct relationship between returns and the real interest rate in this model. It can be isolated by working on the oil producing firm’s first-order condition on extraction:

$$\mu_t = \frac{\alpha_1 P_{o,t} E_t^{(\alpha_1-1)}}{C_{1,t}}$$

This condition equates the marginal cost of extraction ($\mu_t$, the value of oil in the ground) to the marginal benefit of extraction (the value of the marginal product of extracting one barrel). In the absence of asset value, $\mu_t=0$, and it is optimal for the oil producer to extract as much as possible.

Oil price volatility is generally represented as the standard deviation of log price differences ($V_t$):

$$\ln P_{o,t} - \ln P_{o,t-1} = V_t$$

Equations (18) and (19) are used to link directly between $R_{b,t}$ and $V_t$. To make this link for the asset channel, substitute equation (18) into equation (19) and rearrange:

$$V_t = \ln \left\{ \frac{\mu_t}{\alpha_1 E_t^{(\alpha_1-1)}} \right\} + \ln C_{1,t}^{\eta_1} + \ln 1 - \ln P_{o,t-1}$$

Equation (20) shows the impact of changes in $\mu_t$ on changes in the oil price. The value of oil in the ground can now be used to directly link $R_{b,t}$ and $V_t$. Before making this link, one preliminary step is required. Take the first-order condition on Arrow securities in the oil producing region, equation (9), and rearrange:

$$E_t \left\{ \beta_1 \left( \frac{C_{1,t}}{C_{1,t+1}} \right)^{\eta_1} \right\} = \frac{1}{R_{b,t}}$$

This can be substituted for $C_{1,t}$ in equation (20) to make the direct link between $R_{b,t}$ and $V_t$:

$$V_t = \ln \left\{ \frac{\mu_t}{\alpha_1 \beta_1 E_t^{(\alpha_1-1)}} \right\} + \ln E_t \left\{ C_{1,t+1}^{\eta_1} \right\} + \ln \frac{1}{R_{b,t}} - \ln P_{o,t-1}$$

10This security pays 1 unit of final consumption goods, so the gross return will be the inverse of the price.
5.2. Production Channel

The second channel, production value, exists whenever oil is used as an input to production. This is seen through the first-order condition on oil imports in the consuming region:

\[ P_{o,t} = Z_t MP_{Y_o,t} \]  \hfill (23)

\( MP_{Y_o,t} \) is the marginal product of oil as an input to final production. The firm sets the marginal cost of using oil as an input (the price) equal to the marginal benefit of using oil as an input (the value of the marginal product of the final good). Changes in returns due to production value can be seen by substituting equation (23) into equation (19):

\[ V_t = \ln Z_t + \ln MP_{Y_o,t} - \ln P_{o,t-1} \]  \hfill (24)

5.3. The Relationship Between Returns and Oil Price Volatility

Equations (22) and (24) provide the relationship between \( R_{b,t} \) and \( V_t \) in this model. In equilibrium these equations are equivalent, but they are useful in isolating how the addition of \( \mu_t \) changes the relationship between the real interest rate and oil price volatility. The asset value channel provides a direct link, while the production channel gives an indirect one.

Consider an exogenous increase in productivity, \( Z_t \). There is an immediate direct impact on \( V_t \) through equation (24). Price changes will also be indirectly affected through \( MP_{Y_o,t} \). Some of these indirect impacts may be due to the affect of \( R_{b,t} \) on \( MP_{Y_o,t} \), but this is not obvious from the equations. If there is no asset value, these changes capture the total effects of a demand shock on \( V_t \). The role of \( R_{b,t} \) is unclear in this case.

When there is an asset value channel, \( V_t \) is also impacted through equation (22). The technology shock will have impacts on all the variables in that equation, including \( R_{b,t} \). Any changes in \( R_{b,t} \) will have a direct impact on \( V_t \).

In summary, the impact of changes in \( R_{b,t} \) on changes in the oil price can be seen through equations (22) and (24). Asset value provides an endogenous link in the model between interest rates and the oil price that does not exist in its absence. However, the magnitude and direction of the impacts is unclear from the equations, and will require simulations to disentangle. These are taken up in the next section.
6. Simulations and Results

Simulation results are generated for each set of parameter values as mean values over 60,000 time series simulations of both models. In both cases, the planner’s weight is varied until the mean consumption to output ratio is 0.670. The next three sections compare results from both models to data, and conduct some sensitivity experiments.\footnote{Full simulation results are available upon request from the author}

6.1. Comparison with Data

Panel (a) of Table 3 shows that both models do relatively poorly in generating cross-correlations that match the GDP/real interest rate relationship. The model with asset value shows a negative correlation in all cases.\footnote{While this does not fit the current data, the signs are consistent with the statistics reported in Stock and Watson (1998). However, in that case the magnitudes are larger.} The other model shows virtually no correlation.

\begin{table} [h]
\centering
\begin{tabular}{llll}
\hline
 & $R_{b,t}, Q_t$ & $R_{b,t}, Q_{t-1}$ & $R_{b,t}, Q_{t+1}$ \\
\hline
Data & 0.271 & 0.354 & 0.192 \\
Asset Value & -0.114 & -0.083 & -0.080 \\
No Asset Value & 0.001 & 0.000 & 0.006 \\
\hline
\end{tabular}
\caption{$R_{b,t}$ Correlation with $Q_t$, J=1e6}
\end{table}

\begin{table} [h]
\centering
\begin{tabular}{llll}
\hline
 & $V_t, R_{b,t}$ & $V_t, R_{b,t-1}$ & $V_t, R_{b,t+1}$ \\
\hline
Data & -0.056 & 0.246 & -0.404 \\
Asset Value & -0.041 & 0.045 & -0.034 \\
No Asset Value & 0.001 & 0.007 & 0.002 \\
\hline
\end{tabular}
\caption{$V_t$ Correlation with $R_{b,t}$, J=1e6}
\end{table}

\begin{table} [h]
\centering
\begin{tabular}{llll}
\hline
Variables & Description & Data & Asset Value & No Asset Value \\
\hline
$R_b$ & Real Interest Rate & 1.27 & 0.840 & 4.90 \\
$V$ & Log Oil Price Diff & 4.91 & 4.39 & 3.04 \\
\hline
\end{tabular}
\caption{Standard Deviation Relative to Output J=1e6}
\end{table}

\begin{table} [h]
\centering
\begin{tabular}{llll}
\hline
Variables & Description & Data & Asset Value & No Asset Value \\
\hline
$\frac{V}{\pi_c}$ & Relative SD & 3.86 & 7.30 & 0.623 \\
\hline
\end{tabular}
\caption{Standard Deviation of $V$ relative to $R_b$, J=1e6}
\end{table}

Table 2: Simulation Results
The model with asset value does better in accounting for the correlation between returns and the real interest rate, as is shown in panel (b) of Table 2. It is able to account for over 73% of the contemporaneous correlation, and also has the correct signs in the other two cases. The alternative model shows little or no correlation between the two.

The results improve when considering the relative standard deviations shown in panel (c) of Table 2. The model with asset value can account for 66% of the relative standard deviation of $R_{b,t}$. It also accounts for almost 90% of the relative standard deviation of returns. The model without asset value exceeds the relative volatility of the real interest rate, and is less volatile with respect to returns. Adding supply-side responses overstates the relative volatility of returns with respect to the real interest rate as shown in panel (d) of Table 2. This nearly doubles the actual value, while the other model accounts for roughly 16% of this relative volatility.

The model with asset value also outperforms the other model when looking at specific oil market variables. The absolute volatility of returns accounts for 97% (0.056) of the volatility in the data, while the other model accounts for roughly 60%. Finally, the relative volatility of returns with respect to oil production is 8.44. Both models overshoot this; the model with asset value gives a value of 21.60, the other a value of 4.43e5.

The simulations show that adding supply-side responses increases the volatility of returns. At the same time, it also decreases the volatility of the real interest rate. This is the result of adding an asset channel, which allows both variables to directly effect each other. Over a variety of statistics, the model with asset value is able to more closely match characteristics of the observed data than a model without asset value. Importantly, it does better in terms of oil-related data. The next section explores the sensitivity of these results to changes in the oil stock in the producing region, and changes in the oil-related parameters in the consuming region.

6.2. Sensitivity

The sensitivity analysis varies the stock of oil ($J_0$), the CES weight on oil in final production ($\psi$), and the elasticity of substitution in final production ($\omega$).

Experiment 1: Varying the Oil Stock

Panel (a) of Table 3 shows that lowering the stock of oil in the producing region from 100,000 to 1000 makes virtually no difference to the correlation results reported above.
Panel (b) of Table 3 shows that the volatility of the real interest rate falls slightly when the stock is lower. Absolute volatility of returns remains at 5.60%. The model without asset value is also simulated with a lower stock, and its results do not change much either. Although the absolute volatility of returns rise to 3.87%.

**Experiment 2: Varying the CES Weight on Oil**

Panels (a) and (b) of Table 4 show that raising $\psi$ does not change either correlations or relative standard deviations by much. Lowering $\psi$ does have an effect on $R_{b,t}$. It raises the volatility of the real interest rate relative to both output and returns. It also changes the correlation of output and the real interest rate. Absolute volatility of returns does not change when $\psi$ is lowered, but rises to 5.81% when it is raised.
Experiment 3: Varying the Elasticity of Substitution Between Oil and Labour

Panels (a) and (b) of Table 5 show that lowering $\omega$ does not change either correlations or relative standard deviations by much. Raising $\omega$ does have an effect on $R_{b,t}$. It raises the volatility of the real interest rate relative to both output and returns. The absolute volatility of returns rises to 5.75% when $\omega$ is lower, and falls to 5.45% when it is higher.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>$\omega=0.25$</th>
<th>Base</th>
<th>$\omega=0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{b,t}, Q_t$</td>
<td>0.271</td>
<td>-0.108</td>
<td>-0.114</td>
<td>-0.121</td>
</tr>
<tr>
<td>$V_t, R_{b,t}$</td>
<td>-0.056</td>
<td>-0.050</td>
<td>-0.041</td>
<td>-0.053</td>
</tr>
</tbody>
</table>

(a) Correlations Varying $\omega$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>$\omega=0.25$</th>
<th>Base</th>
<th>$\omega=0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$</td>
<td>1.27</td>
<td>0.849</td>
<td>0.840</td>
<td>0.897</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>4.91</td>
<td>4.52</td>
<td>4.39</td>
<td>4.29</td>
</tr>
<tr>
<td>$V_t$</td>
<td>3.86</td>
<td>7.60</td>
<td>7.30</td>
<td>7.00</td>
</tr>
</tbody>
</table>

(b) Relative Standard Deviations Varying $\omega$

Table 5: Variations in $\omega$

Summary of Experiments

The sensitivity analysis shows that the model with asset value is robust to changes in the stock of oil. The results from the model do change when either the CES share of production is altered, or the elasticity of substitution in production between oil and labour is changed. When the CES share is lowered, or when the elasticity of substitution is raised, the volatility of the real interest rate rises.

7. Conclusions and Discussion

This paper finds that the supply-side responses of oil producers to changes in interest rates are quantitatively important for modeling oil price volatility. This is shown by simulating a two-region model with complete asset markets, demand shocks where oil is an input to production, and a fixed stock of oil. Fixing the stock of oil in the ground gives it scarcity value, which forces producers to consider rates of return on other assets. The real interest rate is taken to be an alternative rate of return for producers, and is proxied by the conditional expected rate of return on Arrow securities.

The model is simulated with and without supply-side responses to changes in interest rates. The model with these responses is better able to account for oil price related data, as well as the correlation between...
real interest rates and oil prices than a model without supply-side responses. In particular, giving oil in
the ground value makes the model better able to account for the absolute volatility of returns, the relative
volatility of returns to output, the relative volatility of returns to oil production, and the relative volatility
of returns to the real interest rate.

Modeling supply-side responses makes changes in oil prices less dependent on the exogenous shock process
as well. This is a result of adding a direct channel (the asset value channel) by which changes in the real
interest rate directly effect changes in oil prices. When oil in the ground has no value, there is no direct
relationship between the real interest rate and returns. In that case, the direct relationship between shocks
and returns (the production channel) drives changes in the oil price.

There is a substantive case for incorporating supply-side responses when modeling oil price volatility. The
inclusion of asset value is able to generate links between changes in the oil price and changes in macroeconomic
variables in a more thorough way than if only a production channel existed.

8. References

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**Appendix 1: Data and Model Parameter Values**

**Data**

All data range from January 1982 to March 2010. The data on consumption and GDP are taken from the Federal Reserve Bank of St. Louis’s Federal Reserve Economic Data (FRED) database. Quarterly GDP data
is seasonally adjusted at an annual rate, and in billions of chained 2005 dollars. Quarterly consumption data is personal consumption expenditures seasonally adjusted at an annual rate, and in billions of chained 2005 dollars. The logarithm of these real series are H-P filtered, with a smoothing parameter of 1600, to extract the cyclical components. The reported standard deviations and correlations are based on these cyclical series.

The monthly real interest rate is taken from the Federal Reserve Bank of Cleveland at a monthly frequency. It is converted to a quarterly frequency by taking an average over three months, and is then H-P filtered as above to extract the cyclical components. The reported standard deviation and correlations are based on the cyclical series.

Oil price data is taken from the U.S. Energy Information Administration’s (EIA) Refiner Acquisition Costs at a monthly frequency. This nominal series is converted to January 2005 dollars via the Consumer Price Index (CPI) excluding energy. CPI values are taken from FRED and are seasonally adjusted. Returns are then calculated as the log price differences of the real oil prices from one month to the next. These returns are converted to a quarterly frequency by taking a 3-month average. The quarterly returns are H-P filtered as above, and the reported statistics are based on the cyclical series.

Finally, oil production data is taken from the EIA’s international energy statistics section on petroleum production at an annual frequency. The logarithm of this series is H-P filtered with a smoothing parameter of 100, and the reported statistics are based on the cyclical series. When oil production is compared with returns and GDP, those other series are aggregated to an annual frequency by taking an average over 4 quarters.

Parameter Values

Table 6 summarises parameter values for the baseline simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{c1} = \eta_{c2}$</td>
<td>CRRA $R1/R2$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta_1 = \beta_2$</td>
<td>Discount factor $R1/R2$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Oil share of output $R1$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Oil share of output $R2$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Elasticity of sub in prod $R2$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Planner’s weight on cons $R1$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Persistence of tech shock $R2$</td>
<td>0.979</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Volatility of shock</td>
<td>0.007</td>
</tr>
<tr>
<td>$J_o$</td>
<td>Initial oil stock</td>
<td>1e6</td>
</tr>
</tbody>
</table>
Appendix 2: Model Equations

The problems for the representative firm and consumer in each region are consolidated into one regional problem below. Inequality constraints are also considered in each case.

Baseline Model

In the producing region, the consumer chooses allocations of consumption \((C_{1,t},t)\), labour \((N_{1,t},t)\), next period’s oil stock \((J_{t+1})\), extraction \((E_{t})\), and state-contingent claims \((B_{1,t+1}(s_{t+1}|s^t))\) to maximise expected utility:

\[
\max_{\{C_{1,t},N_{1,t},J_{t+1},E_{t},B_{1,t+1}(s_{t+1}|s^t)\}} \mathbb{E}_{t}\left\{ \sum_{t=0}^{\infty} \beta_t^1 \frac{C_{1,t}^{1-\eta_t}}{1-\eta_t} \right\}
\] (25)

Subject to

\[C_{1,t} \geq 0\] (29)

\[J_{t+1} \geq 0\] (30)

\[E_{t} \geq 0\] (31)

\[0 \leq N_{1,t} \leq 1\] (32)

\[-B_{1,t+1}(s_{t+1}|s^t) > \hat{B}\] (33)

\[J_o \text{ given}\] (34)

\[\lim_{t \to \infty} \beta_t^1 \mu_t J_t = 0\] (35)

Consumption must be greater than zero and the budget constraint will bind. If consumption goes to zero, the marginal utility of consumption is infinite. If the budget constraint did not bind, the possibility of disposal exists. However, the utility function is increasing in consumption, and this can never be the case.

The inequalities on extraction and the oil stock need to be looked at case-by-case and are considered below. Equation (33) bounds debt above some constant \(\hat{B}\). This constraint will not bind in equilibrium. The transversality condition, equation (35), states that either the stock or the discounted marginal utility of the stock \((\beta_t^1 \mu_t)\) must be zero in the very distant future. Finally, \(P_{o,t}\) is the price of oil, \(P_{b,t}(s_{t+1}|s^t)B_{1,t+1}(s_{t+1}|s^t)\) the price of an Arrow security, and \(Y_{o,t}\) oil production. The first order conditions:
\[ P_{b,t}(s_{t+1}|s^t) = \beta_1 E_t \left\{ \frac{C_{1,t}^{\eta_1}}{C_{1,t+1}^{\eta_1}} \right\} \tag{36} \]

\[ \mu_t \leq \beta_1 E_t \{ \mu_{t+1} \} \quad J_{t+1} \geq 0 \tag{37} \]

\[ \mu_t \leq \frac{\alpha_1 P_{o,t} E_t^{(\alpha_1-1)}}{C_{1,t}^{\eta_2}} \quad E_t \geq 0 \tag{38} \]

\[ N_{1,t} = 1 \tag{39} \]

where the first order conditions are for all \( t \), and equation (36) is over all \( s_{t+1} \). Equations (37) and (38) hold with complementary slackness. This leaves four possible cases to consider, two of which can be immediately ruled out. If \( E_t \) is equal to zero (in which case \( J_{t+1} \) is either zero or greater than zero), equation (28) is undefined. This is because there is no oil production, and given that Arrow security holdings are zero in equilibrium, implies no consumption.

This leaves the cases where both extraction and the stock are greater than zero, or where the stock is zero but extraction is greater than zero. The latter can only occur when \( E_t = J_t \), otherwise both will be greater than zero.

The structure of the consuming region is similar. Here, consumers choose allocations of consumption \( (C_{2,t}) \), labour \( (N_{2,t}) \), oil imports \( (Y_{o,t}) \), and Arrow securities \( (B_{2,t+1}(s_{t+1}|s^t)) \) to maximise a CRRA utility function:

\[
\max_{\{C_{2,t}, N_{2,t}, Y_{o,t}, B_{2,t+1}(s_{t+1}|s^t)\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_{2,t}^{1-\eta_2}}{1-\eta_2} \right\} \tag{40} \]

Subject to

\[ C_{2,t} + \int_{s_{t+1}} P_{b,t}(s_{t+1}|s^t) B_{2,t+1}(s_{t+1}|s^t) + Y_{o,t} P_{o,t} \leq Q_t + B_{2,t} \tag{41} \]

\[ Q_t = Z_t(\psi Y_{o,t}^{\rho} + (1-\psi)N_{2,t}^{\rho})^{\frac{1}{\rho}} \tag{42} \]

\[ \ln Z_t = \pi \ln Z_{t-1} + \epsilon_t \tag{43} \]

\[ \epsilon_t \sim i.i.d.N(0, \sigma^2_\epsilon) \tag{44} \]

\[ C_{2,t} \geq 0 \tag{45} \]

\[ Y_{o,t} \geq 0 \tag{46} \]

\[ 0 \leq N_{2,t} \leq 1 \tag{47} \]

\[ -B_{2,t+1}(s_{t+1}|s^t) > \bar{B} \tag{48} \]
As with the first region, consumption must be greater than zero and the budget constraint must bind.
Oil demand must be considered after looking at the first-order conditions. Equation (48) bounds debt above some constant $\bar{B}$. This constraint will not bind in equilibrium. The first order conditions in this region:

$$P_{b,t}(s_{t+1}|s^t) = \beta_2 \mathbb{E}_t \left\{ \frac{C_{b,t}^{1_{\tilde{r}_2}}}{C_{2,t+1}} \right\}$$

$$P_{o,t} \geq \psi Y_{o,t}^{\rho^{(\rho-1)}} Z_t[\psi Y_{o,t}^{\rho} + (1 - \psi)]N_{2,t}^{\rho^{(\frac{1}{2} - 1)}} Y_{o,t} \geq 0$$

$$N_{2,t} = 1$$

The first order conditions are for all $t$, and equation (49) is over all $s_{t+1}$. Equation (50) holds with complementary slackness. However, it can never be optimal to have zero oil demand. This means that equation (50) itself will be undefined. Hence, the only optimal case is where both consumption and oil demand are positive.

The model is closed with two market clearing conditions:

$$Q_t = C_{1,t} + C_{2,t}$$

$$B_{1,t+1}(s_{t+1}|s^t) + B_{2,t+1}(s_{t+1}|s^t) = 0$$

Both are over all $t$, and equation (15) is over all $s_{t+1}$. Equation (52) is the final goods market clearing condition, and equation (53) states that Arrow securities are in zero net supply in every state of the world given some history. Equations (36)-(39), (49)-(51), and (52)-(53), along with the transversality condition (35), budget constraints (26) and (41), debt limits (33) and (48), and exogenous technology process (43) characterise a competitive equilibrium in this model.

**Planner’s Problem**

The planner chooses $C_{1,t}$, $C_{2,t}$, $N_{1,t}$, $N_{2,t}$, and $J_{t+1}$ to maximise expected utility:

$$\max_{\{C_{1,t}, C_{2,t}, N_{1,t}, N_{2,t}, J_{t+1}\}_{t=0}^\infty} \mathbb{E}_t \left\{ \sum_{t=0}^\infty \beta^t \left( \frac{C_{1,t}^{1-\eta_1}}{1-\eta_1} + (1-\tau)\frac{C_{2,t}^{1-\eta_2}}{1-\eta_2} \right) \right\}$$

Subject to

$$C_{1,t} + C_{2,t} \leq Z_t[\psi(J_t - J_{t+1})^{(\rho^{(\rho-1)})} N_{1,t}^{(\rho^{(1-\alpha_1)})} + (1 - \psi)]N_{2,t}^{\rho^{(\frac{1}{2})}}$$

$$\ln Z_t = \pi \ln Z_{t-1} + \epsilon_t$$

$$\epsilon_t \sim i.i.d. N(0, \sigma^2_\epsilon)$$

$$C_{1,t} \geq 0$$

$$C_{2,t} \geq 0$$
\[0 \leq N_{1,t} \leq 1\]  
\[0 \leq N_{2,t} \leq 1\]  
\[J_{t+1} \geq 0\]  
\[J_o \text{ given}\]  
\[\lim_{t \to \infty} \beta^t \mu_t J_t = 0\]

where \(\tau\) is the planner’s weight on consumption in the producing region, and \(\beta\) is assumed to be the same in each region. The same arguments on each constraint can be made as those above.

The first order conditions on this problem:

\[(1 - \tau)C_{1,t} = \tau C_{2,t}\]  
\[
\psi C_{1,t}^{-\rho c_1} (J_t - J_{t+1})^{(\alpha_1 \rho - 1)} Z_t [\psi((J_t - J_{t+1})N_{1,t})^{(\rho \alpha_1)} + (1 - \psi)N_{2,t}^{\rho \alpha_1}]^{\frac{1}{\rho - 1}} = \psi \beta E_t \left\{ C_{1,t+1}^{-\rho c_1} (J_{t+1} - J_{t+2})^{(\alpha_1 \rho - 1)} Z_{t+1} [\psi((J_{t+1} - J_{t+2})N_{1,t+1})^{(\rho \alpha_1)} + (1 - \psi)N_{2,t+1}^{\rho \alpha_1}]^{\frac{1}{\rho - 1}} \right\} \]

\[N_{1,t} = 1\]  
\[N_{2,t} = 1\]

This will give equilibrium allocations for \(C_{1,t}, C_{2,t}, N_{1,t}, N_{2,t},\) and \(J_t\). One can then use the second welfare theorem, refer to the problem above, and find equilibrium allocations for the other variables of interest as shown in the next section.

### Equivalence of Allocations

The competitive equilibrium can be shown to be equivalent to the Pareto optimal solution if the equilibrium characterisations are the same. To show this, first combine equations (36) and (49) with the assumption that discount rates equate to get:

\[E_t \left\{ \frac{C_{1,t}^{-\rho c_1}}{C_{1,t+1}^{\rho c_1}} \right\} = E_t \left\{ \frac{C_{2,t}^{-\rho c_2}}{C_{2,t+1}^{\rho c_2}} \right\} \]

This is equivalent to a manipulation of equation (65), as is shown next. Take equation (65) forward one period to get

\[(1 - \tau)C_{1,t+1} = \tau C_{2,t+1}\]
Now divide equation (65) through by equation (70). This is the same as equation (69). As labour supply is not in the utility function of either region, equations (38) and (51) are the same as equations (67) and (68).

Finally, substitute equation (50) into equation (38), and then substitute this into equation (37).

\[
\frac{\psi Y^{(p-1)}_{o,t} Z_t [\psi Y^p_{o,t} + (1 - \psi) N^p_{2,t}] (\frac{1}{\rho} - 1) E^{(\alpha_1 - 1)}_t}{C^{\rho_1}_{1,t}} = \frac{1}{\rho} - 1
\]

(71)

Next use the fact that \( Y_{o,t} = E^{\alpha_1} = (J_t - J_{t+1})^{\alpha_1} \) and substitute into equation (71). This is equivalent to equation (66), and verifies the equivalence of the Pareto optimal and competitive equilibrium characterisations.