Potentially Harmful International Cooperation on Global Public Good Provision

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**Potentially Harmful International Cooperation on Global Public Good Provision**

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**Abstract:** Recent international climate negotiations suggest that complete agreements are unlikely to materialize. Instead, partial cooperation between like-minded countries appears a more likely outcome. In this paper we analyze the effects of such partial cooperation between like-minded countries. In doing so, we link the literature on partial cooperation with so-called matching approaches. Matching schemes are regarded as providing a promising approach to overcome undersupply of public goods like climate protection. The functioning of matching mechanisms in a setting with an incomplete agreement, i.e. a contract where only a subset of the players participates, has however not been investigated yet. This paper fills this research gap by analyzing incomplete matching agreements in the context of international climate protection. We analyse their effect on both welfare and the global climate protection level. We show that matching coalitions may bring about a decline in global public good provision and a reduction in the welfare of outsiders.

**Keywords:** Coalition formation, public goods, matching, Pareto optimality, partial cooperation  
**JEL Classification:** C78, H41, Q54

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1. Introduction

International cooperation is a prerequisite for the attainment of an efficient outcome in the presence of global public goods. However, international negotiations suffer from divergent national interests, different conceptions of fair burden or effort sharing, and a lack of trust between the negotiating countries. Such obstacles are regularly observed in negotiations on quite different topics such as international trade agreements, regulation of financial markets, development aid and international environmental protection.

A prominent example of the limited success of international negotiations is climate change policy. Negotiators have been unable to achieve a global agreement following the Kyoto Protocol. A grand coalition that credibly commits all polluting parties to domestic emission reductions is out of sight. As a precursor of a global agreement to be reached by 2020 only a rather informal ‘road map’ has been agreed upon at the Conference of Parties in Durban in 2011. In practice, rather than trying to achieve the ideal ‘grand coalition’, it seems more reasonable to strive for partial international agreements amongst some subgroup of like-minded countries, as e.g. the EU member countries.\(^1\)

In this paper we analyze the outcomes of international cooperation in the provision of a global public good like climate protection in which the set of countries participating in an international environmental agreement is of only limited extent. Buchholz, Haslbeck and Sandler (1998) have shown that – due to the offsetting behavior of the outsiders – the incentives to cooperate and to form a coalition might be very weak when the group of ‘willing’ countries is of limited size. In contrast, Vicary (2012) recently has found conditions under which a small group of agents can gain from unilaterally forming a coalition. Neither paper, however, makes reference to explicit equilibrium concepts that permit inferring the optimal design of collective action within some cooperating coalition. In this paper, our treatment of this question links the partial cooperation issue with the matching approach that has gained considerable attention in public theory.

Matching mechanisms through which agents subsidize the public good contributions of other agents were first suggested by Guttman (1978, 1987). They can be seen as a way to exploit the principle of reciprocity to improve efficiency of public good allocation. This ap-

\(^1\) Finus and Tjøtta (2003) find with respect to international environmental agreements that grand coalitions do not necessarily bring about the social optimum and recommend analyzing the formation of sub-coalitions.
proach has been refined in various ways and applied to many national and international pub-
ic goods by Boadway, Pestieau and Wildasin (1989), Danziger and Schnytzer (1991), Al-
(1996), Kirchsteiger and Puppe (1997), Falkinger and Brunner (1999), and Buchholz, Cornes
(2006), and Boadway, Song and Tremblay (2007, 2011) have applied matching particularly in
the context of global environmental protection. In contrast to the standard literature on
matching, our model allows matching rates also to become negative, i.e. the public good
 provision of a coalition is penalized via matching within this group of countries. Practically,
this can be accomplished by introducing a subsidy on fossil fuel use which will trigger higher
levels of greenhouse gas emissions.

Our analysis of matching as the instrument of partial cooperation applies the Aggregative
Game Approach developed by Cornes and Hartley (2003, 2007), which provides a very con-
venient tool in the search for equilibrium outcomes in this context. In particular, we use this
approach to complement the literature on coalition formation (e.g., Carraro and Siniscalco
van Ierland 2005). However, we focus more on the profitability of coalitions and less on its
stability.

We proceed as follows. Section 2 presents our model in which there are two different
groups of countries. One is a coalition of like–minded cooperating countries whose members
are mutually matching their public good provision, and the other consists of outsiders which
- without any matching - act non-cooperatively playing Nash against the coalition. Section 3
applies the Aggregative Game Approach to determine interior Nash equilibria in which all
countries make a strictly positive contribution to the public good and explores the conditions
under which the coalition has an incentive to increase or decrease its matching rate. Section
4 considers corner solutions in which one of the two groups of countries does not make a
positive contribution to the public good. Those corner solutions have to be taken into ac-
count if one wants to determine the matching coalition’s strategically optimal choice of its
matching rate and thus to find the subgame-perfect equilibria of the two stage game in
which the matching rate is chosen at stage 1 and a conventional Nash game in public good
contributions is being played at stage 2. In this section, as throughout the whole paper, we
do not only provide a general analysis of such subgame-perfect equilibria but also illustrate the major results by a simple Cobb-Douglas example. As a specific - and rather unexpected - result we show that partial cooperation of a rather small number of countries may reduce public good supply and thus may be harmful for global environmental quality. Section 5 concludes.

2. The Model

We consider the standard public good economy of Bergstrom, Blume and Varian (1986) and Cornes and Sandler (1986) which we apply to the case of a global public good. There are \( n \) identical countries which all have the same utility function \( u(x_i, G) \) and the same private good endowment (‘income’) \( w \). By \( x_i \) we denote private consumption in country \( i = 1, \ldots, n \) and \( G \) is the quantity of a global public good (i.e., ‘climate protection’). The utility function has the usual properties, i.e. it is strictly monotone increasing and twice continuously differentiable in both arguments, and both the private and the public good are assumed to be non-inferior. The technically given marginal rate of transformation between the public and the private good and thus the marginal costs of greenhouse gas abatement are assumed to be constant and can therefore be normalized to one. Then, the aggregate budget constraint comprising the incomes of all \( n \) countries reads

\[
G = \sum_{i=1}^{n} (w - x_i) \quad \text{or} \quad G + \sum_{i=1}^{n} x_i = nw. \tag{1}
\]

The essential assumption of our analysis now is that the whole world is divided into two groups: One subgroup, the coalition \( K \), consists of \( k \geq 2 \) cooperating countries which jointly provide the public good through reciprocally matching their public good contributions. The applied matching mechanism is a symmetric one. We denote by \( \rho \) the matching rate by which the public good contribution \( g_i \) of each country \( i \in K \) is augmented by the members of this group. This means that the flat contribution \( g_i \) made by any country \( i \in K \) induces \((1 + \rho)g_i\) as public good contribution of the coalition as a whole, where \( \rho g_i \) is a subsidy payment provided by the other coalition members.

In contrast to most of the matching literature we also allow for the possibility of a negative matching rate \( \rho < 0 \) in our model. In this case national public good contributions are not
subsidized but taxed or penalized. In the case of climate protection such a penalty might consist in fossil-fuel support policies, like, e.g., a subsidization of fossil fuel inputs, causing additional greenhouse gas emissions. In OECD countries, among the public fossil-fuel support policies are “direct subsidies, intervention in markets in ways that affect costs or prices, assumption of a part of companies’ financial risks, tax reductions or exemptions, and undercharging for the use of government-supplied goods, services or assets” (OECDb 2011: 3).²

Matching implies that country \( i \in K \) is confronted with the effective public good price \( \mu \), where

\[
\mu = \frac{1}{1 + \rho}.
\]

Clearly, if \( \rho > 0 (< 0) \), then \( \mu < 1 (> 1) \). We can equivalently choose \( \rho \) or \( \mu \) as the variable on which the outcomes produced by matching schemes depend. To facilitate the exposition, we will use the effective public good price \( \mu \) that is induced by the matching mechanism for each country in \( K \) throughout the paper and will call it matching parameter.

The other subgroup \( M \) of non-cooperating outsiders consists of \( m = n - k \geq 1 \) countries which all individually play Nash against the coalition \( K \). Their effective public good price is the technically given marginal rate of transformation between the public and the private good which has been assumed to be 1.

3. Interior Matching Equilibria

We now apply the Aggregative Game Approach (see Cornes and Hartley 2003, 2007) in order to determine public good supply \( \hat{G}(\mu) \) at an interior Nash equilibrium when there is partial matching within the coalition \( K \). Under the matching scheme, the effective public good price for the coalition members is \( \mu \).

Given preferences \( u(x, G) \) let \( e(G, \pi) \) denote the income expansion path of any country when the effective public good price for this country is \( \pi \). Public good supply \( \hat{G}(\mu) \) at an

² In an inventory, the OECD identified more than 250 individual mechanisms that effectively support fossil-fuel production or consumption in the considered 24 OECD countries (OECD 2011a: 3).
interior matching equilibrium, in which all countries in \( K \) and \( M \) make strictly positive flat contributions to the public good, is then characterized by a special version of the aggregate budget constraint (1) (see e.g. Cornes and Hartley 2007, for the general approach and Buchholz, Cornes and Rübbelke 2011, for an application to matching):

\[
\hat{G}(\mu) = k(w - e(\hat{G}(\mu), \mu)) + m(w - e(\hat{G}(\mu), 1)).
\]

Condition (3) uses the facts that in an interior solution all coalition members face the public good price \( \pi = \mu \) as altered by matching, while the public good price for the outsiders still is \( \pi = 1 \). Therefore, in an interior matching equilibrium, the equilibrium allocation of the coalition members must lie on the income expansion path \( e(G, \mu) \) while that of each outsider is on the income expansion path \( e(G, 1) \). For all income levels \( w > 0 \) existence of a public good level \( \hat{G}(\mu) \) which fulfils (3) is implied by the Intermediate Value Theorem if \( \lim_{G \to 0} e(G, \pi) = 0 \) and \( \lim_{G \to \infty} e(G, \pi) = \infty \) for any effective public good price \( \pi \) which clearly holds in the Cobb-Douglas case. Uniqueness is ensured by the strict monotonicity of the income expansion paths which follows from strict normality of the underlying preferences.

In an interior matching equilibrium private consumption is

- \( \hat{x}_K(\mu) = e(\hat{G}(\mu), \mu) \) for countries \( i \in K \),
- \( \hat{x}_M(\mu) = e(\hat{G}(\mu), 1) \) for countries \( i \in M \).

For \( \mu = 1 \), eq. (3) characterizes the standard Nash equilibrium with voluntary public good provision. In the subsequent section we will infer for which matching parameters \( \mu \) \( (\hat{x}_K(\mu), \hat{x}_M(\mu), \hat{G}(\mu)) \) in fact results as a matching equilibrium.

Given non-inferiority, the following comparative statics results hold:

**Proposition 1:** At an interior matching equilibrium

(i) public good supply \( \hat{G}(\mu) \) and private consumption of the outsiders \( \hat{x}_M(\mu) \) rise if the matching rate \( \rho \) is increased and the effective public good price \( \mu \) in the cooperating coalition falls. Private consumption of the coalition members \( \hat{x}_K(\mu) \) is reduced.
(ii) utility of each country in \( K \) is smaller (larger) than utility of a country in \( M \), if \( \rho > 0 \) \((-\rho < 0)\), i.e. \( \mu < 1 \) \((\mu > 1)\).

**Proof:** Both (i) and (ii) follow from the fact that \( e(G, \mu) \) is increasing in \( \mu \) for any level of \( G \).

As a next step we analyze how utility \( \hat{u}_K(\mu) := \hat{u}_K(\hat{x}_K(\mu), \hat{G}(\mu)) \) of a member of the cooperating coalition in an interior matching equilibrium is changed by a marginal variation of the matching parameter \( \mu \). To facilitate notation we adopt the following abbreviations:

\[
\hat{e}_1^K(\mu) := \left. \frac{\partial \hat{e}(G, \mu)}{\partial G} \right|_{G=\hat{G}(\mu)}, \quad \hat{e}_2^K(\mu) := \left. \frac{\partial \hat{e}(\hat{G}(\mu), \mu)}{\partial \mu} \right|_{G=\hat{G}(\mu)}, \quad \hat{e}_1^M(\mu) := \left. \frac{\partial \hat{e}(G, 1)}{\partial G} \right|_{G=\hat{G}(\mu)},
\]

\[
\hat{u}_1^K(\mu) = \left. \frac{\partial \hat{u}(x, G)}{\partial x} \right|_{(x,G)=(\hat{x}_K(\mu),\hat{G}(\mu))} \text{ and } \hat{u}_2^K(\mu) = \left. \frac{\partial \hat{u}(x, G)}{\partial G} \right|_{(x,G)=(\hat{x}_K(\mu),\hat{G}(\mu))}.
\]

For the marginal effect of a variation of the matching parameter on the utility we then get

\[
\frac{\partial \hat{u}_K^K(\mu)}{\partial \mu} = \hat{u}_1^K(\mu) \cdot (\hat{e}_1^K(\mu) + \hat{e}_2^K(\mu)) + \hat{u}_2^K(\mu) \cdot \frac{\partial \hat{G}(\mu)}{\partial \mu}.
\]

Observing that \( \mu \hat{u}_1^K = \hat{u}_2^K \) at an interior matching equilibrium,\(^3\) and that (3) implies

\[
\frac{\partial \hat{G}(\mu)}{\partial \mu} = -\frac{k \hat{e}_2^K(\mu)}{1 + k \hat{e}_1^K(\mu) + m \hat{e}_1^M(\mu)},
\]

it follows from (4) that

\[
\frac{\partial \hat{u}_K^K(\mu)}{\partial \mu} < 0 \quad (\neq 0) \text{ if and only if } \mu k > 1 + m \hat{e}_1^M(\mu) \quad (\neq 0).\]

An immediate implication of condition (6) is:

**Proposition 2:** (i) Starting from the standard voluntary provision equilibrium \( (\mu = 1) \), coalition \( K \) becomes better off by choosing some small positive/negative matching rate \( (\mu < 1 \text{ or } \mu > 1 \text{, but still close to 1}) \), if \( k > 1 + m \hat{e}_1^M(1) \).

(ii) If in an interior solution a local maximum for the utility of the countries in \( K \) is attained for some matching parameter \( \tilde{\mu} \) then

\[^3\text{As in the Nash equilibrium the standard result holds that the marginal rate of substitution between public and private good equals the price ratio between both goods.}\]
\[ \bar{\mu} = \frac{1 + m\hat{\alpha}_i^M(\bar{\mu})}{k}. \]

We illustrate this and several other results using the case of Cobb-Douglas preferences, i.e. \( u(x_i, G) = x_i^\alpha G \) where \( \alpha > 0 \) indicates the preference intensity for the private in relation to that for the public good. Then, given any public good price \( \pi \), the income expansion path becomes

\[ e(G, \pi) = \pi \alpha G. \]

In this special case Proposition 2 reads as follows:

**Proposition 3:** Given \( u(x_i, G) = x_i^\alpha G \):

(i) A small positive matching rate is preferable for coalition \( K \) if and only if \( k > 1 + m\alpha \).

(ii) If in an interior solution a local maximum for the utility of the countries in \( K \) is attained for some matching parameter \( \bar{\mu} \) then

\[ \bar{\mu} = \frac{1 + m\alpha}{k} \quad \text{or} \quad \bar{\rho} = \frac{k}{m\alpha + 1} - 1. \]

**Proof:** From (8) it directly follows that \( \hat{\alpha}_i^M(\mu) = \alpha \) holds for all \( \mu \), which gives (i).

QED

The interpretation of Proposition 2 and Proposition 3 is as follows: A large (small) cooperating coalition \( K \), a small (large) group of outsiders \( M \) and a relatively strong (weak) preference for the global public good, i.e. \( \alpha \) is small (large), favours positive (negative) matching within group \( K \). Consequently, moderate matching increases (reduces) public good supply as compared to the standard private provision equilibrium.

Propositions 2 and 3 reflect the basic result of Buchholz, Haslbeck and Sandler (1998) in the matching context. However, a problem is that the marginal analysis only pertains to small changes of \( \mu \) which preserve interiority of the matching equilibrium (see also Vicary 2012).
4. Subgame-Perfect Equilibria

We now consider the two-stage game in which the members of the coalition cooperatively determine the matching rate at stage 1 and there is a non-cooperative Nash game between the coalition as a whole and the outsiders at stage 2. Then it is possible that corner solutions occur as subgame-perfect equilibria. In such a case either the outsiders or the coalition members would make zero contributions to the global public good. We now, in a first step, describe at a general level how subgame-perfect equilibria are to be determined when corner solutions are taken into account. In a second step we illustrate this procedure for the Cobb-Douglas case.

4.1. General Analysis

To determine the ranges of \( \mu \) for which either interior or corner solutions result, we define two threshold levels \( \underline{\mu} \) and \( \bar{\mu} \) for the matching parameter \( \mu \) by letting

\[
\hat{x}_M(\mu) = e(\hat{G}(\mu), 1) = \hat{x}_K(\bar{\mu}) = e(\hat{G}(\bar{\mu}), \bar{\mu}) = w.
\]

This definition means that in an interior matching equilibrium as defined by (3) all outsiders would stop to make a positive contribution to the public good if the level of the matching parameter were \( \mu = \underline{\mu} \) while the contributions of the coalition would fall to zero if \( \mu = \bar{\mu} \). Decreasing \( \mu \) finally drives the outsiders and increasing \( \mu \) finally drives the coalition members into a free-rider position. If \( \lim_{\mu \to \underline{\mu}} \hat{x}_M(\mu) = \lim_{\mu \to \bar{\mu}} \hat{x}_K(\mu) = 0 \) existence and uniqueness of \( \underline{\mu} \) and \( \bar{\mu} \) follows from Proposition 1 by applying again the Intermediate Value Theorem. In order to characterize the outcomes which result for such relatively high or relatively low values of \( \mu \) we need some additional notation:

- By \( \hat{x}_K(\mu) \) we denote private consumption and by \( \hat{G}_K(\mu) \) public good supply in the ‘standalone’ matching equilibrium of coalition \( K \) when the matching parameter \( \mu \) is given. With matching within group \( K \) this solution would result if the outsider group \( M \) were absent. Applying the Aggregative Game Approach, public good supply \( \hat{G}_K(\mu) \) is defined by \( \hat{G}_K(\mu) + ke(\hat{G}_K(\mu), \mu) = kw \) which, by letting \( m = 0 \), represents a special case of (3). For private consumption then \( \hat{x}_K(\mu) = e(\hat{G}_K(\mu), \mu) \) is obtained. In
a \(x_i\)-\(G\)-diagram in which the position of a country in \(K\) is depicted the point \((\hat{x}_K(\mu), \hat{G}_K(\mu))\) lies on the budget line with slope \(-\frac{1}{\mu}\) passing through \((w,0)\). Then it is a straightforward implication of normality that \(\hat{G}_K(\mu)\) is a decreasing function of the matching parameter \(\mu\). Utility \(\tilde{u}_K(\mu) = u(\hat{x}_K(\mu), \hat{G}_K(\mu))\) of a coalition member is maximized if \(\mu = \frac{1}{k}\) where the efficient symmetric standalone equilibrium for coalition \(K\) is attained. If \(\mu > \frac{1}{k}\), then utility is decreasing, and if \(\mu < \frac{1}{k}\), then utility is increasing in \(\mu\).

- By \((\tilde{x}_M, \tilde{G}_M)\) we denote the standard non-cooperative standalone Nash equilibrium that would be attained by the outsider group \(M\) if the cooperating coalition \(K\) did not exist. With help of the Aggregative Game Approach \(\tilde{G}_M\) is defined by \(\tilde{G}_M + me(\tilde{G}_M, 1) = mw\) and then \(\tilde{x}_M = e(\tilde{G}_M, 1)\). The members of coalition \(K\) then have utility \(\tilde{u}_K = u(w, \tilde{G}_M)\).

Allowing now for corner solutions let \(x^{ma}_K(\mu)\) and \(x^{ma}_M(\mu)\) denote the private consumption of the coalition members and of the outsiders, respectively, in a matching equilibrium and \(G^{ma}(\mu)\) the public good supply given partial cooperation within coalition \(K\). The matching parameter is \(\mu\). Depending on the level of \(\mu\) the matching equilibria are as follows:

**Proposition 4:** (i) If \(\mu \leq \underline{\mu}\), then only coalition \(K\) contributes to the global public good in the unique matching equilibrium and \(x^{ma}_K(\mu) = \tilde{x}_K(\mu), x^{ma}_M(\mu) = w\) and \(G^{ma}(\mu) = \hat{G}_K(\mu)\).

(ii) If \(\mu \geq \overline{\mu}\), then only the outsiders in group \(M\) contribute to the global public good in the unique matching equilibrium and \(x^{ma}_M(\mu) = \tilde{x}_M(\mu), x^{ma}_K(\mu) = w\) and \(G^{ma}(\mu) = \hat{G}_M\).

(iii) If \(\mu \in (\underline{\mu}, \overline{\mu})\), then the unique matching equilibrium is interior and \(x^{ma}_K(\mu) = \hat{x}_K(\mu), x^{ma}_M(\mu) = \hat{x}_M(\mu)\) and \(G^{ma}(\mu) = \hat{G}(\mu)\).
Proof: (i) Since $\hat{G}_k(\mu)$ is a decreasing function of $\mu$ and $\hat{G}_k(\mu) = \hat{G}_k(\mu)$ holds, the optimal reaction of the outsiders in group $M$ is to contribute nothing to the public good if $\mu \leq \mu$ and the coalition’s aggregate public good contribution is $k(w-x_k(\mu))$ which gives rise to public good supply $\hat{G}_k(\mu)$. But if the outsiders make a zero-contribution to the public good, coalition $K$ will – given the matching parameter $\mu$ - choose the standalone matching equilibrium with $\hat{x}_k(\mu)$ and $\hat{G}_k(\mu)$. This is clearly the unique Nash equilibrium when group $M$ contributes nothing to the public good. Now assume that in the case $\mu \leq \mu$ there is another matching equilibrium with public good supply $G^{ma}(\mu)$ in which the outsiders make a positive contribution to the public good. Then $G^{ma}(\mu) < \hat{G}(\mu)$ must hold since otherwise the outsiders would have to make a negative contribution to the public good to reach a position on the income expansion path $e(G,1)$. As $\mu < \mu$ and the income expansion paths are shifted to the left when the effective public good price falls, we have $e(G^{ma}(\mu),\mu) < e(\hat{G}(\mu),\mu)$. This implies that given $\mu$ the coalition $K$ makes a higher contribution to the public good than in the case of $\mu$ so that the aggregate budget constraint (1) would be violated.

(ii) The proof is analogous to that of part (i).

(iii) If $\mu \in (\mu, \bar{\mu})$ it follows from the monotonicity of the income expansion paths that $\hat{x}_k(\mu) < w$ and $\hat{x}_M(\mu) < w$. Then, if each outsider contributes $w-\hat{x}_M(\mu) > 0$ to the public good, the coalition will react by choosing the public good contribution $k(w-\hat{x}_k(\mu))$ because then and only then its members reach an equilibrium position on the income expansion path $e(G,\mu)$. Similarly, if the contribution of the coalition $K$ is $k(w-\hat{x}_k(\mu))$, each outsider country must choose $w-\hat{x}_M(\mu)$ to get on its income expansion path $e(G,1)$ and thus to be at an equilibrium. This shows that $(\hat{x}_k(\mu), \hat{x}_M(\mu), \hat{G}(\mu))$ is an interior matching equilibrium in this case which, as seen in the previous section, is unique among the interior ones. Now assume that $G^{ma}(\mu)$ is public good supply at another matching equilibrium given the same $\mu$ in which only coalition $K$ makes a positive contribution to the public good in this equilibrium. Then we have $G^{ma}(\mu) > \hat{G}(\mu)$ because otherwise the outsiders would still have an incentive to contribute to the public good. But as $\mu > \mu$ also implies that $e(G,\mu) > e(G,\mu)$. 

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for all \( G > 0 \), the contributions of coalition \( K \) at this matching equilibrium would be smaller than with \( \mu \), and the aggregate budget constraint (1) would not be fulfilled. Therefore, a matching equilibrium which has only the members of coalition \( K \) as contributors cannot exist if \( \mu \in (\underline{\mu}, \overline{\mu}) \). The possibility that only the outsiders contribute to the public good in a matching equilibrium when \( \mu \in (\underline{\mu}, \overline{\mu}) \) is excluded in an analogous way. QED

With Proposition 4 as a preparation, we can now infer the strategically optimal choice of the matching parameter \( \mu \) by the coalition \( K \). The basic assumption underlying our general results in this context is that, as a function of the matching parameter \( \mu \), the utility of the coalition members has a single peak in the interior solution. As we will soon show, this condition is fulfilled when the countries have Cobb-Douglas preferences.

**Condition SP:** The function \( \hat{u}_k(\mu) \) is strictly monotone increasing for \( \mu < \bar{\mu} \) and strictly monotone decreasing for \( \mu > \bar{\mu} \).

There are three candidates for the matching parameter \( \mu^* \) which maximizes utility of coalition \( K \): Either \( \mu^* = \bar{\mu} \) when an interior solution is attained at stage 2, or \( \mu^* = \frac{1}{k} \) if only the coalition and \( \mu^* = \bar{\mu} \) if only the outsiders contribute to the public good. The next propositions clarify what considerations determine the chosen value of \( \mu \).

**Proposition 5:** Let condition **SP** be fulfilled and assume \( \bar{\mu} \leq 1 \). The strategically optimal matching parameter \( \mu^* \) for coalition \( K \) then is

- \( \mu^* = \bar{\mu} \) if either \( \underline{\mu} < \frac{1}{k} \) or if \( \frac{1}{k} \leq \mu \leq \bar{\mu} \) and \( \hat{u}_k(\bar{\mu}) > \hat{u}_k(\frac{1}{k}) \).

- \( \mu^* = \frac{1}{k} \) if \( \frac{1}{k} \leq \bar{\mu} \leq \mu \), or if \( \frac{1}{k} \leq \mu \leq \bar{\mu} \) and \( \hat{u}_k(\bar{\mu}) > \hat{u}_k(\frac{1}{k}) \).

**Proof:** If \( \bar{\mu} < 1 \), then **SP** implies that the coalition will not want to choose a matching parameter \( \mu > \bar{\mu} \). If \( \underline{\mu} < \frac{1}{k} \), then **SP** moreover entails that any \( \mu \) with \( \underline{\mu} \leq \mu < \bar{\mu} \) will lead to a lower utility than \( \mu = \bar{\mu} \). Since \( \hat{u}_k(\underline{\mu}) = \hat{u}_k(\mu) \) and \( \hat{u}_k(\mu) \) falls as \( \mu \) is reduced below \( \underline{\mu} < \frac{1}{k} \)
coalition $K$ also does not prefer a matching parameter $\mu \leq \underline{\mu}$ over $\bar{\mu}$. If instead $\frac{1}{k} \leq \bar{\mu} \leq \underline{\mu}$, then it follows from $\text{SP}$ that starting from $\mu = 1$ utility $\hat{u}_K(\mu)$ of the coalition members rises until $\mu = \underline{\mu}$ is reached. Since $\hat{u}_K(\underline{\mu}) = \bar{u}_K(\mu)$ and $\mu > \frac{1}{k}$, utility $\bar{u}_K(\mu)$ will further increase when the coalition further reduces $\mu$ towards the level $\frac{1}{k}$ and fall after that. Therefore, $\mu = \frac{1}{k}$ is the coalition’s best choice in this case. Finally, if $\frac{1}{k} \leq \mu \leq \underline{\mu}$, then $\text{SP}$ yields that the coalition does best by choosing $\bar{\mu}$ among all $\mu \geq \underline{\mu}$. Among all $\mu < \underline{\mu}$ the best choice is $\mu = \frac{1}{k}$. The decision between $\mu = \bar{\mu}$ and $\mu = \frac{1}{k}$ then clearly depends on the comparison drawn between $\hat{u}_K(\bar{\mu})$ and $\bar{u}_K(\frac{1}{k})$. QED

**Proposition 6:** Let condition $\text{SP}$ be fulfilled and assume $\bar{\mu} > 1$. The strategically optimal matching parameter $\mu^*$ for coalition $K$ then is

- $\mu^* = \bar{\mu}$, if $\bar{\mu} < \underline{\mu}$ and $\underline{\mu} \leq \frac{1}{k}$, or if $\underline{\mu} \geq \frac{1}{k}$ and $\bar{u}_K(\bar{\mu}) > \bar{u}_K(\frac{1}{k})$.

- $\mu^* = \underline{\mu}$, if $\bar{\mu} \geq \underline{\mu}$ and $\underline{\mu} \leq \frac{1}{k}$, or if $\underline{\mu} \geq \frac{1}{k}$ and $\bar{u}_K \bar{u}_K(\frac{1}{k})$.

- $\mu^* = \frac{1}{k}$, if $\underline{\mu} \leq \frac{1}{k}$ and $\bar{u}_K(\frac{1}{k}) > \hat{u}(\bar{\mu})$ when $\bar{\mu} \leq \underline{\mu}$, or if $\bar{u}_K(\frac{1}{k}) > \bar{u}_K$ when $\bar{\mu} > \underline{\mu}$.

**Proof:** The proof essentially follows the same lines as those of Proposition 5. We therefore only present an explicit proof for the assertion under the third bullet point: The condition $\text{SP}$ implies that among all $\mu > \underline{\mu}$ the coalition maximizes utility of its members either by choosing $\mu = \bar{\mu}$ or by choosing $\mu = \underline{\mu}$. But the coalition could also choose some $\mu \leq \underline{\mu}$ and would then - given $\mu \geq \frac{1}{k}$ - maximize its utility by letting $\mu = \frac{1}{k}$ among these values of $\mu$. The optimal choice of coalition $K$ then depends on the comparison between the three utility levels $\hat{u}_K(\bar{\mu})$, $\bar{u}_K$ and $\bar{u}_K(\frac{1}{k})$. QED
There is a remarkable asymmetry between Proposition 5 and Proposition 6: From Proposition 2 we know that $\bar{\mu} < 1$ and $\bar{\mu} > 1$ indicate whether the coalition would prefer to have some positive or negative matching as compared to the case without matching. If $\bar{\mu} < 1$, then Proposition 5 says that in the subgame-perfect equilibrium the coalition definitely will choose a positive matching rate and will make higher contributions to the public good than in the stage 2 Nash equilibrium without matching. But as seen from Proposition 6 this is also a possibility if $\bar{\mu} > 1$, i.e. if the marginally negative matching would be in the interest of coalition $K$. Proposition 6 also shows that under certain circumstances the coalition might prefer negative matching as its strategically optimal choice. In the subgame-perfect equilibrium then both public good supply and utility of the outsiders are lower than in the Nash equilibrium without matching. Therefore, in this case partial cooperation within coalition $K$ can be deemed harmful which reminds us of a paradoxical result described by Hoel (1991) in a quite different framework: Unilateral action of a country or a group of countries from which at first sight more greenhouse gas abatement should be expected may in the end have the opposite effect, i.e. entail further environmental degradation. With $\bar{\mu} > 1$ also multiple subgame-perfect equilibria are possible in our model. In the next section we will now exemplify these general results for the case with Cobb-Douglas preferences.

4.2 The Example of Cobb-Douglas Preferences

Applying condition (11) to the Cobb-Douglas example where $u(x_i, G) = x_i^\alpha G$ immediately gives $\mu = 1 - \frac{1}{k\alpha}$ and $\bar{\mu} = 1 + \frac{1}{m\alpha}$. To simplify the treatment of the Cobb-Douglas case we make the empirically plausible assumption that $\alpha \geq 1$, i.e. that the agents do not value the public good higher than the private good. Then clearly $\mu > \frac{1}{k}$. Furthermore we recall from Proposition 3 that $\bar{\mu} > 1$ if $k > m\alpha + 1$ and $\bar{\mu} < 1$ if $k < m\alpha + 1$. Then we obtain the following special version of Proposition 5.

**Proposition 7:** Let $u(x_i, G) = x_i^\alpha G$ with $\alpha \geq 1$ be given. If $k > m\alpha + 1$, then the strategically optimal matching parameter $\mu^*$ for coalition $K$ is
\[ \mu^* = \bar{\mu} = \frac{m\alpha + 1}{k} < 1, \text{ when } k < m\alpha + 1 + \frac{1}{\alpha} \text{ and} \]

\[ \left(1 + \frac{m}{k}\right)^{\alpha+1} > m\alpha + 1 \]

(ii) \[ \mu^* = \frac{1}{k}, \text{ when } k > m\alpha + 1 + \frac{1}{\alpha}. \]

**Proof:** Proposition 5 can be applied since SP is fulfilled in the Cobb-Douglas case (see Appendix A2). Part (i) then follows as it is easily calculated that \( \bar{\mu} > \mu \) follows from \( k < m\alpha + 1 + \frac{1}{\alpha} \).

The other condition stated in (i) is equivalent to

\[ \hat{u}_k(\bar{\mu}) = \left(\frac{\alpha}{k(m\alpha + 1)}\right)^\alpha \left(\frac{(k+m)\nu}{\alpha+1}\right)^{\alpha+1} \left(\frac{\alpha}{\alpha+1}\right)^\alpha kW^{\alpha+1} = \bar{u}_k\left(\frac{1}{k}\right). \]

The calculation of the relevant utility levels is given in Appendix A1. Part (ii) follows from Proposition 5 as \( k > m\alpha + 1 + \frac{1}{\alpha} \) gives \( \bar{\mu} < \mu \).

QED

The two parts of Proposition 7, however, are not equally important. Closer inspection reveals that the condition for interiority is quite hard to fulfil: Since \( k > m\alpha + 1 \) is the basic assumption underlying Proposition 7 in total \( m\alpha + 1 < k < m\alpha + 1 + \frac{1}{\alpha} \) would be required. Thus, if \( \alpha \geq 1 \) and \( m \) is given, an interior solution could at most be obtained for a single natural number \( k \). But then, moreover, it follows from the interiority constraint that we have

\[ m\alpha + 1 < \left(1 + \frac{m}{k}\right)^{\alpha+1} < \left(1 + \frac{1}{\alpha}\right)^{\alpha+1} < 4 \] as \( \alpha \geq 1 \). Therefore, the interior outcome can make the coalition members only better off if the number of outsiders does not exceed 2.

Therefore, the standard outcome in the case \( k > m\alpha + 1 \) will be the optimal standalone allocation for coalition \( K \) where \( \mu^* = \frac{1}{k} \) and the outsiders do not contribute to the public good.

If \( k < m\alpha + 1 \), then the analogue to Proposition 6 for the Cobb-Douglas case is provided by the next proposition.
**Proposition 8:** Let \( u(x_i, G) = x_i^\alpha G \) with \( \alpha \geq 1 \) be given. If \( k < m\alpha + 1 \), then the strategically optimal matching parameter \( \mu^* \) for coalition \( K \) is

(i) \[ \mu^* = \bar{\mu} = \frac{1 + m\alpha}{k} > 1 , \text{ when } k > m\alpha \text{ and } \left(1 + \frac{m}{k}\right)^{\alpha+1} > m\alpha + 1 \text{ hold} \]

(ii) \[ \mu^* = \bar{\mu} = 1 + \frac{1}{m\alpha} , \text{ when } k < \min(m\alpha, \frac{m\alpha}{m\alpha + 1}(1 + \frac{1}{\alpha})^{\alpha+1}) \]

(iii) \[ \mu^* = \frac{1}{k} , \text{ when either } k > m\alpha \text{ and } \left(1 + \frac{m}{k}\right)^{\alpha+1} < m\alpha + 1 \text{ or when } \]

\[ m\alpha \geq k \geq \frac{m\alpha}{m\alpha + 1}\left(1 + \frac{1}{\alpha}\right)^{\alpha+1} \]

**Proof:** Proposition 6 can be applied since \( \text{SP} \) is fulfilled in the Cobb-Douglas case (see Appendix A2). The proof of part (i) then is the same as for part (i) of Proposition 7 since \( \bar{\mu} < \bar{\mu} \) is implied by \( k > m\alpha \). To apply Proposition 6 for the proof of (ii) and (iii) we first observe that \( k \leq m\alpha \) gives \( \bar{\mu} \geq \bar{\mu} \). The other inequalities appearing in Proposition 8 (ii) and (iii) stem from the comparison between \( \hat{u}_k(\bar{\mu}) \) and \( \bar{u}_k(\frac{1}{k}) \) (see Appendix A1 for the calculation of the utility levels).

**QED**

Similar to Proposition 7 an interior solution, i.e. \( \mu^* = \bar{\mu} \) as described by Proposition 8 (i), only results if \( m\alpha < k < m\alpha + 1 \), i.e. at most for one coalition size \( k \) when \( m \) and \( \alpha \) are given.

A straightforward consequence of Proposition 8 (ii) is that harmful partial cooperation, i.e. \( \mu^* = \bar{\mu} \), indeed can occur under specific assumptions. So the set of parameters leading to the outcome of Proposition 8 (ii) is not empty:

- If \( k = 2 \), then the harmful strategic choice \( \mu^* = \bar{\mu} \) is made for any \( \alpha \geq 1 \) and any \( m \geq 3 \).
- If \( k = 3 \), then we get \( \mu^* = \bar{\mu} \) if e.g. \( \alpha = 1 \) and \( m \geq 4 \). In this situation an increase of the coalition size from \( k = 3 \) to \( k = 4 \) would also have much effect on public good supply as then the public good level would more than double from \( \frac{4}{5} \) to 2.
Harmful partial cooperation, however, only results if the size $k$ of the cooperating coalition does not exceed 3, as in $k < \frac{m\alpha}{m\alpha + 1}(1 + \frac{1}{\alpha})^{\alpha+1}$ (which is the condition that yields $\bar{u}_k > \bar{u}(\frac{1}{k})$) the right-hand side is smaller than 4 for each $\alpha \geq 1$.

Finally, Proposition 8 (iii) says that the coalition might also choose its optimal standalone matching, i.e. $\mu^* = \frac{1}{k}$, even in situations where some slight matching would be harmful. Then big changes of the matching parameter entail a Pareto improvement while marginal changes do not.

The results in Proposition 8 can also be applied to show that in some situations there are multiple subgame-perfect equilibria. Consider, e.g., the case where $\alpha = 1$ and $k = m = 3$.

Choosing $\mu = \frac{1}{3}$ yields $\hat{G}(\frac{1}{3}) = \frac{3}{2} w$, $\hat{x}_k(\frac{1}{3}) = \frac{1}{2} w$, $\hat{x}_M(\frac{1}{3}) = w$, $\hat{u}_K(\frac{1}{3}) = \frac{3}{4} w^2$ and $\hat{u}_M(\frac{1}{3}) = \frac{3}{2} w^2$, while $\mu = \bar{\mu} = \frac{4}{3}$ gives $\hat{G}(\frac{4}{3}) = \frac{3}{4} w$, $\hat{x}_k(\frac{4}{3}) = w$, $\hat{x}_M(\frac{4}{3}) = \frac{3}{4} w$, $\hat{u}_K(\frac{4}{3}) = \frac{3}{4} w^2$ and $\hat{u}_M(\frac{1}{3}) = \frac{9}{16} w^2$. In this situation coalition $K$ is indifferent between choosing $\mu = \frac{1}{3}$ and $\mu = \frac{4}{3}$ so that, simultaneously, there is a ‘good’ equilibrium (with $\mu^* = \frac{1}{3}$) where public good supply and utility of outsiders are higher than in the Nash equilibrium without matching and a ‘bad’ equilibrium (with $\mu^* = \frac{4}{3}$) where both are lower. The third possibility appearing in Proposition 8, i.e. $\mu^* = \tilde{\mu}$, has not to be considered separately since $\tilde{\mu} = \bar{\mu} - \frac{4}{3}$ in this specific example.

5. Conclusions

In this paper we have investigated the implications of partial cooperation for the provision of (global) public goods. In this context we have added three novel elements to the existing contributions: We have
considered matching mechanisms as the instrument for cooperation within a coalition of like-minded countries. Matching is by now seen to be a potentially appealing way to improve the level of international environmental protection.

applied the Aggregative Game Approach to characterize matching equilibria in flat contributions. Thereby we linked the theory of coalition formation with some recent developments in public good theory.

explored the coalition’s optimal strategy for setting the matching rate at a first stage, paying special attention to corner solutions at the second stage where either the outsiders or the coalition members do not contribute to the public good.

In particular we have shown that the two types of corner solution are of a very different quality. If only the coalition contributes to the public good, then partial cooperation generates a Pareto improvement as compared to the standard voluntary provision equilibrium: Public good supply and utility of all countries are increased. If, however, only the outsiders contribute, then partial cooperation has harmful effects: Although the utility of coalition members is increased, public good supply and utility of the outsiders fall. In the context of global warming, the coalition lowers its public good contribution (by increasing greenhouse gas emission through subsidization of fossil fuels) as compared to the Nash equilibrium without matching in order to induce the outsiders to increase their contributions. A further result of our analysis has been that, depending on the specific circumstances, it is well possible that the cooperating coalition’s strategically optimal choice of the matching parameter at the first stage of the game steers the economy into either of these two corner solutions. Harmful partial cooperation thus may occur in the subgame-perfect equilibrium.

In the Cobb-Douglas example, moreover, it is precisely the presence of a small coalition that generates the unfavourable outcome. This might provide an additional reason why, even if pragmatic reasons motivate a particular interest in partial cooperation among a small number of countries, it is still important to understand the possibilities afforded by larger coalitions. It is solely large coalitions which definitely avoid the harmful effects of cooperation and ensure a Pareto improvement over the standard voluntary provision equilibrium.
Appendix

A1: Observing the equation for income expansion paths in the Cobb-Douglas case the budget constraint (3) becomes

\[ \hat{G}(\mu) = k(w - \mu x \hat{G}(\mu)) + m(w - \alpha \hat{G}(\mu)) , \]

for any given matching parameter \( \mu \) in coalition \( K \), which directly gives

\[ \hat{G}(\mu) = \frac{(k + m)w}{(k\mu + m)\alpha + 1} \]

and

\[ \hat{x}_k(\mu) = \mu \hat{x}_M(\mu) = \frac{\mu \alpha (k + m)w}{(k\mu + m)\alpha + 1} . \]

Plugging these values into the utility function gives

\[ \hat{u}_k(\mu) = (\mu \alpha)^a \left( \frac{(k + m)w}{(k\mu + m)\alpha + 1} \right)^{\alpha + 1} \]

and

\[ \hat{u}_M(\mu) = \alpha^a \left( \frac{(k + m)w}{(k\mu + m)\alpha + 1} \right)^{\alpha + 1} \]

for the utility levels of the coalition members and the outsiders in an interior matching equilibrium. Inserting

\[ \hat{\mu} = \frac{\maxw + 1}{k} \]

then yields the left-hand side of inequality (x).

Letting \( m = 0 \) in (x) gives

\[ \hat{G}(\mu) = \frac{k w}{k\mu \alpha + 1} \]

for public good supply and

\[ \hat{x}_k(\mu) = \frac{k \mu \alpha \w}{k \mu \alpha + 1} \]

for private consumption of any coalition member in the standalone matching equilibrium when the matching parameter is \( \mu \). Utility then is

\[ \hat{u}_k(\mu) = (\mu \alpha)^a \left( \frac{k w}{k\mu \alpha + 1} \right)^{\alpha + 1} \]

and setting \( \mu = \frac{1}{k} \) yields the right-hand side of inequality (x).

Letting \( k = 0 \) in (x) gives

\[ \hat{G}_M = \frac{m w}{m \alpha + 1} , \hat{x}_M = \frac{m \alpha \w}{m \alpha + 1} \]

and for utility of coalition members in the outsiders’ standalone Nash equilibrium

\[ \hat{u}_k = \frac{m \alpha \w^{\alpha + 1}}{m \alpha + 1} . \]

A2: Taking the derivative of \( \hat{u}_k(\mu) \) w.r.t. \( \mu \) and then dividing the numerator by

\[ \mu^{a-1}((k\mu + m)\alpha + 1)^a((k + m)w)^{\alpha + 1} \]

gives

\[ \alpha((k\mu + m)\alpha + 1) - \mu \alpha \chi (\alpha + 1) = \alpha (\alpha + 1) - \mu \alpha \chi . \]
This expression obviously is greater than 0 (smaller than 0, equal to 0), if
\[ \mu < \tilde{\mu} = \frac{m\alpha + 1}{k} \] \((>, =)\).

References


