Implications of Alternative Banking Systems

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Implications of Alternative Banking Systems *

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Abstract

A significant number of individuals are unwilling to deposit their savings into the banking sector since it does not operate according to their religious beliefs. In this paper we provide a model that aims to answer the following questions: First, under what conditions an alternative banking system would arise? Second, what are the growth, and welfare implications of these banking systems? Our model shows that an alternative banking system would arise if individuals have religious concerns. Moreover, we show that in an economy populated with a certain number of religiously concerned individuals, the existence of an alternative banking system can generate relatively higher growth and improve welfare.

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Keywords: banking; growth; welfare

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1 Introduction

A sizeable portion of the Muslim population is unwilling to deposit their savings into conventional banks that operate not according to their religious principles (see for example Kpodar and Imam [2010]). This in turn affects the level of financial development (and economic growth in general) in many Muslim countries. According to a World Bank report, domestic bank lending is, for example, 3.8% of Afghanistan’s GDP, 29% of Senegal’s GDP, 36% of Nigeria’s GDP and 46% of Pakistan’s GDP.\(^1\) Alternative banking systems (so called Islamic banking systems) aim to provide financial services complying with Islamic principles to serve the financial needs of Muslims who abstain from using the conventional banking system because of their religious concerns.

This paper aims to answer the following two questions: First, under what conditions an Islamic banking system may arise? Second, what are the growth and welfare implications of the Islamic banking system? In order to do so we develop a simple overlapping generations model that incorporates both conventional and Islamic banking systems. Our model economy is populated with two types of individuals who differ in terms of their religiosity.

Economists have realized long ago that individuals’ economic decisions would also depend on factors such as status/social comparison, altruism, social custom/pressure, and self-control issues.\(^2\) In addition, individuals’ religious concerns in general, and their after-life believes in particular, would shape their economic decisions. Many religions including Judaism, Christianity, and Islam emphasize the relationship between an individual’s actions and behavior in his present life and the rewards (or the punishment) in the after-life. Chang [2005] modeled individuals’ after-life concerns: a utility function consists of two types of expenditures, that is, spending for the present life and spending for the after-life. This includes, for example, participating religious activities, donating to the needy, or investing in a way that complies with one’s religious beliefs.

Individuals with standard preferences usually seek investment opportunities according to their level of risk aversion i.e. they choose assets with high returns if they are willing to accept the high risk that follows. However, for individuals with religious/after-life preferences, the risk/return profile of an asset would not be the only factor for investment decisions. For instance, these individuals would choose an asset with a relatively low return if it complies with their religious beliefs. According to the World Islamic Banking Competitiveness Report 2011-2012, the Islamic finance industry is valued at $1.14 trillion and growing at a rate of 10% each year.\(^3\) International banks such as Citibank, HSBC and BNP Paribas are also offering Islamic financial services. This implies Islamic banking is becoming an increasingly visible alternative to the conventional banks in countries with a sizeable Muslim population.

Islamic banks operate under profit sharing and trade agreements since Islam strictly prohib-

\(^1\)http://data.worldbank.org/indicator


its paying and receiving interest. On the deposit side, the bank offers accounts with different risk/return profiles and shares in the profit with the depositors. Hence an Islamic bank’s deposit acts similar to a mutual fund. On the loan side, the bank uses trade and leasing agreements for consumption loans. If the client would, however, require a business loan, an Islamic bank would offer a business partner agreement where both parties share in the profit according to the amount of funds contributed. There are increasing efforts to understand the implications of the Islamic banking system both theoretically and empirically. This study deviates from the earlier studies by providing a framework to explore under what conditions those banks arise. In order to do so, we assume that individuals in our economy differ in terms of their religiosity/after-life concerns. In addition, we analyze the growth, aggregate, and welfare implications of an Islamic banking system. Although the economic implications of the financial intermediation have been well analyzed in the literature, the economic implications of the Islamic banking system is still not known. This paper aims to fill this gap in the literature.

A simple three period overlapping generations model with financial intermediation is developed. We analyze the impact of introducing Islamic banking on economic aggregates and social welfare in an economy with heterogenous population: a portion of the population does not have after-life concerns while the other portion has. Both types of individuals including banks have access to liquid but unproductive and illiquid but productive investment opportunities that produces a capital stock. The capital stock owned by a portion of old individuals and the labor supplied by young individuals are used to produce consumption goods. Young individuals make saving decisions and face a probability that the investment may be early liquidated. As there are many individuals with demand for liquidity and different religious concerns, there is a room for Islamic banks to exist and meet those demands.

Our results show that the Islamic banking system would arise when a portion of the population has after-life concerns. The Islamic banking system causes an increase in the aggregate level of savings and shifts the savings to a more productive channel. This in turn increases the capital stock and hence, social welfare. As a result, introducing an Islamic banking system to an economy with some Muslim population will positively contribute to the development of those countries. We conduct our analysis in two distinct settings: variable savings and non-variable savings. In the variable savings settings we demonstrate that individuals’ after-life preferences impact the level of savings in the economy. In the non-variable savings setting, we are able to show that the presence of an Islamic bank can still benefit the economy even if the level of savings does not vary.

The remainder of the paper is organized as follows. Section 2 provides a literature review. In Section 3 we set up a model with variable savings and lay out the results. In Section 4 we provide a model with non-variable savings and lay out the results. Section 5 concludes. The mathematical details of the solutions in both variable and non-variable savings cases are given in the Appendix.

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4See Khan [1985], Darrat [2002], Yousefi et al. [1997], Hassoune [2002], Yusof and Wilson [2005], Isa and Kaleem [2006], Furqani and Mulyany [2009], Abduh and Chowdhury [2012], and Gosied and Sassi [2012].
2 Literature Review

Our study is related to the three strands of the literature: financial intermediation, behavioral economics, and Islamic finance. Here we provide a brief overview of these strands.

The interaction between the level of financial intermediation and economic growth has gained attention since the early 1990s and hence, a large body of literature has been produced until now. A number of theoretical and empirical studies have looked at the channels where financial intermediation plays a role in stimulating economic growth. Allen [1990] and King and Levine [1993] show that the financial intermediaries undertake the costly process of searching investment possibilities for investors thus they reduce information costs which lead to an improved resource allocation and accelerated growth. Greenwood [1990] explain how financial institutions enable a higher rate of return from asset holdings which in turn increases growth and the development of the financial structure while Kashyap et al. [2002] looks at banks in terms of providing liquidity to meet the needs of individuals. Diamond [1984] shows that the financial intermediaries eliminate the free rider problem since they conduct monitoring for all investors. Demonstrate that the intermediaries improve the governance and reduce the credit rationing which lead to a higher productivity, capital accumulation and growth. De la Fuente and Marin [1996] emphasize that the intermediaries undertake the costly monitoring activities on behalf of investors. This in turn improves credit allocation among competing technology producers and generates a higher economic growth. Saint-Paul [1992], Obstfeld [1994] and Acemoglu and Zilibotti [1997] focus on the risk reducing role played by the financial intermediaries. They state that financial systems mitigate the risks associated with individual projects, firms, industries, regions and countries through diversification. This, in turn increases the amount of savings, generates a better resource allocation to more productive technologies and a higher economic growth. Yet, there are risks that cannot be diversified at a particular point of time such as macroeconomic shocks but can be diversified across generations. Allen and Gale [1997] show that the long-lived intermediaries can facilitate the intergenerational risk sharing by investing with a long-run perspective, for example, offering returns that are relatively low in boom times and high in slack times. Arestis et al. [2001] and Fase and Abma [2003] empirically show that the development of the financial system has a positive effect on economic growth.

There is a fast growing literature that focuses on the motives (besides having more consumption and leisure) that shape individuals’ economic decisions. Falk and Knell [2004] and Kumru and Vesterlund [2010] analyze the effects of social comparison and status, respectively. The former concludes that reference standards increase individuals’ abilities while the latter analyzes the effects of status in a charitable giving environment and demonstrates that status is an important factor determining the level of a public good provision. Andreoni [1990] demonstrates the importance of altruism in the provision level of public goods. A recent empirical study by DellaVigna et al. [2012] show that social pressure is an important determinant of the success of door-to-door fund-raising campaigns. In addition to those motives, individuals’ religiosity in general and their belief in the after-life in particular would be an important determinant of their economic decisions. For instance, Barro and McCleary [2006] look at the
relationship between the level of religiosity and economic development and finds that economic development reduces religious participation and beliefs. Elgin et al. [2013] explain why countries with higher levels of religiosity face higher income inequality. They argue that religion motivates individuals to privately and voluntarily engage in charitable giving and these preferences are reflected in a country’s policy outcome. Religiosity results in lower level of taxes and hence lower levels of spending on both public goods and redistribution. Since measures of income do not fully take into account private transfers, the measured income inequality is higher. Another interesting study by Tao and Yeh [2007] state that a religion promising higher after-life rewards tends to induce more contributions and volunteer frequency. In addition, Shy [2007] analyzes the dynamics behind the demand for secular and religious affiliations by developing a model of religious retention across generations. Conversion patterns are shown to depend on differences in birth rates and different attitudes towards conformism and nonconformism which depend on the teachings and status of that religion in the society. Chang [2005] models the after-life (religious) concerns by assuming individual’s utility function consists of two types of expenditures: spending for the current life and spending for the after-life. After-life spending would include participating religious activities, donating to the needy, and investing in a way that complies with their religious believes. Finally, a recent experimental study reveals that religiosity is an important factor determining individuals’ level of charitable giving (Horioka [2012]).

Although Islamic banks have been developed quite recently, there have been a number of theoretical and empirical studies that investigate the implications of these banks, especially comparing with the conventional interest-based banks in terms of its performance (see Khan [1985], Hassoune [2002], Darrat [2002], Yousefi et al. [1997], Yusof and Wilson [2005], Darrat [2002], and Isa and Kaleem [2006]). There are a few empirical studies that explore the relation between the Islamic banking system and growth. Furqani and Mulyany [2009] examine the dynamic interactions between Islamic banking and growth in Malaysia and show that an increase in GDP causes Islamic banking system to develop but not vice versa. Goaied and Sassi [2012] show that there is no significant positive relationship between Islamic banking and growth. Yet, Abdouh and Chowdhury [2012] investigates the relation between the Islamic banking development and economic growth in Bangladesh and finds a positive and significant relationship. Similarly, Abdouh and Omar [2012] demonstrate a significant bidirectional relationship in both short and long run periods between Islamic banking system and economic growth by using an Indonesian data set.

3 The Model with Variable Savings

3.1 The Economy

Our analysis is based on an overlapping generations model with a financial intermediation. We extend Bencivenga and Smith [1991]’s model by incorporating different types of individuals and banks and compute not only the growth rate but also the steady state values of the economic
aggregates and social welfare. The population consists of two types of individuals: Type 1 individuals are indifferent to the type of bank and they deposit their savings to meet their liquidity needs and their measure is given by \( p_1 \), and type 2 individuals deposit their money into Islamic banks only because of their after-life beliefs and their measure is given by \((1 - p_1)\), where \( p_1 \in [0, 1] \).

All types of individuals live for three periods where time is indexed by \( t \) and population growth rate is constant. The economy consists of young, middle age, old, initial old and initial middle age individuals. The initial old individual is endowed with the initial per capita capital stock \( k_0 \) at \( t = 0 \) while the initial middle age individual is endowed with the initial per capita capital stock \( k_1 \) at \( t = 1 \). There are two types of goods in the economy: a consumption good and a capital good. The consumption good is produced from a capital stock and labor supply. The capital stock is owned by a group of individuals (entrepreneurs). It is assumed that the capital stock depreciates in one period.

A young individual supplies one unit of labor inelastically, receives wage, and makes a consumption-saving decision. At the beginning of the middle age, an individual learns whether he becomes an entrepreneur or not. Old entrepreneurs withdraw all of their savings and use all of the capital stock in the consumption goods production. The level of consumption at time \( t \) is denoted by \( c_t \). Type 1 individual’s utility function can be written as:

\[
    u(c_t, c_{t+1}, c_{t+2} : \phi) = \ln c_t + \ln(c_{t+1} + \phi c_{t+2}),
\]

where \( \phi \) is an individual specific random variable realized at the beginning of \( t + 1 \) and has the following probability distribution:

\[
    \phi = \begin{cases} 
        0 & \text{with probability } 1 - \pi \\
        1 & \text{with probability } \pi,
    \end{cases}
\]

where \( \pi \) takes a value between zero and one. Only individuals with \( \phi = 1 \) become entrepreneurs and operate firms. This formulation of preferences allows demand for liquidity (see for example Diamond and Dybvig [1983]).

Let \( Y_t \equiv \text{output} \); \( L_t \equiv \text{the amount of labor supplied} \); \( k_t \equiv \text{the capital stock per entrepreneur} \); \( \bar{k}_t \equiv \text{the average capital stock per entrepreneur} \); \( w_t \equiv \text{the wage rate} \), and \( A \equiv \text{the level of technology} \). The production function can be written in the form of

\[
    Y_t = AK_t^\delta k_t^\theta L_t^{1-\theta},
\]

where \( \theta \) takes a value between zero and one, and \( \delta = 1 - \theta \) ( \( \delta \) is distinguished from \( 1 - \theta \) notationally as it represents a positive externality in production). Each entrepreneur chooses an employment level to maximize profit:

\[
    \max_{L_t} AK_t^\delta k_t^\theta L_t^{1-\theta} - w_t L_t.
\]

The maximization yields the optimal level of employment for each entrepreneur:
\[
L_t = \left[ \frac{(1 - \theta)A\kappa_t^\delta}{w_t} \right]^{\frac{1}{\theta}} k_t. \tag{5}
\]

Since only a fraction \(\frac{1}{\pi}\) of the old becomes an entrepreneur, equilibrium requires \(L_t = \frac{1}{\pi}\) assuming full employment. Hence the equilibrium wage is given by

\[
w_t = A\kappa_t(1 - \theta)\pi^{\theta}. \tag{6}
\]

The return to capital for entrepreneurs is given by

\[
\theta A\kappa_t^\delta k^\theta L_t^{1-\theta} = \theta A\kappa_t^\delta k^\theta \left[ \frac{(1 - \theta)A\kappa_t^\delta}{w_t} \right]^{\frac{1}{\theta}} k_t \right)^{1-\theta} = \theta A\pi^{\theta-1}k_t = \theta A\psi k_t, \tag{7}
\]

where \(\psi = \pi^{\theta-1}\).

There exists two types of assets in the economy, a liquid but unproductive asset and an illiquid but productive asset. The return from the liquid asset is \(n\) units of consumption goods at either \(t + 1\) or \(t + 2\) while the illiquid asset gives a return of \(R\) units of capital goods at \(t + 2\). However, if the illiquid asset is early liquidated at \(t + 1\), it only gives a return of \(x\) units of consumption goods, where \(x < n\).

Banks receive deposits from individuals and invest them in both liquid and illiquid assets. Let \(q_t\) represent the amount of deposits allocated to illiquid assets and \((1 - q_t)\) represent the amount of deposits allocated to liquid assets to meet individuals’ demand for liquidity. Banks offer a return of \(r_{1t}\) units of consumption good if the withdrawal is made after one period and \(r_{2t}\) units of capital goods plus \(\bar{r}_{2t}\) units of consumption good if the withdrawal is made after two periods. Let \(\alpha_{1t}\) be the fraction of deposits held in liquid assets that will be liquidated after one period and \(\alpha_{2t}\) be the fraction of deposits held in illiquid assets that will be liquidated after one period. Thus, we can write the bank’s first period constraint as follows:

\[
(1 - \pi)r_{1t} = \alpha_{1t}(1 - q_t)n + \alpha_{2t}q_t x
\]

The bank’s second period constraints are given by

\[
\pi r_{2t} = (1 - \alpha_{2t})q_t R \tag{9}
\]

and

\[
\pi \bar{r}_{2t} = (1 - \alpha_{1t})(1 - q_t)n. \tag{10}
\]

Assuming a competitive banking industry with zero profits, banks maximize the utility of a representative depositor by choosing \(r_{1t}, r_{2t}, \bar{r}_{2t}, \alpha_{1t}, \alpha_{2t}\) and \(q_t\). All liquid assets are liquidated after one period and all illiquid assets are held until two periods i.e. \(\alpha_{1t} = 1\) and \(\alpha_{2t} = 0\). This is because early liquidation of illiquid assets could always be improved by increasing the
liquid asset holdings and it is always better to hold illiquid assets for two periods compared to holding liquid assets for two periods. The bank’s constraints are now given by

$$(1 - \pi)r_{1t} = (1 - q_t)n$$

and

$$\pi r_{2t} = q_t R.$$  

3.2 Individuals

3.2.1 Type One

Type 1 individuals do not have preferences towards Islamic banking and deposit their savings into the conventional banking system. When young, they consume a fraction of their wages and deposit the remaining in a bank. A subset of middle aged individuals $(1 - \pi)$ learn that they do not become entrepreneurs. They withdraw their deposits and consume all $r_{1t}$ units of consumption goods they received in return of their one unit deposits. The rest become entrepreneurs. They withdraw their deposits when they become old and receive $r_{2t}$ units of capital goods which is equal to $\theta A \psi k_{t+2}$. Type 1 individuals face the following maximization problem:

$$Max \ln(w_t - d_t) + (1 - \pi) \ln(r_{1t}d_t) + \pi \ln(\theta A \psi r_{2t}d_t),$$

where $k_{t+2} = r_{2t}d_t$. The first order condition results in $d_t = \frac{w_t}{2}$. Because the bank is formed by the young at $t$ and maximizes the expected utility of a representative depositor, we can rewrite the bank’s utility maximization problem as follows:

$$Max \ln(\frac{wu}{2}) + (1 - \pi) \ln(\frac{(1 - q_t)n w_t}{2(1 - \pi)}) + \pi \ln(\frac{\theta A \psi R q_t w_t}{2\pi}).$$

The maximization problem yields that the fraction of deposits invested in illiquid assets is equal to the amount of deposits withdrawn after two periods i.e. $q_t = \pi$.

3.2.2 Type Two

Type 2 individuals’ after-life concerns shape their economic decisions. We follow Chang [2005] when we model those individuals’ preferences: the utility received from spending for the after-life depends on how type 2 individuals use their savings. More precisely, using savings compatible with the after-life beliefs generate a utility gain while using savings without taking after-life concerns into account generate a utility loss. We assume here that this loss is $-\infty$, and hence type 2 individuals never deposit their savings into the conventional banks. A type 2 individual’s utility function can be written as
where $\gamma$ is the type 2 individual’s level of after-life belief. The first two terms on the right hand side represents utility gains from consuming in the present life while the third term represents the after-life utility gains due to making investments compatible with religious concerns.

**Without Islamic Banks** Type 2 individuals do not use the conventional banking system and hence, they invest directly in either liquid or illiquid assets. As described earlier, the liquid asset produces a return of $n$ at both $t+1$ and $t+2$ while the illiquid asset produces a higher return of $R$ at $t+2$. If the individuals liquidate their assets early, they receive a return $x$ which is lower than the return $n$. Type 2 individuals in this setting need to balance the allocation of the two assets according to their liquidity needs. Let $\tilde{q}_t$ be the fraction of the wage allocated to the illiquid asset and $\tilde{z}_t$ be the fraction of the wage allocated to the liquid asset. Type 2 individuals choose $\tilde{q}_t$ and $\tilde{z}_t$ to maximize their utility functions:

$$\max_{\tilde{q}_t, \tilde{z}_t} \ln[w_t (1 - \tilde{q}_t - \tilde{z}_t)] + (1 - \pi) \ln[w_t (x \tilde{q}_t + n \tilde{z}_t)] + \pi \ln[w_t (\theta A \psi R \tilde{q}_t + n \tilde{z}_t)]$$

(14)

The first order conditions return the following equations:

$$\frac{\tilde{q}_t}{\tilde{z}_t} = \frac{n^2 - (1 - \pi) x n - n \pi \theta A \psi R}{x \theta A \psi R - (1 - \pi) n \theta A \psi R - \pi n x}$$

(15)

and

$$\frac{1}{1 - \tilde{q}_t - \tilde{z}_t} = \frac{(1 - \pi) x}{x \tilde{q}_t + n \tilde{z}_t} + \frac{\pi \theta A \psi R}{\theta A \psi R \tilde{q}_t + n \tilde{z}_t}.$$  

(16)

The first equation above gives the ratio of the illiquid asset’s share of wage income to the liquid asset’s share of wage income. The second equation is the ratio of wage income to consumption’s share of wage income. Both ratios take a value between zero and one. Since type 2 individuals do not deposit their savings into a conventional bank, they need to allocate their savings between the liquid and illiquid assets. If the level of consumption when young is high, the less resources will be available to invest in both assets in later periods. This in turn leads to lower levels of consumption during middle and old ages. Yet, if individuals invest relatively more in illiquid assets, they would end up having less resources to meet their liquidity needs. Therefore, when individuals directly invest, they invest less in illiquid assets to meet their future liquidity needs.

**With Islamic Banks** When Islamic banks are present, type 2 individuals deposit their savings $d_t$ into the banking system. In each period, taking $w_t$ and the the returns as given, the young choose the deposit amount $d_t$:

$$\max_{d_t} \ln(w_t - d_t) + (1 - \pi) \ln(r_{1t} d_t) + \pi \ln(\theta A \psi r_{2t} d_t) + \gamma \ln d_t.$$  

(17)
At the optimum, \( d_t = \frac{(1+\gamma) w_t}{(2+\gamma)} \), which is larger than that of the conventional bank setting, \( d_t = \frac{w_t}{2} \). The intuition is simple: Type 2 individuals receive additional utility gain by depositing their savings into the Islamic Banking system. This, in turn, result a higher level of deposits. By substituting \( d_t \) and the constraints, we can rewrite the Islamic bank’s maximization problem as follows:

\[
Max \ln\left(\frac{w_t}{2+\gamma}\right) + (1-\pi) \ln\left(\frac{(1-q_t)n(1+\gamma)w_t}{(2+\gamma)(1-\pi)}\right) + \pi \ln\left(\frac{\theta A\psi Rq_t(1+\gamma)w_t}{(2+\gamma)\pi}\right) + \gamma \ln\left(\frac{(1+\gamma)w_t}{2+\gamma}\right) 
\]

(18)

The Islamic bank’s maximization problem yields that \( q_t = \pi \). This implies that the Islamic and conventional banks use the same rule of thumb when allocating deposits towards illiquid assets. If the Islamic bank provides a lower return than that of the direct investment, type 2 individuals would prefer the direct investment option instead. More precisely, type 2 individuals continue to deposit into the Islamic bank if the level of utility derived from the deposit exceeds that of the direct investment:

\[
\ln\left(\frac{w_t}{2+\gamma}\right) + (1-\pi) \ln\left(\frac{(1-q_t)n(1+\gamma)w_t}{(2+\gamma)(1-\pi)}\right) + \pi \ln\left(\frac{\theta A\psi Rq_t(1+\gamma)w_t}{(2+\gamma)\pi}\right) + \gamma \ln\left(\frac{(1+\gamma)w_t}{2+\gamma}\right) > \ln[w_t(1-q_t-\bar{\pi})] + (1-\pi) \ln[w_t(xq_t + n\bar{\pi})] + \pi \ln[w_t(\theta A\psi Rq_t + n\bar{\pi})].
\]

(19)

### 3.3 Growth Rate

In this section, we calculate the equilibrium growth rates in two different scenarios. In the first scenario, only conventional banks exist in the economy. In the second scenario, on the other hand, both conventional and Islamic banks exist at the same time in the economy.

#### (i) Only Conventional Banks

Type 1 individuals deposit their savings into a conventional bank. Since there is no Islamic banking system that accommodates type 2 individuals’ preferences, these individuals prefer the direct investment option. Thus, the aggregate capital stock is given by

\[
\bar{k}_{t+2} = p_1(r_2 d_t) + (1-p_1)(R\bar{q}_t w_t).
\]

(20)

Substituting \( d_t = \frac{w_t}{2} \), the above equation yields

\[
\bar{k}_{t+2} = p_1\left(\frac{Rw_t}{2}\right) + (1-p_1)(R\bar{q}_t w_t).
\]

(21)

Replacing the value of \( w_t \) the above equation becomes

\[
\bar{k}_{t+2} = p_1\left(\frac{R\bar{k}_t(1-\theta)A\pi^\theta}{2}\right) + (1-p_1)(R\bar{q}_t \bar{k}_t(1-\theta)A\pi^\theta).
\]

(22)
Since the capital formation takes two periods, the equilibrium growth rate is calculated by dividing $\bar{k}_{t+2}$ with $\bar{k}_t$:

$$\frac{\bar{k}_{t+2}}{\bar{k}_t} = p_1(R(1 - \theta)A\pi^\theta) + (1 - p_1)(R\bar{q}_t(1 - \theta)A\pi^\theta). \quad (23)$$

The economy’s growth rate is positively correlated with the labor’s share in output $(1 - \theta)$, the illiquid asset’s return after two periods $(R)$, the probability that both individuals will not withdraw until two periods $(\pi)$, and the fraction of type 2 individual’s savings invested in illiquid assets $(\bar{q}_t)$.

(ii) Dual Banking System

As in above, type 1 individuals continue to deposit their savings into a conventional bank. Yet, type 2 individuals now deposit their savings into an Islamic bank instead of going with a direct investment option. The aggregate capital stock is given by

$$\bar{k}_{t+2} = p_1(r_2d_t) + (1 - p_1)(r_2d_t) \quad (24)$$

Substituting $d_t = \frac{w_t}{2}$ and $d_t = \frac{(1 + \gamma)w_t}{2 + \gamma}$ for the conventional and the Islamic banks respectively yields

$$\bar{k}_{t+2} = p_1\left(\frac{Rw_t}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)w_t}{2 + \gamma}. \quad (25)$$

Replacing the value of $w_t$, the equation above becomes

$$\bar{k}_{t+2} = p_1\left(\frac{R\bar{k}_t(1 - \theta)A\pi^\theta}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)\bar{k}_t(1 - \theta)A\pi^\theta}{2 + \gamma} \quad (26)$$

The equilibrium growth rate is given by dividing $\bar{k}_{t+2}$ with $\bar{k}_t$

$$\frac{\bar{k}_{t+2}}{\bar{k}_t} = p_1\left(\frac{R(1 - \theta)A\pi^\theta}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)(1 - \theta)A\pi^\theta}{2 + \gamma} \quad (27)$$

The economy’s growth rate in this scenario also depends on type 2 individual’s level of after-life believe $(\gamma)$: the higher the level of the after-life belief, the higher the growth rate is. Thus, introducing an Islamic bank to an economy would generate higher growth if some individuals have after-life concerns.

3.4 Steady States

To find the steady state equilibrium values, we set $\delta = 0$ so that the average capital stock per entrepreneur $\bar{k}_t$ has no external effect on production. Each entrepreneur chooses an employment level to maximize the profit:
This yields the optimal level of employment for each entrepreneur:

\[ l_t = \left[ \frac{(1 - \theta)A}{w_t} \right]^\frac{1}{\theta} k_t. \]  

(29)

Since only a fraction \( \frac{1}{\sigma} \) of the old becomes entrepreneurs, equilibrium requires \( l_t = \frac{1}{\sigma} \) under the full employment assumption. Hence the equilibrium wage is given by

\[ w_t = Ak_t^\theta (1 - \theta)\pi^\theta. \]  

(30)

The return to capital (the entrepreneur’s share of the value of production) is given by

\[ \theta Ak_t^\theta l_t^{1-\theta} = \theta Ak_t \left[ \frac{(1 - \theta)A}{w_t} \right]^\frac{1}{\theta} \]  

(31)

We calculate the steady state values of capital stock and wage rate in two scenarios.

(i) Only Conventional Banks

The capital stock is given by

\[ k_{t+2} = p_1(r_w d_t) + (1 - p_1)(R\tilde{q}_t w_t). \]  

(32)

Substituting time \( t \) savings \( d_t \), and wage \( w_t \) into the above equation and dividing it by the fraction of individuals who do not withdraw until \( t + 2 \), the equilibrium per entrepreneur capital stock will be equal to the following:

\[ p_1 \left( \frac{Rq(k_t^\theta(1 - \theta)A\pi^\theta)}{2\pi} \right) + (1 - p_1)(R\tilde{q}_t(k_t^\theta(1 - \theta)A\pi^\theta))). \]  

(33)

The above equation equals the return type 1 individuals receive from depositing in the conventional bank for two periods plus the return type 2 individuals receive for investing in the illiquid asset for two periods. In the steady state equilibrium, \( k_t = k_{t+2} = k^*_t \). By using the conventional bank’s optimal allocation rule for deposits, \( q = \pi \), the steady state capital stock can be written as:

\[ k^*_t = \left[ p_1 \left( \frac{R(1 - \theta)A\pi^\theta}{2} \right) + (1 - p_1)(R\tilde{q}(1 - \theta)A\pi^\theta) \right]^\frac{1}{1-\theta}. \]  

(34)

Hence, the steady state equilibrium wage is given by

\[ w^*_1 = (1 - \theta)A\pi^\theta \left[ p_1 \left( \frac{R(1 - \theta)A\pi^\theta}{2} + (1 - p_1)(R\tilde{q}(1 - \theta)A\pi^\theta) \right)^\frac{1}{1-\theta} \right]. \]  

(35)
(ii) Dual banking system

The capital stock is given by

\[ k_{t+2} = p_1(r_{2t}d_t) + (1 - p_1)(r_{2t}d_t) \]  

(36)

Substituting time \( t \) savings \( d_t \), and wage \( w_t \) into the above equation and dividing it by the fraction of individuals who did not withdraw until \( t + 2 \), the equilibrium per entrepreneur capital stock will be equal to the following:

\[ p_1(Rq(k^\theta(1 - \theta)A^\theta) + (1 - p_1)(1 + \gamma)\frac{Rq(k^\theta(1 - \theta)A^\theta)}{(2 + \gamma)}) \]  

(37)

The above equation equals the return type 1 individuals receive from depositing in the conventional bank for two periods plus the return type 2 individuals receive from depositing in the Islamic bank for two periods. In the steady state equilibrium, \( k_t = k_{t+2} = k^*_2 \). By using both the conventional bank and Islamic bank’s optimal allocation rules for deposits, \( q = \pi \), the steady state capital stock can be written as:

\[ k^*_2 = \left[ p_1\left(\frac{R(1 - \theta)A^\theta}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)(1 - \theta)A^\theta}{(2 + \gamma)}\right]^{\frac{1}{1 - \theta}} \]  

(38)

Hence, the steady state equilibrium wage is written as follows:

\[ w^*_2 = (1 - \theta)A^\theta\left[ p_1\left(\frac{R(1 - \theta)A^\theta}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)(1 - \theta)A^\theta}{(2 + \gamma)}\right]^{\frac{\theta}{1 - \theta}} \]  

(39)

A comparison of scenarios leads that \( k^*_2 > k^*_1 \) if

\[ \left[ p_1\left(\frac{R(1 - \theta)A^\theta}{2}\right) + (1 - p_1)\frac{R(1 + \gamma)(1 - \theta)A^\theta}{(2 + \gamma)}\right]^{\frac{1}{1 - \theta}} > \left[ p_1\left(\frac{R(1 - \theta)A^\theta}{2}\right) + (1 - p_1)(R\tilde{q}(1 - \theta)A^\theta)\right]^{\frac{1}{1 - \theta}} \]  

(40)

or

\[ \frac{(1 + \gamma)}{(2 + \gamma)} > \tilde{q}. \]  

(41)

This implies that the presence of Islamic banks increases the steady state capital stock if the savings deposited in the Islamic banks are larger than those directly invested in illiquid assets.

3.5 Numerical Results

In the earlier section, we draw intuitive conclusions regarding the effects of introducing Islamic banking into an environment where individuals have after-life preferences. In this section we
compare steady state values of the economic aggregates and welfare in two scenarios as above: (i) only conventional banks and (ii) a dual banking system.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$p_1$</th>
<th>$A$</th>
<th>$\theta$</th>
<th>$\pi$</th>
<th>$\gamma$</th>
<th>$x$</th>
<th>$n$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.5</td>
<td>5</td>
<td>0.3</td>
<td>0.25</td>
<td>0.1</td>
<td>1</td>
<td>1.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

The parameters values used in our experimental exercises are given in Table 1. First, we set the population share of type 2 individuals at 50% and then vary this value. The technology parameter is set freely at 5. The capital share of output is equal to 0.3 and the probability that individuals will not withdraw until two periods is equal to 0.25. The strength of the after-life belief parameter is set to 0.1. The returns of illiquid and liquid assets are set according to the condition $x < n < R$. The return from the liquid asset in each period is set at 1.5 units of consumption goods while the return from the illiquid asset after two periods is set at 4.5 units of capital goods. If the illiquid asset is early liquidated, its scrap value after one period will be equal to 1 unit of consumption good.

<table>
<thead>
<tr>
<th>Scenario One</th>
<th>Scenario Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^*$</td>
<td>0.512</td>
</tr>
<tr>
<td>$w^*$</td>
<td>1.889</td>
</tr>
</tbody>
</table>

Table 2: Steady State Values of Capital Stock and Wage Rate in Two Scenarios

Table 2 provides the steady state values of capital stock and wage rate in two scenarios. The steady state values of both capital stock and wage rate are higher in the second scenario, where there is an Islamic bank that caters the needs of the religious people. In the first scenario, half of the population do not benefit from the banking system since it does not comply with their religious beliefs and go with direct investment options. Since the direct investment option generates a lower return, religious individuals save at lower rate in this scenario. This, in turn generates a lower level of capital stock and wage rate in the equilibrium.

<table>
<thead>
<tr>
<th>Type One</th>
<th>Type Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*$</td>
<td>0.944</td>
</tr>
<tr>
<td>$c_1^*$</td>
<td>0.944</td>
</tr>
<tr>
<td>$c_2^*$</td>
<td>1.417</td>
</tr>
<tr>
<td>$c_3^*$</td>
<td>3.365</td>
</tr>
<tr>
<td>welfare</td>
<td>0.507</td>
</tr>
<tr>
<td>total welfare</td>
<td>0.404</td>
</tr>
</tbody>
</table>

Table 3: Steady State Savings, Consumption, and Welfare of Each Type of Individuals in Scenario One

Table 3 presents the steady state values of savings, consumption, and aggregate welfare for
both types in Scenario 1. Since type 2 individuals have after-life preferences, they do not deposit their savings in conventional banks but invest in both liquid and illiquid assets directly. The conventional banks can manage the depositors' liquidity needs better due to their economies of scale. Yet, type 2 individuals can not benefit from the same economies of scale and end up with investing less in illiquid assets. As a result, type 2 individuals receive relatively lower returns during middle and old ages. This in turn implies relatively lower levels of consumption in middle and old ages and a lower welfare.

<table>
<thead>
<tr>
<th></th>
<th>Type One</th>
<th>Type Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^* )</td>
<td>1.186</td>
<td>1.242</td>
</tr>
<tr>
<td>( c_1^* )</td>
<td>1.186</td>
<td>1.129</td>
</tr>
<tr>
<td>( c_2^* )</td>
<td>1.778</td>
<td>1.863</td>
</tr>
<tr>
<td>( c_3^* )</td>
<td>4.224</td>
<td>4.425</td>
</tr>
<tr>
<td>welfare</td>
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<td>0.982</td>
</tr>
<tr>
<td>total welfare</td>
<td></td>
<td>0.972</td>
</tr>
</tbody>
</table>

Table 4: Steady State Savings, Consumption, and Welfare of Each Type of Individual in Scenario Two

Table 4 provides the steady state values of savings, consumption, and aggregate welfare for both types in Scenario 2, where both conventional and Islamic banks are present. Religious individuals in this scenario increase their savings because they can get additional utility by depositing their savings in a banking system which is compatible with their beliefs. Thus, type 2 individuals save relatively more and consume relatively less in the first period. Higher savings in the fist period generate relatively higher consumption levels for second and third periods and relatively higher welfare (both in comparison to welfare and consumption levels of type 1 individuals in Scenario 2 and their own consumption and welfare levels in Scenario 1). Since, the overall levels of capital stock and wage rate are higher in Scenario 2 as shown in Table 2, aggregate levels of welfare are higher as well.

Figure 1. Proportion of Type 2 Individuals and Total Welfare in Scenario 1
Figure 1 shows the impact of a change in the proportion of type 2 individuals on total welfare in Scenario 1. An increase in the number of type 2 individuals means that more savings are left out of the banking system and directly invested in liquid assets. This results in lower levels of capital stock, wage rate, consumption, and total welfare.

Figure 2. Proportion of Type 2 Individuals and Total Welfare in Scenario Two

Figure 2 shows the impact of a change in the proportion of type 2 individuals on total welfare in Scenario 2. An increase in the number of type 2 individuals increases total welfare. Intuition is as follows: In the Scenario 2, type 2 individuals not only save more but also deposit their savings into the banking system. This in turn, generates higher steady state values for the capital stock, wage rate, and consumption leading to a higher total steady state welfare.

4 The Model with Non-variable Savings

Here, we assume that the entire income is saved. Type 1 individual’s utility function in this section can be written as follows:

\[ u(c_t, c_{t+1}, c_{t+2} : \phi) = \ln(c_{t+1} + \phi c_{t+2}), \]  

where \( \phi \) is an individual specific random variable realized at the beginning of \( t + 1 \). It has the following probability distribution:

\[ \phi = \begin{cases} 
0 & \text{with probability } 1 - \pi \\
1 & \text{with probability } \pi, 
\end{cases} \]  

where \( \pi \) takes a value between zero and one. Only individuals with \( \phi = 1 \) become entrepreneurs and operate firms. Because all income is saved, we assume that individuals do not value consumption when young. All other features of the economy are the same as in Section 3.
4.1 Individuals

4.1.1 Type One

Type 1 individuals face the following maximization problem:

\[
\max_{q_t} (1 - \pi) \ln(\frac{(1 - q_t)nw_t}{(1 - \pi)}) + \pi \ln(\frac{\theta A\psi Rq_tw_t}{\pi})
\]  

(44)

As in Section 3, the maximization problem yields that the fraction of deposits invested in illiquid assets is equal to the amount of deposits withdrawn after two periods i.e. \(q_t = \pi\).

4.1.2 Type Two

Type 2 individual’s utility function can be written as follows:

\[
u(c_{t+1}, c_{t+2}, w_t : \phi, \gamma) = \ln(c_{t+1} + \phi c_{t+2}) + \gamma \ln w_t.
\]  

(45)

Without Islamic Banks Type 2 individuals face a similar maximization problem as in Section 3:

\[
\max_{\tilde{q}_t} (1 - \pi) \ln[(x\tilde{q}_t + n(1 - \tilde{q}_t))w_t] + \pi \ln[(\theta A\psi R\tilde{q}_t + n(1 - \tilde{q}_t))w_t].
\]  

(46)

By taking the first order condition, we find the optimal fraction of the wage allocated to the illiquid asset,

\[
\tilde{q}_t = \frac{n^2 - nx + nX\pi - n\pi\theta A\psi R}{n^2 - nx + x\theta A\psi R - n\theta A\psi R^2}.
\]  

(47)

Since the entire income is saved, type 2 individuals have more resources to allocate towards illiquid assets compared to the variable saving case.

With Islamic Banks The Islamic bank faces the following maximization problem:

\[
\max_{q_t} (1 - \pi) \ln\left(\frac{(1 - q_t)nw_t}{(1 - \pi)}\right) + \pi \ln\left(\frac{\theta A\psi Rq_tw_t}{\pi}\right) + \gamma \ln[w_t]
\]  

(48)

As in above and Section 3, the Islamic banks follow the same rule of thumb and set \(q_t = \pi\). As in Section 3, type 2 individuals continue to deposit their savings in the Islamic Banks if the following condition holds:

\[
(1 - \pi) \ln\left(\frac{(1 - q_t)nw_t}{(1 - \pi)}\right) + \pi \ln\left(\frac{\theta A\psi Rq_tw_t}{\pi}\right) + \gamma \ln[w_t] > (1 - \pi) \ln[(x\tilde{q}_t + n(1 - \tilde{q}_t))w_t] + \pi \ln[(\theta A\psi R\tilde{q}_t + n(1 - \tilde{q}_t))w_t].
\]  

(49)
4.2 The Economy’s Growth Rate

As in Section 3, we calculate the equilibrium growth rate in two scenarios: (i) only conventional banks and (ii) a dual banking system.

(i) Only Conventional Banks

The economy’s capital stock is equal to the sum of savings:

\[ k_{t+2} = p_1(r_2d_t) + (1 - p_1)(R\bar{q}_t w_t). \]  

(50)

Since savings are non-variable, type 1 individuals save their entire income i.e. \( d_t = w_t \) which implies

\[ k_{t+2} = p_1(r_2 w_t) + (1 - p_1)(R\bar{q}_t w_t). \]  

(51)

Replacing the value of \( w_t \) it becomes

\[ k_{t+2} = p_1(R\bar{k}_t(1 - \theta)A\pi^\theta) + (1 - p_1)(R\bar{q}_t k_t(1 - \theta)A\pi^\theta). \]  

(52)

As capital formation takes two periods, the equilibrium growth rate is defined by dividing \( \bar{k}_{t+2} \) with \( \bar{k}_t \)

\[ \frac{\bar{k}_{t+2}}{\bar{k}_t} = p_1(R(1 - \theta)A\pi^\theta) + (1 - p_1)(R\bar{q}_t(1 - \theta)A\pi^\theta). \]  

(53)

The economic growth rate increases with an increase in the labor’s share in output \( (1 - \theta) \), the illiquid asset’s return after two periods \( (R) \), the probability that both individuals do not withdraw until two periods \( (\pi) \), and the fraction of type 2 individual’s savings invested in illiquid assets \( (\bar{q}_t) \).

(ii) Dual Banking System

In this scenario, type 1 individuals deposit their savings into a conventional bank while type 2 individuals deposit their savings into an Islamic bank. The capital stock is given by

\[ k_{t+2} = p_1(r_2d_t) + (1 - p_1)(r_2d_t). \]  

(54)

Both type of individuals save their entire income. Substituting \( d_t = w_t \) implies that

\[ k_{t+2} = p_1(Rw_t) + (1 - p_1)(Rw_t). \]  

(55)

Replacing the value of \( w_t \) it becomes

\[ k_{t+2} = p_1(R\bar{k}_t(1 - \theta)A\pi^\theta) + (1 - p_1)(R\bar{q}_t(1 - \theta)A\pi^\theta). \]  

(56)
The equilibrium growth rate is given by dividing $\overline{k}_{t+2}$ with $\overline{k}_t$:

$$\frac{\overline{k}_{t+2}}{\overline{k}_t} = p_1(R(1-\theta)A\pi^\theta) + (1-p_1)(R(1-\theta)A\pi^\theta).$$

The growth rate in the second scenario is larger than that of the first scenario if

$$p_1(R(1-\theta)A\pi^\theta) + (1-p_1)(R(1-\theta)A\pi^\theta) > p_1(R(1-\theta)A\pi^\theta) + (1-p_1)(R\tilde{q}_t(1-\theta)A\pi^\theta)$$

or

$$1 > \tilde{q}_t.$$

This means that introducing Islamic banking increases the growth rate unless type 2 individuals invest all of savings into illiquid assets in the absence of the Islamic banking.

### 4.3 Steady State Values

We follow the same methodology as in Section 3 to calculate steady state values.

#### (i) Only Conventional Banks

The capital stock is given by

$$k_{t+2} = p_1(r_2 d_t) + (1-p_1)(R\tilde{q}_t w_t).$$

Since the entire income is saved, $d_t = w_t$. To calculate the equilibrium per entrepreneur capital stock, substitute $d_t$ with the equilibrium value of $w_t$ in the above equation and divide it by the optimal fraction of individuals who does not withdraw until $t+2$:

$$p_1\left(\frac{R\tilde{q}(k_t^\theta(1-\theta)A\pi^\theta)}{\pi}\right) + (1-p_1)(R\tilde{q}_t k^\theta_t(1-\theta)A\pi^\theta).$$

As in Section 3, the above equation equals the return type 1 individuals receive from depositing in the conventional bank for two periods plus the return type 2 individuals receive for investing in the illiquid asset for two periods. In the steady state equilibrium, $k_t = k_{t+2} = k^*_3$. Using the conventional bank’s rule of thumb for the optimal allocation of deposits ($q = \pi$), the steady state capital stock can be written as

$$k^*_3 = \left[p_1(R(1-\theta)A\pi^\theta) + (1-p_1)(R\tilde{q}(1-\theta)A\pi^\theta)\right]^{1/\theta}.$$ 

Hence the steady state equilibrium wage is given by

$$w^*_3 = (1-\theta)A\pi^\theta[p_1(R(1-\theta)A\pi^\theta) + (1-p_1)(R\tilde{q}(1-\theta)A\pi^\theta)]^{\frac{\theta}{1-\theta}}.$$ 

Compared to the variable savings case, the steady state capital stock is now higher since both type of individuals save their entire income. As a result, the steady state equilibrium
wage is also higher.

(ii) Dual Banking System

The capital stock is given by

\[ k_{t+2} = p_1(r_2 d_t) + (1 - p_1)(r_2 d_t) \]  

(63)

To calculate the equilibrium per entrepreneur capital stock, substitute \( d_t \) with the equilibrium value of \( w_t \) in the above equation and divide it by the optimal fraction of individuals who does not withdraw until \( t + 2 \):

\[ p_1 \left( \frac{R q(k_t^0(1 - \theta)A \pi^0)}{\pi} \right) + (1 - p_1) \left( \frac{R q(k_t^0(1 - \theta)A \pi^0)}{\pi} \right) \]  

(64)

The above equation equals the return type 1 individuals receive from depositing in the conventional bank for two periods plus the return type 2 individuals receive from depositing in the Islamic bank for two periods. In the steady state equilibrium, \( k_t = k_{t+2} = k_4^* \). Using both conventional and Islamic banks’ optimal deposit allocation rule (\( q = \pi \)), the steady state capital stock can be written as

\[ k_4^* = [p_1(R(1 - \theta)A \pi^0) + (1 - p_1)(R(1 - \theta)A \pi^0)]^{\frac{1}{1-\pi}}. \]  

(65)

This leads to a steady state equilibrium wage rate as follows:

\[ w_4^* = (1 - \theta)A \pi^0[p_1(R(1 - \theta)A \pi^0) + (1 - p_1)(R(1 - \theta)A \pi^0)]^{\frac{\pi}{1-\pi}}. \]  

(66)

The steady state capital stock of Scenario 2 is larger than that of Scenario 1 if

\[ [p_1(R(1 - \theta)A \pi^0) + (1 - p_1)(R(1 - \theta)A \pi^0)]^{\frac{1}{1-\pi}} \]

(67)

\[ > [p_1(R(1 - \theta)A \pi^0) + (1 - p_1)(R \tilde{q}(1 - \theta)A \pi^0)]^{\frac{1}{1-\pi}} \]

or

\[ 1 > \tilde{q} \]

(68)

Having an Islamic bank in the economy increases the equilibrium capital stock if type 2 individuals do not invest all of their savings in the illiquid asset directly.

4.4 Numerical Results

Similar to our analysis in Section 3, in this section we compare steady state values of the economic aggregates and welfare in two scenarios: (i) only conventional banks and (ii) a dual
banking system. Parameter values are given in Table 1. Our results here are in the same directions as those in Section 3.

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^*$</td>
<td>2.020</td>
</tr>
<tr>
<td>$w^*$</td>
<td>2.852</td>
</tr>
</tbody>
</table>

Table 5: Steady State Values of Capital Stock and Wage Rate in Two Scenarios

Table 5 provides the steady state values of capital stock and wage rate in two scenarios. The steady state values of both capital stock and wage rate are higher in the second scenario where there is an Islamic bank that caters the needs of the religious people. In the first scenario, half of the population do not benefit from the banking system since it does not comply with their religious beliefs and go with direct investment options. Since, the direct investment option generates lower return, religious individuals save at a lower rate in this scenario. This, in turn generates a lower level of capital stock and wage rate in the equilibrium. Notice that both the steady state capital stock and wage rate is higher than in Section 3 since we assume that the individuals save their entire incomes.

<table>
<thead>
<tr>
<th>Type One</th>
<th>Type Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*$</td>
<td>2.852</td>
</tr>
<tr>
<td>$c^*_1$</td>
<td>–</td>
</tr>
<tr>
<td>$c^*_2$</td>
<td>4.277</td>
</tr>
<tr>
<td>$c^*_3$</td>
<td>10.159</td>
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<tr>
<td>welfare</td>
<td>1.670</td>
</tr>
<tr>
<td>total welfare</td>
<td>1.507</td>
</tr>
</tbody>
</table>

Table 6: Steady State Savings, Consumption, and Welfare of Each Type of Individuals in Scenario One

Table 6 presents the steady state values of savings, consumption, and aggregate welfare for both types in Scenario 1. Since type 2 individuals have after-life preferences, they do not deposit their savings in conventional banks but invest in both liquid and illiquid assets directly. The conventional banks can manage the depositors’ liquidity needs better due to their economies of scale. Yet, type 2 individuals cannot benefit from the same economies of scale and end up with investing less in illiquid assets. As a result, type 2 individuals receive relatively lower returns during middle and old ages. This in turn implies relatively lower levels of consumption in middle and old ages and a lower welfare. As both individuals save their entire incomes, after-life preferences do not have an impact on type 2 individual’s level of savings in this section. This in turn reduces the differences between type 1 and 2 individuals’ consumption and welfare levels.

Table 7 shows that when there is an Islamic bank in the economy, type 2 individuals deposit their money into the banking system leading to a faster capital accumulation. The
Table 7: Steady State Savings, Consumption, and Welfare of Each Type of Individuals in Scenario Two

<table>
<thead>
<tr>
<th></th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*$</td>
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<tr>
<td>$c_1^*$</td>
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<tr>
<td>$c_2^*$</td>
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<tr>
<td>$c_3^*$</td>
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<td>total welfare</td>
<td>1.887</td>
<td></td>
</tr>
</tbody>
</table>

Coexistence of both banks increases the steady state values of all economic aggregates and makes everyone better off. As now both individuals deposit their money into the banking system, their consumption levels are the same but type 2 individuals have a higher welfare due to their after-life concerns.

Notice that in this section, we assume that individuals do not value consumption when young and they save their entire income. As a result, the individuals’ after-life concerns do not increase their level of savings when the Islamic bank is present. In this environment, the existence of Islamic bank has only one role which is channelling religious individuals’ savings into the banking system. Even with this limited role, the existence of the Islamic banking system is welfare improving.

Figure 3. Proportion of Type 2 Individuals and Total Welfare in Scenario One

Figure 3 shows the impact of a change in the proportion of type 2 individuals on total welfare in Scenario 1. An increase in the number of type 2 individuals means that more savings are left out of the banking system and directly invested in liquid assets. This results in lower levels of capital stock, wage rate, consumption, and total welfare. Compared to Figure 1, this figure is more flat. The intuition is as follows: In Scenario 1 of Section 3, increasing the number of type 2 individuals causes total welfare to decrease due to two sources. First, type 2
individuals save less when directly investing since they do not get additional utility from their after-life preferences like depositing in an Islamic bank. Second, by directly investing, more savings are left out of the banking system which can manage the depositors’ liquidity needs better due to their economies of scale. Type 2 individuals hence receive a lower return during middle and old ages. This leads to lower consumption in both periods, and lower welfare. By assuming that individuals save their entire income, the decrease in total welfare when increasing the number of type 2 individuals is only due to the second source. As a result, total welfare decreases at a lower rate.

Figure 4 shows the impact of a change in the proportion of type 2 individuals on total welfare in Scenario 2. An increase in the number of type 2 individuals increases total welfare. Intuition is as follows: In the Scenario 2, type 2 individuals deposit their savings into the banking system. This in turn, generates higher steady state values for the capital stock, wage rate, and consumption leading to a higher total steady state welfare. Compared to Figure 2, total welfare increases less when we increase the proportion of type 2 individuals as the after-life concern does not play a role in increasing their savings.

5 Conclusion

A sizeable portion of the Muslim population is unwilling to deposit their savings into conventional banks that operate not according to their religious principles. This in turn affects the level of financial development (and economic growth in general) in many Muslim countries. Alternative banking systems (so called Islamic banking systems) aim to provide financial services complying with Islamic principles to serve the financial needs of Muslims who abstain from using the conventional banking system because of their religious concerns.

This paper aims to answer the following two questions: First, under what conditions an Islamic banking system may arise? Second, what are the growth and welfare implications of the
Islamic banking system? We conduct our analysis in two distinct settings: variable savings and non-variable savings. In the variable savings settings we demonstrate that individuals’ after-life preferences impact the level of savings in the economy. In the non-variable savings setting, we are able to show that the presence of an Islamic bank can still benefit the economy even if the level of savings does not vary. To conclude, the Islamic banking system would arise when a portion of the population has after-life concerns. It causes an increase in the aggregate level of savings and shifts the savings to a more productive channel. This in turn increases the capital stock and hence, social welfare. Introducing an Islamic banking system to an economy with a sizeable Muslim population will positively contribute to the development of those countries.

Islamic banks operate under profit sharing since Islam strictly prohibits paying and receiving interest. The bank offers deposit accounts with different risk/return profiles and shares in the profit with the depositors. Hence an Islamic bank’s deposit acts similar to a mutual fund where some periods the return is high and some periods the return is low. An important point is that due to this nature, guaranteeing the rate of return on deposits will be necessary to avoid type 2 individuals from withdrawing their savings. In other words, the expected rate of return that Islamic banks offer should not be less than directly investing in the liquid and illiquid asset. The bank could create reserve funds by deducting some profit from good performing periods to compensate depositors in bad performing periods. This in turn smooths the returns the bank offers to depositors. However, if the bank cannot guarantee its rate of return, it could face such withdrawal risks. The impact of this risk on the economy is an interesting topic we hope to address in future research.

Appendix

The details of the solutions to both variable and non-variable savings cases are given here.

I. The Model with Variable Savings

A. Type One Individuals

Type 1 individuals are neutral to the type of banks and will deposit their savings into the banking system. They maximize the following utility function:

$$Max \ln(w_t - d_t) + (1 - \pi) \ln(r_{1t}d_t) + \pi \ln(\theta A\psi r_{2t}d_t).$$

Taking the first order condition with respect to $d_t$ results in

$$-\frac{1}{w_t - d_t} + \frac{(1 - \pi)r_{1t}}{r_{1t}d_t} + \frac{\pi \theta A\psi r_{2t}}{\theta A\psi r_{2t}d_t} = 0$$

$$-\frac{1}{w_t - d_t} + \frac{(1 - \pi)}{d_t} + \frac{\pi}{d_t} = 0$$
\[
\frac{1}{d_t} = \frac{1}{w_t - d_t}
\]

\[2d_t = w_t\]

\[d_t = \frac{w_t}{2} .\]

Substituting \(d_t\) back into the bank’s utility maximization problem along with the bank’s constraint implies the following maximization problem:

\[
\max_{q_t} \ln(\frac{w_t}{2}) + (1 - \pi) \ln(\frac{w_t}{2(1 - \pi)} + \pi \ln(\frac{\theta A\psi R q_t w_t}{2\pi})
\]

Taking the first order condition with respect to \(q_t\) results in

\[-(1 - \pi) \frac{2(1 - \pi)}{(1 - q_t)w_t} + \pi \frac{2\pi}{\theta A\psi R q_t w_t} = 0\]

\[-\frac{(1 - \pi)}{(1 - q_t)} + \frac{\pi}{q_t} = 0\]

\[\frac{\pi}{q_t} = \frac{(1 - \pi)}{(1 - q_t)}\]

\[q_t = \pi.\]

B. **Type Two Individuals**

**a. Without Islamic Banks** We assume that type 2 individuals do not use the banking system if it is not Islamic. Hence, these individuals directly invest in either liquid or illiquid assets. Type 2 individuals choose \(\hat{q}_t\) and \(\hat{z}_t\) in order to maximize their utility according to the following:

\[
\max_{\hat{q}_t, \hat{z}_t} \ln[w_t(1 - \hat{q}_t - \hat{z}_t)] + (1 - \pi) \ln[w_t(x\hat{q}_t + n\hat{z}_t)] + \pi \ln[w_t(\theta A\psi R\hat{q}_t + n\hat{z}_t)]
\]

Taking the first order condition with respect to \(\hat{q}_t\) results in

\[-\frac{w_t}{w_t(1 - \hat{q}_t - \hat{z}_t)} + \frac{(1 - \pi)x w_t}{w_t(x\hat{q}_t + n\hat{z}_t)} + \frac{\pi \theta A\psi R w_t}{w_t(\theta A\psi R\hat{q}_t + n\hat{z}_t)} = 0,\]

which implies that

\[\frac{1}{(1 - \hat{q}_t - \hat{z}_t)} = \frac{(1 - \pi)x}{x\hat{q}_t + n\hat{z}_t} + \frac{\pi \theta A\psi R}{(\theta A\psi R\hat{q}_t + n\hat{z}_t)}.\]

Taking the first order condition with respect to \(\hat{z}_t\) results in
\[-\frac{w_t}{w_t(1-\bar{q}_t - \bar{z}_t)} + \frac{(1-\pi)n w_t}{w_t(x\bar{q}_t + n\bar{z}_t)} + \frac{\pi n w_t}{w_t(\theta A\psi R\bar{q}_t + n\bar{z}_t)} = 0,\]

which implies that
\[
\frac{1}{(1-\bar{q}_t - \bar{z}_t)} = \frac{(1-\pi)n}{(x\bar{q}_t + n\bar{z}_t)} + \frac{\pi n}{(\theta A\psi R\bar{q}_t + n\bar{z}_t)}.\]

By using the result of the first order condition with respect to \(\bar{q}_t\) we can derive the followings:
\[
\frac{(1-\pi)x}{(x\bar{q}_t + n\bar{z}_t)} + \frac{\pi \theta A\psi R}{(\theta A\psi R\bar{q}_t + n\bar{z}_t)} = \frac{(1-\pi)n}{(x\bar{q}_t + n\bar{z}_t)} + \frac{\pi n}{(\theta A\psi R\bar{q}_t + n\bar{z}_t)}
\]
\[
(1-\pi)x(\theta A\psi R\bar{q}_t + n\bar{z}_t) + \pi \theta A\psi R(x\bar{q}_t + n\bar{z}_t)
\]
\[
= (1-\pi)n(\theta A\psi R\bar{q}_t + n\bar{z}_t) + \pi n(x\bar{q}_t + n\bar{z}_t)
\]
\[
(1-\pi)x\theta A\psi R\bar{q}_t + (1-\pi)x n\bar{z}_t + \pi \theta A\psi Rx\bar{q}_t + \pi\theta A\psi Rn\bar{z}_t
\]
\[
= (1-\pi)n\theta A\psi R\bar{q}_t + (1-\pi)n^2\bar{z}_t + \pi n x\bar{q}_t + \pi n^2\bar{z}_t
\]
\[
(1-\pi)x\theta A\psi R\bar{q}_t + \pi \theta A\psi Rx\bar{q}_t - (1-\pi)n\theta A\psi R\bar{q}_t - \pi n x\bar{q}_t
\]
\[
= (1-\pi)n^2\bar{z}_t + \pi n^2\bar{z}_t - (1-\pi)x n\bar{z}_t - \pi \theta A\psi Rn\bar{z}_t
\]

\[
\frac{\hat{\bar{q}}_t}{\bar{z}_t} = \frac{(1-\pi)n^2 + \pi n^2 - (1-\pi)x n - \pi \theta A\psi Rn}{(1-\pi)x\theta A\psi R + \pi \theta A\psi Rx - (1-\pi)n\theta A\psi R - \pi n x}
\]

\[
\frac{\hat{\bar{q}}_t}{\bar{z}_t} = \frac{n^2 - (1-\pi)x n - \pi n\theta A\psi R}{x\theta A\psi R - (1-\pi)n\theta A\psi R - \pi n x}
\]

b. With Islamic Banks  
When an Islamic bank is present, type 2 individuals deposit their savings into the banking system. In each period, young individuals taking \(w_t\) and the deposit returns as given, decide how much to deposit into the banks \((d_t)\) by maximizing the following utility function:

\[
\max_{d_t} \ln(w_t - d_t) + (1-\pi)\ln(r_{1t}d_t) + \pi \ln(\theta A\psi r_{2t}d_t) + \gamma \ln d_t.
\]

Taking the first order condition with respect to \(d_t\) results in:
\[
-\frac{1}{w_t - d_t} + \frac{(1-\pi)r_{1t}}{r_{1t}d_t} + \frac{\pi \theta A\psi r_{2t}}{\theta A\psi r_{2t}d_t} + \frac{\gamma}{d_t} = 0
\]
\[- \frac{1}{w_t - d_t} + \frac{(1 - \pi)}{d_t} + \frac{\pi}{d_t} + \frac{\gamma}{d_t} = 0 \]

\[\frac{1 + \gamma}{d_t} = \frac{1}{w_t - d_t} \]

\[(1 + \gamma)w_t - (1 + \gamma)d_t = d_t \]

\[(1 + \gamma)w_t = d_t + (1 + \gamma)d_t \]

\[d_t = \frac{(1 + \gamma)}{(2 + \gamma)w_t}. \]

Substituting \(d_t\) back into the Islamic bank's maximization problem along with its constraints gives the following maximization problem:

\[\text{Max} \ q_t \left[ - \frac{w_t}{(2 + \gamma)} + (1 - \pi) \ln\left( \frac{(1 - q_t)n(1 + \gamma)w_t}{(2 + \gamma)(1 - \pi)} \right) + \pi \ln\left( \frac{\theta A \Psi R q_t(1 + \gamma)w_t}{(2 + \gamma)\pi} \right) + \gamma \ln\left( \frac{(1 + \gamma)w_t}{(2 + \gamma)} \right) \right] \]

Taking the first order condition with respect to \(q_t\) results in

\[-(1 - \pi) \frac{(2 + \gamma)(1 - \pi)}{(1 - q_t)n(1 + \gamma)w_t (2 + \gamma)(1 - \pi)} n(1 + \gamma)w_t + \pi \frac{(2 + \gamma)\pi}{\theta A \Psi R q_t(1 + \gamma)w_t} \theta A \Psi R(1 + \gamma)w_t = 0 \]

\[-(1 - \pi) + \frac{\pi}{q_t} = 0 \]

\[\frac{\pi}{q_t} = \frac{(1 - \pi)}{(1 - q_t)} \]

\[q_t = \frac{(1 - \pi)}{(1 - q_t)} \]

II. The Model with Non-variable Savings

A. Type One Individuals

Type 1 individuals are neutral to the type of banks and deposit their savings into the banking system. Here we assume that individuals do not value consumption when young and hence, they save their entire incomes. Substituting \(d_t = w_t\) back into the bank's utility maximization problem along with the bank's constraint gives the following maximization problem:

\[\text{Max} \ q_t (1 - \pi) \ln\left( \frac{(1 - q_t)n w_t}{(1 - \pi)} \right) + \pi \ln\left( \frac{\theta A \Psi R q_t w_t}{\pi} \right) \]
Taking the first order condition with respect to $q_t$ results in:

\[-(1 - \pi) \frac{(1 - \pi)}{(1 - q_t)n w_t} n w_t + \pi \frac{\pi}{\theta A \psi R_q t w_t} \theta A \psi R w_t = 0\]

\[-\frac{(1 - \pi)}{(1 - q_t)} + \frac{\pi}{q_t} = 0\]

\[\frac{\pi}{q_t} = \frac{(1 - \pi)}{(1 - q_t)}\]

\[q_t = \pi.\]

B. Type Two Individuals

a. Without Islamic Banks  Type 2 individuals do not use the banking system if it is not Islamic. Thus, these individuals have to invest directly in either liquid or illiquid assets. Type 2 individuals choose $\hat{q}_t$ in order to maximize their life-time utility as follows:

\[\text{Max}(1 - \pi) \ln[w_t(xq_t + n(1 - \hat{q}_t))] + \pi \ln[w_t(\theta A \psi R \hat{q}_t + n(1 - \hat{q}_t))].\]

Taking the first order condition with respect to $\hat{q}_t$ results in

\[\frac{(1 - \pi)(w_t x - w_t n)}{w_t(xq_t + n(1 - \hat{q}_t))} + \frac{\pi(w_t \theta A \psi R - w_t n)}{w_t(\theta A \psi R \hat{q}_t + n(1 - \hat{q}_t))} = 0\]

\[\frac{(1 - \pi)(x - n)}{(xq_t + n(1 - \hat{q}_t))} + \frac{\pi(\theta A \psi R - n)}{(\theta A \psi R \hat{q}_t + n(1 - \hat{q}_t))} = 0\]

\[\frac{(xq_t + n(1 - \hat{q}_t))}{(1 - \pi)(x - n)} + \frac{(\theta A \psi R \hat{q}_t + n(1 - \hat{q}_t))}{\pi(\theta A \psi R - n)} = 0\]

\[\frac{xq_t}{(1 - \pi)(x - n)} + \frac{n}{(1 - \pi)(x - n)} - \frac{n \hat{q}_t}{(1 - \pi)(x - n)} + \frac{\theta A \psi R \hat{q}_t}{\pi(\theta A \psi R - n)} + \frac{n \hat{q}_t}{\pi(\theta A \psi R - n)} = 0\]

\[\frac{xq_t}{(1 - \pi)(x - n)} - \frac{n \hat{q}_t}{(1 - \pi)(x - n)} + \frac{\theta A \psi R \hat{q}_t}{\pi(\theta A \psi R - n)} - \frac{n \hat{q}_t}{\pi(\theta A \psi R - n)} = 0\]

\[\hat{q}_t = \frac{-n \pi (\theta A \psi R - n) - n(1 - \pi)(x - n)}{(x - n)(\theta A \psi R - n) + (\theta A \psi R - n)(1 - \pi)(x - n)}\]
\[
= \frac{n^2 - nxx + nxx - n\pi A\psi R}{n^2 - nxx + x\theta A\psi R - n\theta A\psi R}
\]

b. With Islamic Banks  When an Islamic bank is present, type 2 individuals deposit their savings into the banking system. Substituting \(d_t = w_t\) back into the bank’s utility maximization problem along with the bank’s constraint gives the following maximization problem:

\[
\text{Max}_{q_t} (1 - \pi) \ln\left(\frac{(1 - q_t)nw_t}{(1 - \pi)} \right) + \pi \ln\left(\frac{\theta A\psi Rq_tw_t}{\pi} \right) + \gamma \ln[w_t]
\]

Taking the first order condition with respect to \(q_t\) results in

\[
-(1 - \pi) \frac{(1 - \pi)}{(1 - q_t)nw_t (1 - \pi)} + \pi \frac{\pi}{\theta A\psi Rq_tw_t} \frac{\theta A\psi Rw_t}{\pi} = 0
\]

\[
-(1 - \pi) + \frac{\pi}{q_t} = 0
\]

\[
\frac{\pi}{q_t} = \frac{(1 - \pi)}{(1 - q_t)}
\]

\[
q_t = \pi
\]

References


