VOTING ON INFRASTRUCTURE INVESTMENT: THE ROLE OF PRODUCT MARKET COMPETITION

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ABSTRACT. In spatial competition, public infrastructure plays a crucial role in determining product market outcomes. In our model, consideration of infrastructure’s impact on the product market drives the voting behavior of consumers in their dual role as voter/taxpayers. The spatial heterogeneity of consumers produces conflicting political interests and in many cases inefficient outcomes. However across both exogenous and endogenous market environments product market competition consistently leads to higher levels of publicly funded infrastructure than monopoly/collusion. Furthermore, competition’s boost to the popular support for infrastructure investment is often excessive while monopoly leads to underinvestment.

Keywords: Spatial Competition, Infrastructure Investment, Salop’s circular city, Voting, Referendum.

JEL Codes: D43, L13, H40, H54

1. INTRODUCTION

One of the original interpretations of transportation costs in the spatial competition framework is as a reflection of transport infrastructure:

“These particular merchants would do well, instead of organising improvement clubs and booster associations to better the roads, to make transportation as difficult as possible.” Hotelling (1929, page 50).

Implicit in this quote is a recognition of the pro-competitive nature of the transport infrastructure in the model. Since the transport costs determine participation, substitutability and hence competition in a market, one can interpret infrastructure quite broadly as being physical (e.g. roads and telecommunications) as well as institutional (e.g. trade liberalization, contract enforcement, anti-trust regulation and banking sector reforms).

Although the spatial competition literature shows the impact of infrastructure on individual welfare, it typically treats the level of infrastructure/transport costs as exogenous. See, for example, Eaton and Lipsey (1978), Salop (1979), Eaton and Wooders

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In this paper we develop a political economy framework which shows how individual voting can be used to determine the public provision of infrastructure in Salop’s “circular city” spatial market.

In our approach, citizens play a dual role as both consumers and voters/taxpayers; as a result, their endogenous voting preferences depend intimately on the details of competitive conditions in the product market. Infrastructure investment has two effects in the product market: it directly lowers costs to consumers and it indirectly affects market power. These market-based effects of infrastructure investments on consumers are heterogeneous because consumer locations are heterogeneous. The tax-based effects on citizens are unambiguously negative because taxes must increase in order to pay for the investment (we preclude the bundling of redistribution policies with the infrastructure funding).

Voting is analyzed through two related political paradigms — (i) the pairwise voting process in a representative democracy, which produces a Condorcet winner when individuals vote sincerely for their preferred level of infrastructure, and (ii) what appears to be a new set-based approach to represent a referendum in a representative democracy where individuals vote yes or no for a proposed increase from the status quo level of infrastructure provision.

Almost by definition, infrastructure improves the performance of individual markets and hence, in aggregate, the performance of an economy. Empirical studies are typically not at the level of the individual consumers and firms considered in our model; nonetheless, macro empirical estimates indicate that the effects of infrastructure can be large. In a seminal paper, Aschauer (1989) showed that public capital, especially core infrastructure (roads, utility networks, etc.) had a strong role in determining productivity. Similarly, Fernald (1999) found that the construction of the interstate road network had a large one-off impact on growth. Röller and Waverman (2001) showed that about one-third of growth in OECD countries over the period 1971–90 can be attributed to telecommunications investment. Czernich et al (2011) found that a 10 percentage point increase in broadband penetration raised annual per capita growth by 0.9–1.5 percentage points.

These empirical models, though sophisticated in their treatment, are too macroscopic to show who benefits from infrastructure and how these individual benefits result in government investment decisions. In line with the theoretical move to augment the traditional social planner approach with a more realistic political economy approach (see, for example, Persson and Tabellini, 2000; Winer and Hettich, 2004), a recent empirical literature has considered the political dimensions of public infrastructure expenditure. Papers such as Knight (2004) and Cadot et al (2006) provide significant evidence that government expenditure on infrastructure is directed by
self-interested politicians to please key voters rather than to maximize social welfare.\(^1\)

Analyzing voting over infrastructure, we find that when market structure is exogenous, product market competition boosts popular support for infrastructure—often excessively so—while monopoly/collusion leads to underinvestment. An infrastructure trap—a situation in which no investment in infrastructure is made despite the existence of social welfare enhancing investment—is common under monopoly/collusion. A robust positive implication of the above findings is that product market competition consistently leads to higher levels of publicly funded infrastructure than monopoly/collusion.

Infrastructure appears prominently in microeconomic models of economic geography and regulation. Transportation infrastructure plays an important role in the land use and core-periphery models of economic geography (see, for example, the excellent overviews in Fujita, Krugman and Venables, 1999 and Fujita and Thisse, 2002). Although this large literature also uses spatial techniques, and occasionally political economy, it is most definitely not a branch of oligopoly theory and hence is mute on the competitive aspects of infrastructure which we investigate here. In the regulatory literature, Aghion and Schankerman (2004) develop a model of lobbying by producers with asymmetric costs over competition-enhancing infrastructure.

Fernandez and Rodrik (1991) show how individual specific uncertainty can stall reforms and generate status quo bias even if a majority benefits from the reforms ex-post. Status quo bias is also considered in Majumdar and Mukand (2004). Political economy forces also generate this bias in their model, but the driver of inefficient policy choice is a government’s reputation rather than voter heterogeneity. Key to our infrastructure traps—akin to status quo bias—are the details of market environment which have little role to play in these papers.

While spatial models are used extensively in the industrial organization literature, the underlying infrastructure provision, as well as the institutional details determining the provision, are treated as exogenous. On the other hand, the public economics literature, despite its richness in tax and voting structures, has typically not analyzed spatial markets. By embedding voting over infrastructure in spatial oligopoly models we provide an explicit link between market environment and infrastructure.

In the subsequent sections, in all scenarios, there exist strictly positive investment levels that increase aggregate surplus. This suggests that the results arise for political economy reasons rather than from the existence of fixed costs or increasing returns. Though it is well known in general that political outcomes can differ from

\(^1\)Evidence that policy is distorted to meet the interests of powerful lobby groups is not conclusive for the simple reasons that lobbying is typically hard to observe and the interests of firms are hard to identify.
the social optimum, to our knowledge, our work is the first to explore how the difference between the two depends on the subtleties of the market environment within a voting setup.

2. A Model of Infrastructure Investment

We now develop a model of spatial competition explicitly extended to include public investments in infrastructure. Following Salop (1979), assume that a unit mass of consumers is uniformly distributed around a circle $C$ of circumference 1 with density 1, which can be interpreted either geographically or as a type space. The locations of consumers $y$ are described in a clockwise manner starting from 12 o’clock. Assume there are $n$ firms, with the location of firm $i$ denoted by $x_i$. We will make the standard assumption that firms are evenly dispersed around the circle and are numbered consecutively in a clockwise direction starting with firm 1 at 12 o’clock.\(^2\)

Assume that the $n (> 1)$ firms produce a product with marginal cost $m \geq 0$ and fixed cost $K$. Each consumer buys either zero or one unit of the product which yields gross utility of $A$ per unit of consumption. If a consumer living at $y$ purchases from firm $i$ then he incurs a price of $p_i$ and a transport cost or utility loss of $t|y - x_i|^{\beta}$ ($\beta \geq 1$). Consumer $y$’s net utility from consumption of good $i$, denoted by $v_i(y)$ is given by

\begin{equation}
  v_i(y) = A - p_t - t|y - x_i|^{\beta},
\end{equation}

where the distance $|y - x_i|$ is measured around the circumference of the circle. The consumers have a generic outside option, whose utility we normalize to zero, and they choose whichever option yields the highest net utility. This implies that a consumer $y$ purchases product $i$ as long as $v_i(y) \geq 0$ and $v_i(y) \geq v_j(y), j \neq i$.

We interpret the transport cost parameter $t$ as an index of infrastructure. More specifically, we consider a reduction in $t$ as resulting from an investment in infrastructure. The interpretation is quite natural in the geographical context where improvements in roads or rail connections, or the construction of a freeway system, lead to lower physical transportation costs. More generally, we might think of the consumers being located in a characteristic space with $t$ as an index of competition facilitating infrastructure.

We assume $t$ is determined by consumers/voters through a political process, which we describe below. Starting from an initial $t_0$, an investment of $I \geq 0$ reduces transport cost to $t_0 - I$. An investment of amount $I$ costs $\frac{I^2}{2}$ and is financed by a lumpsum tax of $g$ per consumer. Since there is a unit mass of consumers, the total tax revenue is also $g.1 = g$. We assume that the proceeds from the lumpsum tax cannot be used for redistributive purposes. This implies that in equilibrium $g = \frac{t^2}{2}$. The tax $g$ or equivalently the level of investment $I$ is determined by political process.

\(^{2}\) Economides (1989) shows that this is the unique symmetric equilibrium in a location-then-price game.
The sequence of events is as follows. Given some status quo $t_0$, the political process determines the level of infrastructure investment $I$ which determines transport cost $t = t_0 - I$. Subsequently, firms set prices, and consumers then make their purchasing decisions.

In order to focus on the voting behavior of consumers, we assume that all profits accrue to a measure zero elite. In the absence of shareholding by consumers, the surplus of a consumer $y$, denoted by $s(y, I)$, is the indirect utility from consumption less tax, i.e.

$$s(y, I) = \max\{v_1(y), ..., v_n(y), 0\} - \gamma I^2.$$

**2.1. Aggregate Surplus Measures.** Though the individual surplus measure determines the voting behavior of an individual, the cost-benefit comparison requires aggregate measures. Two aggregate surplus measures are introduced below. The measures are defined generally so that they can be used for comparison in the later sections. The first measure, denoted by $S(I)$ is simply the sum of consumer surplus for all $y$:

$$S(I) = \int_C s(y, I) dy.$$

The second measure, aggregate social surplus, denoted by $W(I)$, is the sum of aggregate consumer surplus $S(I)$ and aggregate profits $\Pi$:

$$W(I) = S(I) + \Pi(I).$$

Note $\Pi(I) = \sum_{i=1}^n \pi_i(I)$, where $\pi_i(I)$ denotes firm $i$’s profit for a given investment level $I$.

### 3. Political Economy

At regional or local levels, or even at a country level (especially if the country is small), proposals are often put forward in a popular vote or referendum (see Catt, 1999). For example, in September 2003, the residents of Hampton Roads and Northern Virginia voted on whether to raise sales tax to fund the improvements and extension of existing roads in the area. In September 2002, Mexico City voted on a double deck road plan which promised to relieve the traffic crisis by building elevated freeways over a crosstown artery. Examples of referendum also exist on telecommunication related issues in Slovenia, and electricity liberalization in Switzerland. We use referendum in our analysis, not only because some of the decision making or decision approval occur in reality in this fashion, but also because theoretically it provides a useful refinement of the set of proposals in the absence of a specific political/electoral mechanism.
In the current context, the referendum on infrastructure works as follows. A positive level of income tax \( g = \frac{\gamma I}{2} \) is proposed to finance an infrastructure investment of amount \( I \) which lowers the transport cost from \( t_0 \) to \( t_0 - I \). The proposal is passed in the referendum if at least 50% of the consumers/voters vote in favor of the proposal against the status quo \( I = 0 \).

A consumer \( y \) votes in favor of a proposed investment level \( I_1 \) over the alternative \( I_2 \) if and only if \( s(y, I_1) \geq s(y, I_2) \). Let \( m(I_1, I_2) \) denote the measure of consumers who vote in favor of the proposal \( I_1 \) over the alternative investment level \( I_2 \). We define \( R^0 \) as the set of investment levels which a majority of voters favour over the status quo \( I_2 = 0 \), i.e.

\[
R^0 = \{ I : m(I, 0) \geq \frac{1}{2} \}.
\]

We refer to \( R^0 \) as the referendum viable investments. To understand the extent of distortion in the political outcomes, we consider two benchmarks based on the surplus measures \( S(I) \) and \( W(I) \) introduced previously.

\[
S^0 = \{ I : S(I) - S(0) \geq 0 \}
\]

\[
W^0 = \{ I : W(I) - W(0) \geq 0 \}
\]

The set \( S^0 \) (\( W^0 \)) consists of investment levels for which the aggregate consumer surplus (social surplus) is higher compared to the status quo.

Following the standard practice in the voting literature, we use the concept of a Condorcet winner in the pairwise voting scenario. Excluding abstention, \( I^* \) is a Condorcet winner if \( m(I^*, I) \geq \frac{1}{2} \) for all \( I \neq I^* \). To determine whether political outcomes yield “underprovision” or “overprovision” of investment, we compare \( I^* \) with aggregate consumer surplus maximizing investment level

\[
I_s = \arg \max_{I \geq 0} S(I)
\]

and social surplus maximizing investment level

\[
I_w = \arg \max_{I \geq 0} W(I).
\]

Two common features across the models in different sections are that (i) \( S(I) \) and \( W(I) \) are continuous in \( I \) and (ii) \( S^0 \) and \( W^0 \) are compact, which guarantee the existence of \( I_s \) and \( I_w \).

4. Spatial Competition

In this section, we assume that the number and locations of firms are fixed, which is appropriate for analyzing situations involving sunk costs, entry barriers or the short run. Furthermore, throughout this section we assume that \( n \geq 2 \). The spatial competition between firms arising from locational differences links equilibrium prices
to the level of infrastructure. As a consequence, when voting, a consumer not only has to consider the effect of infrastructure investment on transport costs but also its effect on prices.

4.1. **Price Equilibria:** We assume that the gross utility from consuming a variety, $A$, is high enough (or equivalently $t_0$ is low enough) such that each consumer buys some variety and firms directly compete with their neighbors.\(^3\) We also assume equally spaced firms on the circle, with $1 \leq \beta \leq 6.2$, in order to guarantee the existence of the unique symmetric price equilibrium (see Anderson et al., 1992, p. 177):

\[
p^*(I) = c + \frac{\beta 2^{1-\beta} (t_0 - I)}{n^\beta}.
\]

Note that $p^*(I)$ is decreasing in $I$ reflecting the fact that an increase in investment level, i.e. a reduction in $t$, creates more competition among the existing firms, which in turn leads to lower equilibrium prices.

4.2. **Political Economy Results.** Recall the individual surplus measure, $s(y, I)$, introduced in section 2. Substituting $p = p^*(I)$ from equation (4.1), for a consumer $y \in C$ we have:

\[
s(y, I) = A - p^*(I) - (t_0 - I)|y - x^*_i|^\beta - \frac{\gamma I^2}{2},
\]

where $x^*_i$ is the location of the firm nearest to consumer $y$.

Since the $n$ firms are equally spaced around the circle and the equilibrium prices are identical, it suffices to consider a mass of $\frac{1}{2n}$ consumers all located on one side of a representative firm whose location is normalized to 0. A consumer $y \in [0, \frac{1}{2n}]$ votes against the status quo if

\[
s(y, I) - s(y, 0) = [p^*(0) - p^*(I)] + Iy^\beta - \frac{\gamma I^2}{2} \geq 0.
\]

Observe that $s(y, I)$ satisfies the single crossing property. Thus by an application of Gans and Smart (1996), the voting behavior of the median voter is sufficient to determine the voting behavior of the majority.\(^4\) Noting that $|y| = \frac{1}{4n}$ is the median consumer, the set of investment levels that beats the status quo in pairwise voting is given by:

\[
R^0 = \left\{ I : s\left(\frac{1}{4n}, I\right) - s\left(\frac{1}{4n}, 0\right) \geq 0 \right\}.
\]

\(^3\)If $A$ is low, then each firm becomes a local monopolist. Monopoly power is considered in section 5.

\(^4\)Suppose $y \in [0, \frac{1}{2n}]$. The single crossing property of consumer/voter preferences holds when the following statement is true: If $I > I'$ and $y' > y$ or $I < I'$ and $y' < y$ then $s(y, I) \geq s(y, I') \implies s(y', I) \geq s(y', I')$. If voter preferences satisfy the single crossing property then a Condorcet winner exists and coincides with the preferred policy of the voter of the median type. See for example, p. 23, Chapter 2 in Persson and Tabellini (2000).
Solving this inequality for $I$ characterizes the referendum viable investment levels, i.e., investment levels which will win in a referendum. It also follows from the single crossing property and Gans and Smart (1996) that the most preferred investment level of the median consumer is the unique Condorcet winner. The results are summarized in the following proposition.

**Proposition 1.** Voting on infrastructure investment yields the Condorcet winner $I^*$, where

$$I^* = \frac{\beta 2^{1+\beta} + 1}{4\beta \gamma n^\beta}.$$

The set of referendum viable investments which dominate the status quo is $R^0 = [0, 2I^*]

By inspection, $I^*$ is decreasing in $\gamma$ and $n$. $\gamma$ determines the rate at which marginal cost increases, thus quite naturally as the marginal cost of infrastructure increases, the equilibrium choice decreases.

Increased $n$, an exogenous increase in the number of firms, lowers the distance travelled by the median consumer, which in turn reduces the direct marginal benefit from $I$. The indirect benefit of increased $I$, which operates through price reduction, i.e. $\frac{d(p^*(0)-p^*(I))}{dI} = \frac{\beta 2^{1-\beta}}{n^\beta}$, is decreasing in $n$. Hence on both counts, the incentive to invest becomes smaller as the number of firms increases.

Finally, we turn to comparative statics with respect to $\beta$, the convexity of the transport cost function. The direct marginal benefit of an increase in $I$ is $(4n)^{-\beta}$, which is decreasing in $\beta$. This is reinforced by the indirect effect, of price reduction, $\frac{d(p^*(0)-p^*(I))}{dI} = \frac{2\beta}{(2n)^\beta}$, which becomes smaller as $\beta$ increases. Thus $I^*$ is decreasing in $\beta$.

### 4.3. Welfare Results.

Substituting $s(y, I)$ as given by equation (4.2) into the definitions of $S$ and $W$ gives

\begin{align}
S(I) &= A - p^*(I) - \frac{t_0 - I}{(2n)^\beta(1 + \beta)} - \frac{\gamma I^2}{2}, \\
W(I) &= A - c - \frac{t_0 - I}{(2n)^\beta(1 + \beta)} - \frac{\gamma I^2}{2}.
\end{align}

We begin by determining $S^0$ and $W^0$ which are respectively the sets of $I$ that improve aggregate consumer surplus and welfare compared to the status quo. Using equations (4.5) and (4.6) gives the following proposition.
Proposition 2. The investment levels which maximize consumer surplus $I_s$ and welfare $I_w$ are

\begin{align}
I_s &= \frac{2\beta(1 + \beta) + 1}{(2n)^\beta(1 + \beta)\gamma}, \\
I_w &= \frac{1}{(2n)^\beta(1 + \beta)\gamma}.
\end{align}

The set of investments which increases consumer surplus, over the status quo, is $S^0 = [0, 2I_s]$ and the set of investments which increases welfare is $W^0 = [0, 2I_w]$

Comparing $W^0$ and $S^0$ it follows that $W^0 \subset S^0$. The reasoning is simple. An increase in investment level increases $S(I)$ through two channels: reduction in equilibrium prices and reduction in aggregate transport costs. Changes in price do not affect $W(I)$. This implies that, corresponding to any change in $I$, the increase in $W(I)$ is less than the increase in $S(I)$ and accordingly any investment level that increases aggregate social surplus increases aggregate consumer surplus as well. In other words, $W^0 \subset S^0$. This argument, appropriately modified, applies to marginal changes in $I$ too. Since the marginal increase in $W(I)$ is less than that of $S(I)$, and $W(I)$ and $S(I)$ are strictly concave, it follows that $I_w < I_s$. A complete comparison of welfare and equilibrium outcomes is given by the following proposition.\(^\text{5}\)

Proposition 3. In a circular city model with $n \geq 2$ firms, voting leads to over investment:

\begin{align}
I_w &< I^* \leq I_s, \\
W^0 &\subseteq R^0 \subseteq S^0
\end{align}

where equality holds only for $\beta = 1$.

Due to the convexity of transport costs, the savings in transport costs for the median consumer, as a result of improved infrastructure, is less than the average savings. This implies that there are investment levels $I$ which increase $S(I)$ but are not favored by the median consumer, and accordingly not referendum viable. Hence $R^0 \subseteq S^0$. Since the savings are valued similarly in $W^0$ and $S^0$, the argument described above would suggest that $R^0 \subseteq W^0$ as well. However, recall that the change in aggregate social surplus, $W(I) - W(0)$, does not take into account the beneficial effect of price reduction due to improved infrastructure. This enlarges the set $R^0$, and in fact for the specification chosen, it turns out that $W^0 \subset R^0$. Similar arguments can be used to establish the ordering of the investment levels in (4.9).

\(^\text{5}\)Qualitatively $I_s$ (or $I_w$) vary with $n$, $\beta$ and $\gamma$ in the same way was as $I^*$ does and the arguments are similar to those presented immediately after Proposition 3.
5. Monopoly

Consider again Salop’s circular city model described in section 2. Retain all features of the model except competition. In particular, consider a monopolist with \( n \) outlets evenly dispersed around the circle. Given the underlying symmetric structure of the model, there is a unique monopoly price, which is increasing in \( I \) (as the following lemma shows).

**Lemma 1.** With \( n \geq 2 \) outlets and \( A \geq c + \frac{(1+\beta)}{(2n)^{\beta}} \) the unique monopoly price is

\[
p_m(I) = A - (t_0 - I)(\frac{1}{2n})^\beta.
\]

This price is just high enough to reduce the utility of a marginal consumer — midway between two outlets — to zero. The gross value of the product \( A \) is sufficiently high, relative to costs, to make it unattractive for the monopolist to set a price which excludes any consumer from the market.

Under monopoly, for pivotal consumers, the loss from increased price exactly offsets the gain from transport cost savings. All other consumers are strictly worse off because they face the same increase in price but they benefit less from the improvement in infrastructure. This exploitative aspect of infrastructure under monopoly leads to the following:

**Proposition 4.** Monopoly causes an infrastructure trap:

\[
R^0 = S^0 = \{0\} \subset W^0
\]

\[
I^* = I_s = 0 < I_w
\]

The above analysis applies equally for collusion as well once we view the \( n \) outlets owned by the monopolist as \( n \) independent firms, who collude on price. The collusive price is the same as the monopoly price in Lemma 1, assuming implicitly that the discount rates are high enough to sustain collusion.

Comparing Proposition 4 and Propositions 1-3 highlights the importance of market reforms in generating popular support to undertake infrastructure improvements. Even though welfare-improving changes exist, those changes might not be politically viable in the absence of competition. For many years, global institutions, such as the World Bank, have pushed for market reforms before providing any aid in terms of infrastructure improvements. Our framework provides an explicit link between the two and suggests that indeed market structure (or more generally market environment) has important bearings on support for infrastructure provision.

A robust positive implication of our analysis with fixed \( n \) is that the level of infrastructure investment is always higher under competition than under monopoly. Allowing for endogenous determination of \( n \) — by incorporating free entry under competition and choice of outlets under monopoly — we show that the same conclusion holds.
A new feature under free entry is that investment levels under competition can be efficient (see Proposition 5) although that is not always the case (see subsection 6.3).

6. Free Entry

6.1. Free Entry Equilibrium. In our analysis so far, the number and locations of firms were assumed to be fixed. The assumption is appropriate for short run analysis, but, in the long run, firms can change locations and furthermore, entry and exit may occur in the industry. To incorporate these features into our framework and to examine the consequent effects on the voting outcome, we consider a free entry version of our model.

On the production side, in addition to constant marginal cost, we also assume positive fixed cost of production $K > 0$. Consider a sequential game, where corresponding to a given level of infrastructure provision $t = t_0 - I$, a firm $i$ first decides whether to enter and subsequently it chooses location $(x_i)$ and then price $(p_i)$. If firms choose simultaneously at each stage and $n$ firms have entered in the first stage, the location and price of firm $i$ in a symmetric equilibrium, denoted by $\bar{x}_i$ and $\bar{p}_i$ respectively, satisfy the following (see Economides, 1989 and Anderson et al, 1992):

\begin{equation}
|\bar{x}_i - \bar{x}_{i+1}| = |\bar{x}_i - \bar{x}_{i-1}| = \frac{1}{n} \tag{6.1}
\end{equation}

\begin{equation}
\bar{p}_i(n) = \bar{p}(n) = c + \beta 2^{1-\beta}(t_0 - I)(\frac{1}{n})^\beta. \tag{6.2}
\end{equation}

Treating $n$ as a continuous variable, the free-entry number of firms corresponding to a given level of investment $I$, denoted by $n^*(I)$ is obtained by solving the zero profits condition $(\bar{p}(n) - c)\frac{1}{n} = K$. This yields

\begin{equation}
n^*(I) = \left(\frac{\beta 2^{1-\beta}(t_0 - I)}{K}\right)^{\frac{1}{1+\beta}}. \tag{6.3}
\end{equation}

For a given $I \geq 0$, a subgame perfect Nash equilibrium outcome of the three-stage game — entry (stage 1), location choice (stage 2) and price competition (stage 3) — can be summarized by a triplet $(n^*(I), \{x_i^*(I)\}_{i=1}^{n^*(I)}, p^*(I))$ where $n^*(I)$ is as in equation (6.3), and $x_i^*(I)$ and $p^*(I)$ are $\bar{x}_i$ and $\bar{p}_i$ respectively evaluated at $n = n^*(I)$.

6.2. Welfare and Uncertainty. Suppose the initial level of infrastructure provision in the economy is $t = t_0$. While voting for $I \geq 0$, a consumer $y$ correctly anticipates $n^*(I)$ and $p^*(I)$. However, since any equispaced location of $n^*(I)$ firms constitutes an equilibrium, a consumer computes the expected utility over all possible distances $|y - x_i^*(I)|$, where $x_i^*(I)$ denotes the location of the nearest firm. Assuming a uniform prior

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6 Note that if fixed costs are sunk on entry, the short run analysis is the same as the long run analysis because infrastructure investment increases competition which lowers profits.

7 Any equi-spaced configuration of the $n$ firms on the circle is an equilibrium.
for equilibrium distance $|y - x^*_i(I)|$ over the support $[0, \frac{1}{2n^*(I)}]$, the expected surplus from an investment $I \geq 0$ is:

$$
E[s(y, I)] = A - p^*(I) - (t_0 - I)2n^*(I) \int_y^{y + \frac{1}{2n^*(I)}} |y - x_i|^\beta dx_i - \frac{\gamma I^2}{2}
$$

(6.4)

$$
= A - p^*(I) - \frac{t_0 - I}{(2n^*(I))^{\beta(1 + \beta)}} - \frac{\gamma I^2}{2},
$$

$$
= \bar{S}(I).
$$

We use a constrained optimal approach to welfare in considering free entry—constrained in the sense that we take as given the way in which market forces determine equilibrium prices and the equilibrium number of firms. This seems a natural way to examine in isolation the distortions caused by the political process in determining infrastructure investments.

Since $E[s(y, I)] = \bar{S}(I)$ for all $y$ on the circle $C$, and there is a unit mass of consumers, it follows that $S(I) = \bar{S}(I)$. Moreover, since profits are zero in free-entry equilibrium, the two aggregate surplus measures are equivalent: $W(I) = S(I) = \bar{S}(I)$ for all $I \geq 0$. This equivalence in turn implies that for all $\beta \geq 1$,

$$
W^0 = S^0 \supset \{0\},
$$

(6.5)

$$
I_w = I_s = \arg \max_{I \geq 0} \bar{S}(I) > 0.
$$

(6.6)

As in the previous sections, the existence of strictly positive, surplus enhancing $I$, follows from the observation that infinitesimally small levels of $I$ have zero cost and $W(I)$ and $S(I)$ are continuous and increasing in $I$ for all $I \geq 0$.

All consumers have the same expected utility, $\bar{S}(I)$, for a given $I \geq 0$. Hence an investment is surplus enhancing, for each individual, if and only if each individual would vote for it in a referendum. This alignment of welfare and political economy considerations leads immediately to the following proposition.

**Proposition 5 (Optimality Under Free Entry).** With free entry the welfare improving investments are exactly the referendum viable investments, $R^0 = S^0 = W^0$, and the Condorcet winner is socially optimal, $I^* = I_s = I_w > 0$.

The uncertainty over the possible equi-spaced firm locations in equilibrium produces a stochastic smoothing making all consumers identical in their assessment of investments, irrespective of initial conditions.\(^8\)

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\(^8\)Alternatively, one might consider a situation in which initially the number of firms, locations and prices are given by $n^*(0), \{x^*_i(0)\}_{i=1}^{n^*(0)}$ and $p^*(0)$ respectively. If there is no investment this equilibrium is maintained, otherwise any positive investment produces the uncertain relocations described above. This specification, with initial conditions, is less appealing because it confounds the pure impact of infrastructure on transport costs with the welfare effects of uncertainty. See Appendix B of the working paper Ghosh and Meagher (2014).
6.3. **Deterministic exit: an example.** As an alternative to uncertain exit, we consider an illustrative example where firms’ exit pattern is pre-determined and surviving firms stay put.\(^9\)

Consider a simplified model where the investment choice is binary: \(I = 0\) or \(I = \frac{3}{4} t_0 \equiv \hat{I}\). The free-entry number of firms corresponding to \(I = 0\) and \(I = \hat{I}\) are \(n^*(0) = \sqrt{\frac{t_0}{K}} \equiv m\) and \(n^*(\hat{I}) = \sqrt{\frac{t_0}{4K}} = \frac{m}{2}\) respectively. Thus, only half of the \(m\) firms that exist for \(I = 0\) survive with an investment of \(\hat{I}\).

Suppose \(m\) is even and all \(m\) firms are evenly dispersed around the circle, starting with firm 1 located at 12 o’clock. Rather than assuming that all firms are equally likely to exit when \(I\) changes from 0 to \(\hat{I}\), here we assume that every second firm exits so that the odd numbered firms 1, 3, ..., \(m - 1\) stay put and even-numbered firms 2, 4, 6, ..., \(m\) exit.

To determine whether \(\hat{I}\) is referendum viable and welfare enhancing, we first compute the surplus for consumers under \(I = 0\) and \(I = \hat{I}\). Rather than considering all consumers, focus on any representative mass of \(\frac{1}{m}\) consumers uniformly distributed between two adjacent firms. Without loss of generality, consider firms 1 and 2 who are located at \(x_1 = 0\) and \(x_2 = \frac{1}{m}\) respectively.

A consumer \(y \in [0, \frac{1}{2m}]\) buys from firm 1 in equilibrium for both \(I = 0\) and \(I = \hat{I}\). She votes for \(\hat{I}\) if and only if

\[
s(y, \hat{I}) - s(y, 0) \geq 0 \iff p^*(0) - p^*(\hat{I}) - \frac{\gamma \hat{I}^2}{2} + \hat{I} y \geq 0,
\]

or equivalently,

\[
y \geq \max\{0, \frac{\gamma \hat{I}^2 - (p^*(0) - p^*(\hat{I}))}{\hat{I}}\} \equiv y_L.
\]

Though the price reduction and tax is the same for all consumers in \([0, \frac{1}{2m}]\), the reduction in transport cost is more for distant consumers. Accordingly, among all consumers who buy from firm 1, only those located at a distance \(y_L\) or more from firm 1 vote in favour of \(I = \hat{I}\).\(^10\)

For a consumer \(y \in [\frac{1}{2m}, \frac{1}{m}]\) the change in surplus due to investment is

\[
s(y, \hat{I}) - s(y, 0) = p^*(0) - p^*(\hat{I}) - \frac{\gamma \hat{I}^2}{2} + \hat{I} y - 2 t_0 (y - \frac{1}{2m}).
\]

Observe that the difference in surplus for \(y \in [\frac{1}{2m}, \frac{1}{m}]\) is the same as that for \(y \in [0, \frac{1}{2m}]\) except for the negative term \(-2 t_0 (y - \frac{1}{2m})\). The extra negative term captures the loss

\(^9\)There is no unique way to think about exit in this class of models; indeed this is an open issue in the Industrial Organization literature. One of the few papers to consider entry with fixed locations is Anderson and Engers (2001). They consider fixed firm locations in a growing market where entry occurs in exponentially growing waves.

\(^10\)Note that for suitably low values of \(\gamma, y_L\) could be zero, i.e., all consumers irrespective of the distance could vote for \(\hat{I}\). Similarly, for suitably high values of \(\gamma, y_L\) could exceed \(\frac{1}{2m}\) implying that no \(y \in [0, \frac{1}{2m}]\) votes for \(\hat{I}\).
in utility for consumers in \([\frac{1}{2m}, \frac{1}{m}]\), all of whom have to travel at least \(\frac{1}{2m}\) further when their nearest firm exits. The increase in distance, and consequently the loss in surplus due to investment, increases with proximity to firm 2. Accordingly, we find that a consumer \(y \in [\frac{1}{2m}, \frac{1}{m}]\) votes in favor of \(\hat{I}\) if and only if she is sufficiently close to firm 1, i.e.

\[ y \leq \max\left\{ \frac{1}{2m} \cdot \frac{\gamma \hat{I}^2}{2} - \frac{(p^*(0) - p^*(\hat{I}))}{\hat{I}} \right\} \equiv y_H. \]

There are two sets of consumers who prefer \(I = \hat{I}\) to \(I = 0\): \(y \in [y_L, \frac{1}{2m}]\) and \(y \in [\frac{1}{2m}, y_H]\). Thus \(\hat{I}\) beats the status quo in pairwise voting if these two sets of consumers constitute a majority:

\[ \hat{I} \in R^0 \iff y_H - y_L \geq \frac{1}{2m} \iff \gamma < \frac{2((p^*(0) - p^*(\hat{I})) + \frac{\eta^0}{64m})}{\hat{I}^2} \equiv \gamma_R. \]

Now we turn to welfare analysis. Comparing welfare \((W)\) between \(I = \hat{I}\) and \(I = 0\) we get:

\[ \hat{I} \in W^0 \iff W(\hat{I}) - W(0) \geq 0 \iff \gamma < \frac{2((p^*(0) - p^*(\hat{I})) + \frac{\eta^0}{8m})}{\hat{I}^2} \equiv \gamma_W. \]

Comparing (6.7) and (6.8) it follows that \(\gamma_R > \gamma_W\). Therefore, there exists \(\gamma \in (\gamma_W, \gamma_R)\) such that the only available positive investment level \(\hat{I}\) is excessive in the sense of being referendum viable but not welfare enhancing: \(\hat{I} \in R^0\) but \(\hat{I} \notin W^0\).

In the free entry environment with uncertainty we found that the a consumer votes for an investment level \(I > 0\) if and only if it increases welfare. Thus, all politically viable investments level were socially desirable and vice versa. However, as the deterministic version of the model show, excessive investment present in the exogenous market structure can continue to hold in the free entry environment. These differences notwithstanding, a common theme emerges from all scenarios under competition (with exogenous and endogenous market structure): there is always political support for positive levels of investment in infrastructure.

### 7. Monopoly with Endogenous Number of Outlets

We extend the monopoly case analyzed in section 5 by allowing the profit-maximizing monopolist to choose the number of outlets in addition to price. As before, the unique monopoly price for exogenously given \(n\) and \(I\) is: \(p^m(n, I) = A - (t_0 - I)(\frac{1}{2m})^\beta\). Maximizing \((p^m(n, I) - c) - nK\) with respect to \(n\) yields the optimal number of outlets for a monopolist:

\[ n^m(I) = \left( \frac{\beta(t_0 - I)}{2^\beta K} \right)^{\frac{1}{1+\beta}}. \]
Evaluating at $n = n^m(I)$, the optimal monopoly price turns out to be

\[(7.1) \quad p^m(I) = p^m(n^m(I), I) = A - (t_0 - I)^{\frac{1}{1+\beta}} (\frac{K}{2\beta})^{\frac{\beta}{1+\beta}}.\]

The following results are immediate from the expressions of $p^m(I)$ and $n^m(I)$.

**Lemma 2.** Monopoly price, $p^m(I)$ is increasing in $I$ and the profit-maximizing number of monopoly outlets, $n^m(I)$, is decreasing in $I$.

Any equispaced location of $n^m(I)$ outlets constitute an equilibrium, and hence a consumer computes the expected utility over all possible distances. Assuming a uniform prior for equilibrium distances over the support $[0, 2n^m(I)]$, we find that the expected surplus for any individual from an investment $I > 0$ is:

\[E[s(y, I)] = A - p^m(I) - \frac{t_0 - I}{(2n^m(I))^{\beta(1+\beta)}} - \frac{\gamma I^2}{2} = S^m(I).\]

Since there is a unit mass of consumers and no aggregate uncertainty, $S(I)$, i.e., aggregate consumer surplus for an investment level $I \geq 0$, is same as $S^m(I)$. We find that

\[(7.2) \quad S(I) - S(0) = \bar{S}^m(I) - \bar{S}^m(0) = \frac{\beta}{1+\beta} (p^m(0) - p^m(I)) - \frac{\gamma I^2}{2} \leq 0.\]

where equality holds only if $I = 0$. The two indirect effects of infrastructure investment are negative for consumers: higher prices and greater average distance to an outlet. The only positive effect for consumers is the reduction in per unit transport cost $t$. It turns out that the negative effects dominate for consumers, Thus, there is no $I > 0$ that yields higher aggregate consumer surplus compared to the status quo. Consequently,

\[(7.3) \quad S^0 = \{0\}, \quad I^* = 0.\]

As in section 6, all consumers have the same expected utility, $S^m(I)$, for a given $I \geq 0$. Hence an investment is surplus enhancing, for each individual, if and only if each individual would vote for it in a referendum. This alignment of welfare and political economy considerations implies the following:

\[R^0 = \{0\}, \quad I^* = 0.\]

While consumer surplus worsens, welfare can still improve with an investment in infrastructure. The negative effect is lower for welfare as loss in consumer surplus due to increased price is exactly offset by increase in monopoly profits. Moreover, there is an additional positive welfare effect due to the reduction in fixed costs with fewer outlets. In Appendix, we show that there exists $I > 0$ such that the welfare gain from fixed cost savings outweighs the welfare loss from increased distance. Thus
Collecting the welfare and political economy results, we can summarise our findings as follows:

**Proposition 6.** Consider the circular city model with monopoly and an endogenous number of outlets. While there are welfare enhancing investments, positive investments are neither referendum viable nor do they increase aggregate consumer surplus:

\[
R^0 = S^0 = \{0\},
\]

\[
I^* = I_s = 0,
\]

\[
I_w > 0, \quad W^0 \supset \{0\}.
\]

Infrastructure traps or zero investment arise under monopoly even with endogenous determination of the number of outlets. As in the fixed \( n \) scenario, investment level continues to be lower under monopoly than under competition. To summarize, in a democratic market economy, a lack of competition jeopardizes welfare enhancing infrastructure investments.

### 8. Conclusion

We consider a spatial competition model in which we interpret the transport cost parameter as an index of infrastructure. By incorporating voting over infrastructure by consumers, we provide an explicit political economy foundation for infrastructure investment. As one might expect, political processes do not necessarily generate socially optimal or efficient outcomes. What is novel in our analysis is how such inefficiencies (induced by voting) depends crucially on competition (or lack of it).

Across the models, competition is infrastructure promoting (sometimes excessively so) while collusion or monopoly leads to infrastructure traps. Thus, the investment level is strictly higher under competition. The implication could be tested simply by observing the difference in investment levels between two regions with different market structure or between pre and post deregulation in the same region. A reduced form empirical analysis could investigate broadly the over/under investment in infrastructure as it relates to competition or monopolistic market conditions. As always, sharp quantitative insights would require a more structural approach and would depend on the subtle details of the model.

Throughout the analysis we have consumers do not hold any shares in the firm. The results are qualitatively unchanged if a small fraction of the shares are held by the consumers and there is no heterogeneity (across consumers) in shareholding. However, the analysis is cleaner if all the shares belong to a measure zero elite, as we assume to be the case in the main text. The assumption on shareholding accords well with the findings in developing countries where shareholding is extremely skewed. In developed countries, many people who hold shares do so through pension funds or
unit trusts due to the cost saving of delegated diversification or regulation. One observed consequence of the diffusion of ownership and the ensuing free rider problems is that most individuals do not exercise any effective influence over the management of the firms in which they hold shares.

By focusing on consumers and voting, we have ignored the other side of the story: producers and the political apparatus they employ to protect their profits — lobbying. In the applied literature, the presence of lobbying is often captured by considering weighted social surplus as the objective function, with profits being assigned higher weights than aggregate consumer surplus. See, for example, Grossman and Helpman (1994) and Mitra (2001) for a microfoundation of this approach. Our preliminary investigation suggests that inefficiencies and the possibility of underinvestment exist under this setting as well.

By endogenizing the transport cost parameter as a politically determined infrastructure investment, we allow consumers, in their dual role as voters, to partially determine the environment they face when they make purchasing decisions. This approach, of allowing consumers a role in choosing the “rules of the game”, appears to produce a rich framework without a great deal of additional technical complexity. Our results highlight how market environment and political economy concerns can subtly impact public investment in infrastructure.  

11Though we covered some distance in the analysis of market environments — competition, monopoly and free entry— on the political economy front we have been more selective. Two recent advances in modeling electoral competition, which we do not consider, are the citizen-candidate framework, à la Besley and Coate (1997) or Osborne and Slivinski (1996) and the party competition approach of Roemer (2001) and Osborne and Tourky (2004). However, we would like to highlight the novelty our analysis offers by considering both point outcomes (e.g. electoral competition) as well as set outcomes (e.g. referendum outcomes). While coarse, the set based notion is appealing as it does not rely on the details of position selection mechanism.
APPENDIX A

Proof of Proposition 1. Substituting equation (4.2) into equation (4.4) gives

\[
[p^*(0) - p^*(I)] + \frac{I}{4n^\beta} - \frac{\gamma I^2}{2} \geq 0. 
\]

(8.1)

Substituting the equilibrium prices from equation (4.1) gives

\[
I \left[ \beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(4n)^\beta} - \frac{\gamma I}{2} \right] \geq 0 \iff I \in \left[ \frac{\beta^{2^{1+\beta}} + 1}{2^{1+\beta} n^\beta \gamma} \right]. 
\]

(8.2)

As the discussion preceding Proposition 1 shows the voting outcome is the median voter’s preferred policy, which is given by the following:

\[
I^* = \arg \max_{I \in R^0} (s(I) - s(0)) = \frac{1}{\gamma} \left( (\beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(4n)^\beta}) \right) = \frac{\beta^{2^{1+\beta}} + 1}{(4n)^\beta \gamma}.
\]

Observe that \(\frac{\beta^{2^{1+\beta}} + 1}{2^{1+\beta} n^\beta \gamma} = 2I^*\) which explains why \(R^0 = [0, 2I^*]\).

Proof of Proposition 2. By definition \(S^0 := \{ I : I \geq 0, S(I) - S(0) \geq 0 \}\). Using (4.5) it follows that

\[
S(I) - S(0) = [p(0) - p^*(I)] + I\left( \frac{1}{2n^\beta (1 + \beta)} - \frac{\gamma I}{2} \right)
\]

(8.3)

\[
= I\left( \beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(2n)^\beta (1 + \beta)} - \frac{\gamma I}{2} \right),
\]

(8.4)

which is positive for all \(I \leq \frac{2}{\gamma} (\beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(2n)^\beta (1 + \beta)})\). Hence

\[
S^0 := \{ I : 0 \leq I \leq \frac{2}{\gamma} (\beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(2n)^\beta (1 + \beta)}) \} = \{ I : 0 \leq I \leq 2I_s \}
\]

where

\[
I_s = \arg \max_{I \in S^0} S^0 = \frac{1}{\gamma} \left( \beta^{2^{1-\beta}} \left( \frac{1}{n} \right)^\beta + \frac{1}{(2n)^\beta (1 + \beta)} \right) = \frac{2\beta (1 + \beta) + 1}{(2n)^\beta (1 + \beta) \gamma}.
\]

Similarly \(W^0 := \{ I : I \geq 0, W(I) - W(0) \geq 0 \}\). Thus using equation (4.6) we find that

\[
W^0 := \{ I : 0 \leq I \leq \frac{2}{(2n)^\beta (1 + \beta) \gamma} \} = \{ I : 0 \leq I \leq 2I_w \}
\]

where

\[
I_w = \arg \max_{I \in W^0} W^0 = \frac{1}{(2n)^\beta (1 + \beta) \gamma}.
\]
Proof of Proposition 3. Direct substitution of $\beta = 1$ yields $I^* = 5/(4n\gamma) = I_s$, from which the equality result follows immediately.

From propositions 1 and 2 the upper boundaries of the appropriate sets are simply double the corresponding $I$ value (with lower boundaries all zero). Hence it suffices to establish the ranking of the $I$’s. First comparing $I^*$ and $I_w$ from propositions 1 and 2:

\[ I^* = \frac{\beta 2^{1+\beta} + 1}{4^\beta n^\beta \gamma} \geq \frac{1}{(2n)^\beta (1+\beta) \gamma} = I_w \]

\[ \iff \beta 2^{1+\beta} + 1 \geq \frac{1}{2^\beta (1+\beta)} \]

\[ \iff (\beta 2^{1+\beta} + 1)(1+\beta) > 2^\beta. \]

which holds for all $\beta \geq 1$ since $\beta 2^{1+\beta} + 1 > 2^\beta$.

Comparing $I^*$ and $I_s$ from propositions 1 and 2:

\[ I^* = \frac{\beta 2^{1+\beta} + 1}{4^\beta n^\beta \gamma} \leq \frac{2\beta(1+\beta) + 1}{(2n)^\beta (1+\beta) \gamma} = I_s \]

\[ \iff 2\beta + \frac{1}{2^\beta} \leq 2\beta + \frac{1}{1+\beta}, \]

which holds because $\beta \geq 1$. ■

Proof of Lemma 1. Suppose $n$ firms cooperatively set a single price $p$ to maximize industry profits. The value of $p$ that solves this maximization problem is the unique collusive price.

As firms have identical costs and are evenly dispersed around the circle $C$, maximizing industry profits is equivalent to maximizing profit per firm. For a given $p$ and $t$, each firm’s output and profit respectively are

\[ q(p,t) = 2\min\{((A-p)/t)^{1\beta}, 1/2n\}, \]

\[ \pi(p,t) = (p-c)q(p,t), \]

where $\min\{((A-p)/t)^{1\beta}, 1/2n\} = (A-p/t)^{1\beta}$ if market is not completely covered and $1/2n$ otherwise.

First consider $p \in [A - \frac{t}{(2n)^\beta}, A]$ for which $q(p,t) = (A-p/t)^{1\beta}$ and consequently $\pi(p,t) = (p-c)(A-p/t)^{1\beta}$. We have that

\[ \frac{d\pi(p,t)}{dp} = -(1+\beta)(A-p)^{1\beta-1}[p - \beta A + c]/[1+\beta]. \]

We find that $A \geq c + \frac{t(1+\beta)}{(2n)^\beta} \Rightarrow A - \frac{t}{(2n)^\beta} > \frac{\beta A + c}{1+\beta} \Rightarrow p - \frac{\beta A + c}{1+\beta} > 0 \iff \frac{d\pi(p,t)}{dp} < 0$ for all $p \in [A - \frac{t}{(2n)^\beta}, A]$. This implies $p = A - \frac{t}{(2n)^\beta}$ maximizes $\pi(p,t)$ in the interval $p \in [A - \frac{t}{(2n)^\beta}, A]$. Now consider $p \leq A - \frac{t}{(2n)^\beta}$ for which $q(p,t) = \frac{1}{n}$. As $q(p,t)$ is not affected by $p$, $\pi(p,t)$ is maximized at the highest possible price that satisfy $p \leq A - \frac{t}{(2n)^\beta}$.
\[ A - \frac{t}{(2n)^\alpha} \] which is \( p = A - \frac{t}{(2n)^\alpha} \). Thus at any given \( t \), \( p = A - \frac{t}{(2n)^\alpha} \) maximizes \( \pi(p, t) \) which immediately implies that for all \( I \geq 0 \), \( A - \frac{t_0 - I}{(2n)^\alpha} \equiv p^c(I) \) is the unique collusive price. 

Substituting the collusive price \( p^c \) into equation (4.3) and simplifying we find that an individual \( y \) votes in favor of positive investment level if and only if the following holds

\[
-I\left[ \left( \frac{1}{2n} \right)^\beta - y^\beta \right] - \frac{\gamma I^2}{2} \geq 0.
\]

Since \( y \in [0, \frac{1}{2n}] \) we have that \( \left( \frac{1}{2n} \right)^\beta \geq y^\beta \) which in turn implies that no \( I > 0 \) satisfies the above inequality. Therefore all consumers are hurt by infrastructure improvements and hence \( R^0 = S^0 = \{0\} \) and \( I^* = I_s = 0 \). Notice the collusive price is just sufficient to ensure that the most distant (lowest surplus from consumption) consumers still purchase. Thus under collusion all consumers still purchase and hence the effects of infrastructure improvements on social welfare are the same as under competition just with a different distribution of benefits. Thus, as in proposition 2, \( W^0 \neq \{0\} \) and \( I_w > 0 \).

**Proof of Proposition 5:** The proof is immediate from the discussion preceding the proposition. 

**Proof of Proposition 6.** In the main text we have already proved that \( R^0 = S^0 = \{0\} \) and \( I^* = I_s = 0 \) (see between Lemma 7.1 and Proposition 6). That \( W^0 \supset \{0\} \) and \( I_w > 0 \) follows from noting that

\[
\frac{dW(I)}{dI} \bigg|_{I=0} = \frac{1}{(2n^m(0))^{\beta}(1+\beta)} - \frac{\beta(t_0 - I)}{2^\beta(1+\beta)(n^m(0))^{1+\beta}} \frac{d n^m(I)}{dI} \bigg|_{I=0},
\]

\[
= \frac{1}{(2n^m(0))^{\beta}(1+\beta)} - \frac{\beta K}{1+\beta} \frac{d n^m(I)}{dI} > 0,
\]

where the inequality follow from the fact that \( \frac{dn^m(I)}{dI} < 0 \) (Lemma 7.1).


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