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On firm choice between online and physical markets*

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Abstract

Consumers buying goods online often cannot physically inspect the products prior to purchase. Thus an online market may turn what is usually regarded as a search good into an experience good. We investigate how this feature, together with other features of the marketplace, affects a firm’s choice between online and physical markets. Using a simple yet flexible framework, we show that the choice of a marketplace can be used to disclose or hide product quality. If the production cost is convex with respect to quality, the firm’s choice will be characterized by a cutoff quality level, below which the firm will choose the online market, and above which the firm will choose the physical market. However, if the production cost of quality is concave, there are situations where the highest qualities pool with the lowest ones in the online market, leaving the physical market to intermediate qualities.

Keywords: Online vs. physical markets, quality signalling, market choice
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1 Introduction

When consumers buy goods such as toys, clothing, and furniture from an online market, they cannot inspect the product’s quality prior to purchase. In such cases what is usually regarded as a search good in a physical market is, at least partially, turned into an experience good in the online market. In this paper we investigate on the demand side how this feature affects consumers’ perceptions about product quality in online and physical markets, and on the supply side how it affects a firm’s choice between the marketplaces.

We conducted the study in three steps. First, we developed a benchmark model, where a firm chooses between an online market and a physical market. The product’s quality is observable to consumers only if it is sold in the physical market. We showed that the firm’s choice is characterized by a cutoff quality level above which it chooses the physical market and below which it sells online. The benchmark model is flexible and can be extended to incorporate more features of the markets. We then extended the model to allow for an online consumer review

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system and a larger online market. We showed that both features lead to a higher cutoff quality than that of the benchmark model. That is, they attract higher quality into the online market.

Last, based on the first two steps, we developed the full model, where the production cost depends on product quality. We showed that to choose the marketplace the firm weighs the online benefit, which is the benefit from choosing the online market over the physical market, and the online cost, which is the relative cost to switch from the physical market to online. The online benefit consists of three parts. First, the firm can benefit from pooling lower qualities with higher qualities and selling them at a price higher than it could command otherwise. Second, the firm pays a lower fixed cost to sell online than it would have if it were using the physical market. Third, the firm can serve a larger market when it goes online, and this is more attractive to higher qualities than to lower ones because the consumer reviews allow them to price higher once the qualities are revealed.

In the full model, we studied the firm’s choice in two situations, first with the production cost of quality being strictly convex, and second with the cost being strictly concave. The convex case can be justified by the law of diminishing returns, but we argue that it is also possible for the production cost of quality to be concave. For example, for fashion products such as clothing and perfume, the quality may depend more on the design than on the amount of materials that are used in production, so it is possible that, with a creative idea that arrives as a random shock, the quality improves significantly while the actual cost increases only slightly.

We showed that if the production cost of quality is convex, then the main results in the previous two steps carry through. However, if the cost of quality is concave, then there are situations where high-end qualities pool with low-end ones in the online market, leaving the physical market to intermediate qualities. Firms purveying low-end qualities choose the online market for reasons different from those of purveyors of high-end qualities. Firms purveying low qualities are driven by the incentive to pool them with higher qualities and to save the fixed costs. However, for high qualities, as the rate of increase of the online cost drops, the online benefit that stems from the larger consumer base and consumer reviews keeps increasing and finally outweighs the online cost, thus driving high-end qualities to also choose the online market.

This paper is related to the theoretical literature on the link between online and offline business, especially those on the impact of consumer demand, including Dinlersoz and Pereira (2007), Kocas and Bohlmann (2008), and Loginova (2009). While these studies take into account that consumers buying online cannot inspect products before purchase, they place firms’ marketplaces exogenous to product quality. By contrast we endogenize the firm’s market choice with respect to product quality. The analytical framework that we use is simple and flexible, allowing us to

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1 See Lieber and Syverson (2011) for a comprehensive review of the literature on online vs. offline competition.
2 Dinlersoz and Pereira (2007) study physical retailers’ adoption of e-commerce in a technology-adopt-race framework where some customers have loyalty for particular firms while others buy from the lowest-price firm. Using another model with loyal consumers versus price-sensitive "switchers", Kocas and Bohlmann (2008) study price dispersion between firms with homogeneous products. Loginova (2009) studies the strategic interactions between online and offline markets in a Salop (circular city) model.
3 In Dinlersoz and Pereira (2007) the online good’s quality differs from the offline good by an exogenous constant. In Loginova (2009) firms’ choice of market type is also exogenous to the product quality, which is identical across firms. Consumers’ uncertainty about an online product’s quality is modelled via her uncertainty about her own type, which determines how well the product fits her and before purchase can only be found out by visiting a physical store.
examine the features of the marketplaces in sequence. The theme is thus closer to that of Jin and Kato (2007). Whereas they develop a model for a specific case study on sports cards, we aim at a more general framework that is flexible enough to be applied to a wider range of products. Moreover, our finding in the full model may contrast with their results, which predict that online goods have lower quality than those sold in the physical market.\footnote{Jin and Cato (2007) suggest that "[sport] cards sold in the online graded segment must have quality no worse than those sold in the retail ungraded segment, and card quality sold in these two segments must be no worse than those sold online as ungraded."}

We have adopted the analytical framework from the literature of voluntary information disclosure (see Milgrom 2008), which shows that when disclosure is costly, only sellers with product quality above a threshold will disclose it (Grossman and Hart 1980; Jovanovic 1982). While the literature is focused on a single market, in this paper we show that with multiple marketplaces, the market choice per se can be used to (costly) disclose or hide product quality. While the result from our benchmark model is akin to the standard result, the change from a single market to multiple markets allows us to arrive at a contrasting result, where the highest qualities pool with the lowest ones in the online market.

The rest of the paper is organized as follows. We develop the benchmark model in section 2 before we investigate the effects of market size and consumer reviews in section 3. We study the full model with the production cost of quality in section 4. Section 5 is the conclusion. The appendix contains proofs of a lemma and two propositions.

2 Benchmark Model

2.1 Set-up

A firm produces a product, of which the quality $q$ is a realization from a uniform distribution on $[0, \bar{q}]$. The firm observes the product’s quality, and then chooses between two markets in which to sell the product: an online market and a physical market. Meantime the firm announces a price $p$. We assume that the fixed cost of selling on the physical market, including rent and utility expenses, is larger than the fixed cost of selling online. For simplicity, we assume the fixed cost for the physical market is $F$, and the fixed cost for the online market is zero. The marginal cost of production is constant and normalized to 0.

There is a unit mass of consumers with unitary demand. Consumers can verify $q$ prior to purchase if the product is sold in the physical market, but they can only find out $q$ after purchase if they buy from the online market. If consumers buy the product at the price $p$, they derives utility $u(q, p) = q - p$ from consumption. Consumers have an outside option of not buying the product, which generates utility 0.

For notational clarity, we use the letters $v$ and $r$ to distinguish virtual (online) and real (physical) variables, e.g., $p^v$ and $p^r$ referring to the virtual market and the real market, respectively.

The firm’s strategy thus is to choose a marketplace and a price given its product quality. The strategy can be represented by a function $S : [0, \bar{q}] \to \{r, v\} \times [0, \infty)$. Consumers’ strategy is to decide if they will buy at the announced price given the firm’s marketplace and the observability of product quality. Since the situation is essentially a dynamic game with incomplete information,
we use Perfect Bayesian Equilibrium (PBE) as the solution concept, which consists of a strategy profile and a consumer-belief system about the product quality such that each agent’s strategy is sequentially rational with respect to the belief system, and the belief system is consistent with the strategy profile.

2.2 Analysis

If the firm sells the product on the physical market and sets price $p^r$, since consumers can observe $q$, they will buy the product if the quality is greater than or equal to the price, i.e. $q \geq p^r$. Therefore, if the firm sells in the physical market, it will set $p^r = q$.

In equilibrium denote the set $Q \subset [0, \hat{q}]$ such that the firm will choose the online market if $q \in Q$. Denote $Q = E[q | q \in Q]$. In the online market, since consumers cannot observe $q$ prior to purchase, they will buy the product if and only if the expected utility from purchase is non-negative, i.e., $p^v \leq Q$. Consequently, in equilibrium if it sells online the firm will set $p^v = Q$.

Because of $F$, we can rule out the possibility of an equilibrium where the firm always chooses the physical market: If the firm chooses the physical market for all $q$, then at $q < F$ the firm’s profit is $q - F < 0$, and it can be better off by moving to the online market: regardless of the consumers’ off-equilibrium-path belief, the minimum profit there is 0.

Next, the lemma below will facilitate our search for the firm’s equilibrium decision on marketplaces.

**Lemma 1** In equilibrium if the firm with a quality $\hat{q}$ chooses the online market, then it will choose the online market for all $q < \hat{q}$.

**Proof.** Suppose the firm with $\hat{q}$ chooses the online market, then it must be that $p^v \geq p^r - F$, i.e. $p^v \geq \hat{q} - F$, which implies that $p^v > q - F$ for all $q < \hat{q}$, and thus for all $q < \hat{q}$ the firm will choose the online market. ■

Following the above analysis, we can restrict our attention to the equilibrium where there exists a cutoff quality $q^*$ such that the firm will choose the online market for all $q \leq q^*$, and the physical market for all $q > q^*$.

**Proposition 1** (i) If $F < \hat{q}/2$, then in equilibrium the firm will choose the online market for $q \leq 2F$, and the physical market for $q > 2F$.

(ii) If $F \geq \hat{q}/2$, then in equilibrium the firm will always choose the online market.

**Proof.** Suppose there exists a $q^* < \hat{q}$ such that the firm will choose the online market for $q \leq q^*$, and the physical market for $q > q^*$. Then $p^v = E[q | q \leq q^*] = q^*/2$. Then we can derive $q^*$ using the condition that the firm will be indifferent between the two markets at $q = q^*$:

\[ p^v(q^*) = p^r(q^*) - F, \]

which leads to

\[ q^*/2 = q^* - F, \]
which implies

\[ q^* = 2F \]

Thus if \( 2F < \tilde{q} \), then in equilibrium the firm will choose the online market for \( q \leq 2F \), and the physical market for \( q > 2F \). If \( 2F \geq \tilde{q} \), then in equilibrium the firm will always choose the online market.

The intuition of Proposition 1 is as follows: in equilibrium when \( F \) is low, the firm with high qualities (\( q > 2F \)) will choose the physical market, since this allows the firm to signal its quality and thus charge a higher price than \( p^v = F \) in the online market. For low qualities (\( q \leq 2F \)), however, there are two forces that drive the firm to choose the online market: For very low qualities (\( q < F \)), choosing the online market allows the firm to pool the lower qualities with the higher and thus sell at a price higher than the low quality would otherwise command. For the intermediate qualities (\( F \leq q \leq 2F \)), however, the online price is lower than the quality. In this case the incentive to avoid the higher fixed cost in the physical market drives the firm to choose the online market.

The result in Proposition 1 is akin to a result that is standard in the literature of costly information disclosure (see Milgrom 2008): when disclosure is costly, only firms with high-quality products reveal information to consumers. The literature, however, focuses on firms in a single market, whereas this paper studies firms’ choices between multiple marketplaces. Proposition 1 therefore contributes to the literature by showing that the choice of marketplace per se can be used to disclose or hide product quality.

3 Consumer Reviews and Market Size

In this section we extend the benchmark model to incorporate two more features of the marketplaces: First, consumers may publicly review online products’ quality and thus provide information to future customers. To capture this we extend the model to a two-period setting; in Period 1 online consumers review the product quality after purchase. Second, to physically inspect the product, consumers have to be geographically close to the physical market, while the online market can serve consumers in a much larger area. To incorporate this feature, we assume that the consumer base of the online market is larger than that of the physical market.

3.1 Setup

Time is discrete with two periods \( t = 1, 2 \). The firm’s technology and product quality are given at the beginning of period 1 as described in the benchmark model and fixed in both periods. The firm will choose between online and physical markets at the first period and will use the same marketplace in the second period. After choosing the market, the firm decides the price \( p \) and quantity \( x \) in each period. That is, the firm can change the price across periods. For simplicity we assume no discounting of future payoffs. For clarity we will use the subscript \( t = 1, 2 \) on variables, such as \((p_1, p_2)\) and \((x_1, x_2)\) to denote the corresponding periods.

Each period there is a continuum of consumers, who are active for one period and have the same preference as in the benchmark model. The size of the consumer base of the physical market
is $\alpha$, with $0 < \alpha < 1$, while the size of the online market is 1.

At the end of period 1, there exists an online rating system for the first-period online consumers to review product quality. The period-2 consumers read the reviews before purchase. While it seems straightforward to assume that consumers simply reveal $q$ when providing feedbacks, in reality a consumer’s review may reflect a mixture of objectivity and subjectivity. The objective reviews describe the attributes of the product, but the subjective reviews may be affected by the consumers’ personalities and emotions; they may thus be biased with respect to the product quality. In aggregation, however, the subjective opinions may cancel each other out as the number of reviewers increases. In other words, the accuracy of the rating system should increase with the number of reviewers. Cabral and Hortacsu (2010) show that the frequency of reviews provides a good proxy for frequency of transactions. Hence it is reasonable to assume that the probability that $q$ is revealed in the second period is represented by a function $(x_v^1)$, where $(0) = 0$, $(1) = 1$, and $(0) > 0$. With probability $1/(x_v^1)$ the rating system is uninformative and thus period-2 consumers hold the same expectation about the product’s quality as their counterpart prior to purchase in period 1. For simplicity we assume $(x_v^1) = x_v^1$.

### 3.2 Analysis

If the firm sells on the physical market, then its pricing strategy in both periods will be the same as in the benchmark model, i.e. $p_v^1(q) = p_v^2(q) = q$.

If the firm sells online, Period 2 consumers, given $(x_v^1) = x_v^1$, will expect the quality of the online product to be $x_v^1q + (1 - x_v^1)p_v^1$. Therefore if the firm chooses the online market, in period 2 it will set the price $p_v^2 = x_v^1q + (1 - x_v^1)p_v^1$. Hence the profit of an online firm with quality $q$ is

$$\psi(x_v^1, x_v^2; q) = p_v^1x_v^1 + (x_v^1q + (1 - x_v^1)p_v^1)x_v^2.$$  

From the expression of $\psi(x_v^1, x_v^2; q)$ we can see that an online firm will optimally set $x_v^2 = 1$, which then implies that it is optimal to set $x_v^1 = 1$. This suggests that in equilibrium the online rating system will truthfully reveal the product quality, which implies $p_v^2 = q$.

Analogous to the argument in the benchmark model, we can rule out the possibility of an equilibrium in which the firm will always choose the physical market. We can similarly show that Lemma 1 also holds in the current setting, and thus we can restrict our attention to the equilibrium where there is a cutoff quality, below which the firm will choose the online market and above which the firm will choose the physical market.

**Proposition 2** If $\alpha > 3/4$ and $4F_{\alpha-3} < \bar{q}$, then the firm will choose the online market for $q \leq 4F_{\alpha-3}$, and the physical market for $q > 4F_{\alpha-3}$; otherwise the firm will always choose the online market.

**Proof.** Suppose, with some abuse of notations, there exists a $q^* < \bar{q}$ such that the firm will choose the online market for $q \leq q^*$, and the physical market for $q > q^*$. Thus $p_v^1 = E[q|q \leq q^*] = q^*/2$. Then we can derive $q^*$ using the condition that the firm will be indifferent between the two markets at $q = q^*$:

$$p_v^1(q^*) + p_v^2(q^*) = 2\alpha p^*(q^*) - 2F,$$
which leads to
\[ \frac{q^*}{2} + q^* = 2\alpha q^* - 2F, \]
which implies
\[ q^* = \frac{4F}{4\alpha - 3}. \]
Thus if \( \alpha > 3/4 \) and \( \frac{4F}{4\alpha - 3} < \bar{q} \), then the firm will choose the online market for \( q \leq \frac{4F}{4\alpha - 3} \), and the physical market for \( q > \frac{4F}{4\alpha - 3} \). While if \( \alpha \leq 3/4 \) or \( \frac{4F}{4\alpha - 3} \geq \bar{q} \), then in equilibrium the firm will always choose the online market. \( \blacksquare \)

Proposition 2 indicates that both the larger number of consumers and the consumer reviews help the online market attract higher qualities in this model than in the benchmark model. To see the effect of consumer reviews, note that even if the two markets have the same size, i.e. \( \alpha = 1 \), the cutoff quality level increases from \( 2F \) in Proposition 1 to \( 4F \) in Proposition 2. That is, firms with \( q \in [2F, 4F] \) now switch from the physical market to the online market. This is because in period 2 the consumer review allows the firm to reveal the quality, so it can sell at the same price as in the physical market, yet avoid paying the higher fixed cost.

To see the impact of the market size, note that the cutoff quality level \( \frac{4F}{4\alpha - 3} \) in Proposition 2 decreases with \( \alpha \). In other words, the larger the relative size of the online market, the higher the cutoff quality that will choose that market. The result is also intuitive: the smaller physical market limits demand for the firm’s products, so in Period 1 firms with higher qualities will have to sell at a price lower than appropriate for their products’ quality, so the online market becomes preferable.

4 Production Cost of Quality

Previous sections assume that production cost is independent of product quality, and the key result is that only products with low quality are sold online. By relaxing this assumption, this section shows that if the production cost is convex with respect to quality, the main results in the previous sections carry through. However, if they are concave, then there are situations where the highest qualities pool with the lowest ones in the online market, leaving the physical market to intermediate qualities.

4.1 Setup

Based on the two-period model in the previous section, now suppose that in each period the production cost associated with quality \( q \) is denoted by \( C(x, q) = C(q)x \). Thus the marginal cost of production is \( C(q) \), which we will call "production cost of quality". We assume that \( C'(q) > 0 \), implying that the marginal cost increases as product quality improves. Moreover, we assume that \( q > C(q) \) for \( q > 0 \) and \( C(0) = 0 \), so that it is socially desirable to produce the good.
4.2 Analysis

First, if the firm chooses the physical market, then its total profit will be

$$\pi^r(q) = 2\alpha(q - C(q)) - 2F;$$

if the firm chooses the online market and sells at price $p^v_1$ in period 1, its total profit will be

$$\pi^v(q) = \max_{x^v_1, x^v_2} \psi(x^v_1, x^v_2; q)
\text{ s.t. } x^v_1 \in [0, 1], x^v_2 \in [0, 1],$$

where, as in the previous section,

$$\psi(x^v_1, x^v_2; q) = \{(p^v_1 - C(q))x^v_1 + [(x^v_1q + (1 - x^v_1)p^v_1) - C(q)]x^v_2\}.$$

The firm with quality $q$ will choose the online market if $\pi^v(q) \geq \pi^r(q)$.

**Lemma 2** 1) If the firm chooses to sell online, it will choose $x^v_1^* = x^v_2^* = 1$, and so its quality will be fully revealed in the second period.

2) The firm with quality $q$ chooses the online market iff

$$0 < 2\alpha(q - C(q)) - 2F \leq [p^v_1 - C(q)] + [q - C(q)].$$

(1)

**Proof.** See Appendix. ■

We will study the firm’s choice in two situations: (i) $C'' > 0$, i.e. $C$ is strictly convex, and (ii) $C'' < 0$ (strictly concave). The strict convexity of $C$ is consistent with the standard textbook law of diminishing returns. However, since $C$ is the production cost of quality, we argue that it is also compelling for $C$ to be concave. For example, the quality of products such as clothing, furniture, and perfume, may depend more on the conceptual design than on the amount of materials actually used in production[5] So it is possible that, with a good idea that occurs as a random shock, there is a significant improvement in quality while the actual cost increases only modestly.

As in the benchmark model, denote $Q$ as the set of qualities at which the firm will choose the online market. The proposition below shows that if $C$ is strictly convex then the results in Propositions 1 and 2 extend to the current setting.

**Proposition 3** If $C''(q) > 0$, then there exists a unique $q^* \in (0, q]$ such that $Q = [0, q^*].$

**Proof.** In the Appendix. ■

To see the intuition of Proposition 3, note that the second inequality of (1) can be written as

$$p^v_1^* + (1 - 2\alpha)q + 2F \geq 2(1 - \alpha)C(q).$$

(2)

[5] For examples of fashion clothing, see a New York Magazine article in the References. Also cited there is a newspaper Daily Mail article for examples of perfume.
The left side of (1) represents the "online benefit", which is the benefit of choosing the online market over the physical market, while the right side measures the "online cost". The online benefit consists of three parts. First, for very low qualities \((q < p^1)\), in Period 1 the firm is able to sell at a price higher than otherwise warranted by the quality of the product. Second, \((1 - 2\alpha)q\) indicates the additional profit the firm may make from the online market, which, as discussed in the previous section, arises from the larger online consumer base and the consumer reviews. Last, by choosing the online market the firm pays less fixed cost, which is \(2F\) in total.

On the other hand, the online cost is \(2(1 - \alpha)C(q)\), because with the marginal cost of production \(C(q)\), the larger consumer base of the online market implies that the firm incurs higher total production cost.

Figures 1 and 2 illustrate the online benefit and the online cost, corresponding to \(\alpha < 1/2\) and \(\alpha > 1/2\). When \(\alpha < 1/2\), the total demand in the physical market, \(2\alpha\), is less than the period-2 demand in the online market. Since in period 2 the online firm can charge the same price as in the physical market, \(\alpha < 1/2\) implies that the online benefit is strictly positive and increases with \(q\). In contrast, when \(\alpha > 1/2\) the online benefit decreases with \(q\).

The main results in Proposition 1 and 2 carry through when \(C\) is strictly convex, as shown in Figures 1 and 2. In both cases, there exists a cutoff quality \(q^*\), such that the firm will choose the online market when \(q \leq q^*\), and higher-quality products are sold in the physical market. The intuition is as follows: The online benefit is a linear function of \(q\), whereas \(C'' > 0\) implies that the online cost increases fast as \(q\) goes up. Consequently for all qualities above \(q^*\), the online cost exceeds the online benefit. In other words, for high qualities the larger consumer base and the lower fixed cost of the online market cannot offset the increasingly higher production cost. Therefore the firm with \(q > q^*\) chooses the physical market.

In contrast to Propositions 1 to 3, if \(C\) is strictly concave then it is possible that not only the lowest qualities but also the highest qualities will choose the online market, leaving the physical market to the intermediate qualities, as Proposition 4 shows.

**Proposition 4** Suppose \(C''(q) < 0\), then \(Q = [0, \bar{q}] \cup [\tilde{q}, \tilde{q}]\) with \(0 < \tilde{q} < \tilde{q} < \bar{q}\) if and only if (i)
\[ p_{1*}^v = \frac{(\bar{q}^2 + q^2 - \bar{q}^2)}{2(\bar{q} + \bar{q} - q)} \] and (ii) \[ p_{1*}^v + (1 - 2\alpha)q + 2F = 2(1 - \alpha)C(q) \] at \( q = \bar{q} \) and \( q = \hat{q} \).

**Proof.** In the Appendix. 

In Proposition 4 the two sets of qualities \([0, \bar{q}]\) and \([\hat{q}, \bar{q}]\) choose the online market for different reasons. At \( q = 0 \), the online benefit is \( p_{1*}^v + 2F \) while the online cost is 0. So the firm is purely driven by the incentive to pool with higher qualities and to save the fixed cost. Analogous to the case where \( C \) is strictly convex, as \( q \) increases the online cost catches up, and beyond \( \bar{q} \) it starts exceeding the online benefit. However, as shown in Figure 3, since \( C \) is strictly concave, the rate of increase of \( C \) keeps dropping. Meanwhile, when the physical market is small (\( \alpha < 1/2 \)), the online benefit that stems from the larger consumer base and the consumer reviews, i.e. \((1 - 2\alpha)q\), increases at a constant rate. Above \( \bar{q} \) the attraction of the larger consumer base in the online market is high enough to outweigh the online cost, hence the firm switches to the online market even though it has to pool with lower qualities in the first period. Moreover it can be shown that, in contrast to the lowest qualities, for qualities in \([\hat{q}, \bar{q}]\) the firm necessarily makes a loss in the first period. That is, within \([\hat{q}, \bar{q}]\) not only \( q > p_{1*}^v \) but also \( C(q) > p_{1*}^v \). Therefore with the highest qualities the firm’s market choice is purely driven by the prospect of selling to a larger consumer base in period 2.

![Figure 3](image-url)  
**Figure 3:** \( C''(q) < 0, \alpha < \frac{1}{2} \)

**Example 1** Suppose \( C(q) = \ln(q + 1) \), \( \alpha = 0.3 \), \( \bar{q} = 5.3415 \) and \( F = 1.5641 \times 10^{-2} \). Applying conditions (i) and (ii) in Proposition 4, we have \( \hat{q} = 0.6733 \) and \( \bar{q} = 5.33 \), and thus \( p_{1*}^v = (\bar{q}^2 + q^2 - \bar{q}^2) / [2(\bar{q} + \bar{q} - q)] = 0.4206 \).
5 Conclusion

When consumers buy products from an online market, they usually cannot physically inspect the products prior to purchase. With this in mind, we have endogenized the firm’s market choice with respect to product quality, adopting the analytical framework from the literature of information disclosure and extending it to a multiple-market setting. We show that market choice per se can be used to disclose or hide information about quality. If the production cost of quality is strictly convex, the firm’s choice of marketplace will be characterized by a cutoff quality level, but if the production cost of quality is strictly concave, the online market may attract both high-end and low-end products. Endogenizing the firm’s choice of product quality has been shown to be worthwhile, and the flexibility of the analytical framework not only allows us to examine the features of the marketplaces in sequence, it shows the potential for undertaking further studies on the links between virtual and physical markets. For example, the model can be extended to allow the firm to choose online, physical, or both markets, and even to an oligopolistic setting for studying competition between online and physical stores.

6 Appendix

6.1 Proof of Lemma 2

Proof. Note

\[
\frac{\partial \psi(x_1^v, x_2^v; q)}{\partial x_1^v} = p_1^v - C(q) + x_2^v(q - p_1^v)
\]

\[
\frac{\partial \psi(x_1^v, x_2^v; q)}{\partial x_2^v} = p_1^v + (q - p_1^v)x_1^v - C(q)
\]

\[
= q - C(q) + (1 - x_1^v)(p_1^v - q)
\]

and that the Hessian Matrix is given by

\[
\begin{pmatrix}
0 & q - p_1^v \\
q - p_1^v & 0
\end{pmatrix}
\]

Since \(-(q - p_1^v)^2 < 0 \) when \( q \neq p_1^v \), the problem has no interior maximizer if \( q \neq p_1^v \).

Now we discuss the solutions in different cases.

**Case 1: \( C(q) < p_1^v \)**

First, note \( \frac{\partial \psi(x_1^v, x_2^v; q)}{\partial x_2^v} > 0 \), so \( x_2^v = 1 \). Hence, \( \frac{\partial \psi(x_1^v, x_2^v; q)}{\partial x_1^v} = q - C(q) > 0 \), and \( x_1^v = 1 \).

**Case 2: \( p_1^v < C(q) \)**

We first prove that \( x_2^v \in (0, 1) \) is not possible. If this is true, \( \frac{\partial \psi(x_1^v, x_2^v; q)}{\partial x_1^v} = q - C(q) + (1 - x_1^v)(p_1^v - q) = 0 \), and this expression implies \( x_1^v \in (0, 1) \) since \( q - C(q) > 0 \) and \( p_1^v - C(q) < 0 \).

But as we already show, there is no interior maximizer if \( q \neq p_1^v \) since the second order condition cannot be satisfied for any interior point.
Hence, either \( x_{2}^{*} = 1 \) or \( x_{2}^{*} = 0 \) holds. If \( x_{2}^{*} = 1 \), then \( \frac{\partial \psi(x_{1}^{*},x_{2}^{*};q)}{\partial x_{1}} = q - C(q) > 0 \), so \( x_{1}^{*} = 1 \); if \( x_{2}^{*} = 0 \), \( \frac{\partial \psi(x_{1}^{*},x_{2}^{*};q)}{\partial x_{1}} = p_{1}^{*} - C(q) < 0 \), so \( x_{1}^{*} = 0 \). Thus, we only need to compare \( \psi(1, 1; q) \) and \( \psi(0, 0; q) \). If \( \psi(1, 1; q) \geq \psi(0, 0; q) \), which is equivalent to \( p_{1}^{*} \geq C(q) - (q - C(q)) \), then \( x_{1}^{*} = x_{2}^{*} = 1 \); if \( \psi(1, 1; q) < \psi(0, 0; q) \), which is equivalent to \( p_{1}^{*} < C(q) - (q - C(q)) \), then \( x_{1}^{*} = x_{2}^{*} = 0 \).

The case \( p_{1}^{*} < C(q) - (q - C(q)) \) can never happen because then the firm makes zero profit by selling online, which is strictly dominated by selling on the real market, where the firm can make strictly positive profit.

**Case 3: \( p_{1}^{*} = C(q) \)**

When \( p_{1}^{*} = C(q) \), \( \frac{\partial \psi(x_{1}^{*},x_{2}^{*};q)}{\partial x_{1}} = x_{2}^{*}(q - p_{1}^{*}) \) and \( \frac{\partial \psi(x_{1}^{*},x_{2}^{*};q)}{\partial x_{2}} = (q - p_{1}^{*})x_{1}^{*} \). Since \( q > C(q) = p_{1}^{*} \), the maximizer is given at \( x_{1}^{*} = 1 \) and \( x_{2}^{*} = 1 \).

The lemma follows immediately from the above discussions.

### 6.2 Proof of Proposition 3

**Proof.** First, we show that in equilibrium if \( Q \) is non-empty, then it is convex: Denote the function

\[
G(q) = p_{1}^{*} + q - 2C(q) - \{2\alpha[q - C(q)] - 2F\} \\
= p_{1}^{*} + (1 - 2\alpha)q + 2F - 2(1 - \alpha)C(q),
\]

where \( p_{1}^{*} \) denotes the equilibrium period 1 online price. Since \( G''(q) = -2(1 - \alpha)C''(q) < 0 \), \( G(q) \) is strictly concave. Note that \( q \in Q \) if and only if \( G(q) \geq 0 \). Pick two arbitrary \( q_{1} \) and \( q_{2} \) from \( Q \). For any \( \theta \in (0, 1) \), the strict concavity of \( G \) implies that \( G(\theta q_{1} + (1 - \theta)q_{2}) > \theta G(q_{1}) + (1 - \theta)G(q_{2}) \geq 0 \). Therefore \( \theta q_{1} + (1 - \theta)q_{2} \in Q \). Hence \( Q \) is convex.

Second, since \( G(0) = p_{1}^{*} + 2F > 0 \), we have \( 0 \in Q \). Then the convexity of \( Q \) implies that if \( Q \) is not a singleton, there exists a \( q^{*} \in (0, \bar{q}) \) such that \( q \in Q \) if \( q < q^{*} \). Consequently, \( p_{1}^{*} = \frac{1}{2}q^{*} \).

So next we prove the existence and uniqueness of \( q^{*} \).

Denote

\[
H(q) = \frac{1}{2}q + q - 2C(q) - \{2\alpha[q - C(q)] - 2F\} \\
= \frac{3}{2} - 2\alpha)q + 2F - 2(1 - \alpha)C(q).
\]

Note that \( H(q) \) essentially replaces \( p_{1}^{*} \) in \( G(q) \) by \( \frac{1}{2}q \). There are

\[
H'(q) = \frac{3}{2} - 2\alpha - 2(1 - \alpha)C'(q), \\
H''(q) = -2(1 - \alpha)C''(q) < 0, \\
H(0) = 2F > 0
\]

Below we break the rest of the proof into two cases: (i) \( H'(0) = \frac{3}{2} - 2\alpha - 2(1 - \alpha)C'(0) \leq 0 \), and (ii) \( H'(0) > 0 \).
If \( H'(0) \leq 0 \), i.e. \( C''(0) \geq (\frac{3}{2} - 2\alpha)/(2(1 - \alpha)) \), then \( H''(q) < 0 \) implies that \( H'(q_2) < H'(q_1) \) for all \( q_2 > q_1 > 0 \) (i.e. \( H \) is decreasing at an increasing rate). Since \( H(0) = 2F > 0 \), the Intermediate Value Theorem implies that there exists a unique \( q^{**} > 0 \) such that \( H(q^{**}) = 0 \). Hence if \( q^{**} < \tilde{q} \), then \( Q = [0, q^{**}] \) with \( q^* = q^{**} \); if \( q^{**} \geq \tilde{q} \), then \( Q = [0, q^{*}] \) with \( q^* = \tilde{q} \).

If \( H'(0) > 0 \), i.e. \( C''(0) < (\frac{3}{2} - 2\alpha)/(2(1 - \alpha)) \), then since \( C''(q) > 0 \), there are two possibilities:
(a) For all \( q > 0 \), \( C'(q) < (\frac{3}{2} - 2\alpha)/(2(1 - \alpha)) \), i.e., \( H'(q) > 0 \), and thus \( H(q) > 0 \) for all \( q \). In this case, the firm will always choose the online market, and so \( Q = [0, q^{*}] \) with \( q^* = \tilde{q} \). (b) There exists a unique \( q^{*} > 0 \) such that \( C'(q^{*}) = (\frac{3}{2} - 2\alpha)/(2(1 - \alpha)) \), i.e. \( H'(q^{*}) = 0 \). Since \( H(0) = 2F > 0 \) and \( C'(q) < (\frac{3}{2} - 2\alpha)/(2(1 - \alpha)) \) for all \( q < q^{*} \), \( H(q^{*}) > 0 \). Then because \( H(q^{*}) > 0 \), \( H'(q^{*}) = 0 \) and \( H''(q) < 0 \), the Intermediate Value Theorem implies that there exists a unique \( q^{***} > q^{*} \) such that \( H(q^{**}) = 0 \). Hence if \( q^{**} < \tilde{q} \), then \( Q = [0, q^{**}] \) with \( q^* = q^{**} \); if \( q^{**} \geq \tilde{q} \), then \( Q = [0, q^{*}] \) with \( q^* = \tilde{q} \). \( \blacksquare \)

### 6.3 Proof of Proposition 4

**Proof.** [If part] Suppose (i) \( p_1^{**} = (\tilde{q}^2 + \tilde{q}^2 - \tilde{q}^2)/2(\tilde{q} + \tilde{q} - \tilde{q}) \) and (ii) \( p_1^{**} + (1 - 2\alpha)q + 2F = 2(1 - \alpha)C(q) \) at \( q = \tilde{q} \) and \( q = \tilde{q} \) with \( 0 < \tilde{q} < \tilde{q} < \tilde{q} \).

As introduced in the proof of the previous proposition, denote the function
\[
G(q) = p_1^{**} + q - 2C(q) - \{2\alpha[q - C(q)] - 2F\} = p_1^{**} + (1 - 2\alpha)q + 2F - 2(1 - \alpha)C(q),
\]
thus \( q \in Q \) if and only if \( G(q) \geq 0 \). Note that now \( G(q) \) is strictly convex because \( G''(q) = -2(1 - \alpha)C''(q) > 0 \).

Below we will show that (i) for \( q \in (\tilde{q}, \tilde{q}) \), \( G(q) < 0 \), and (ii) for \( q \in [0, \tilde{q}] \cup [\tilde{q}, \tilde{q}] \), \( G(q) \geq 0 \).

First, for every \( q \in (\tilde{q}, \tilde{q}) \) there exists a \( \theta \in (0, 1) \) such that \( q = \theta\tilde{q} + (1 - \theta)\tilde{q} \). The strict convexity of \( G \) implies that \( G(\theta\tilde{q} + (1 - \theta)\tilde{q}) < \theta G(\tilde{q}) + (1 - \theta)G(\tilde{q}) \). For every \( q \in (\tilde{q}, \tilde{q}) \), there exist a \( \theta \in (0, 1) \) such that \( q = \theta\tilde{q} + (1 - \theta)\tilde{q} \).

Second, pick an arbitrary \( q^# \in (\tilde{q}, \tilde{q}) \). For every \( q \in [0, \tilde{q}] \), there exist a \( \theta \in (0, 1) \) such that \( q = \theta\tilde{q} + (1 - \theta)\tilde{q} \). The strict convexity of \( G \) then implies that \( G(q) = G(\theta q + (1 - \theta)\tilde{q}) < \theta G(q) + (1 - \theta)G(\tilde{q}) \), which, since \( G(\tilde{q}) = 0 \) and \( G(q^#) < 0 \), implies that \( \theta G(q) > -(1 - \theta)G(q^#) > 0 \). Hence \( G(q) > 0 \). Similarly, pick an arbitrary \( q^# \in (\tilde{q}, \tilde{q}) \). For every \( q \in (\tilde{q}, \tilde{q}) \), there exist a \( \theta \in (0, 1) \) such that \( q = \theta\tilde{q} + (1 - \theta)\tilde{q} \). The strict convexity of \( G \) then implies that \( G(q) = G(\theta q + (1 - \theta)\tilde{q}) < \theta G(q) + (1 - \theta)G(\tilde{q}) \), which, since \( G(\tilde{q}) = 0 \) and \( G(q^#) < 0 \), implies that \( \theta G(q) > -(1 - \theta)G(q^#) > 0 \). Hence \( G(q) > 0 \). Therefore, for \( q \in [0, \tilde{q}] \cup [\tilde{q}, \tilde{q}] \), \( G(q) \geq 0 \), and thus the firm will choose the online market.

[Only If part] Suppose \( Q = [0, \tilde{q}] \cup [\tilde{q}, \tilde{q}] \) with \( 0 < \tilde{q} < \tilde{q} < \tilde{q} \).

First, from the perspective of period-1 consumers, the expected value of the online good and thus their highest willingness to pay is
\[
E[q|q \in Q] = \frac{1}{\tilde{q} + \tilde{q} - \tilde{q}} (\int_{0}^{\tilde{q}} q dq + \int_{\tilde{q}}^{\tilde{q}} q dq) = \frac{\tilde{q}^2 + \tilde{q}^2 - \tilde{q}^2}{2(\tilde{q} + \tilde{q} - \tilde{q})}
\]
and thus in equilibrium the online firm will charge \( p_{1}^{*} = (\bar{q}^2 + \bar{q}^2 - \bar{q}^2)/2(\bar{q} + \bar{q} - \bar{q}) \).

Secondly, suppose toward a contradiction that at \( \bar{q} \), \( p_{1}^{*} + (1 - 2\alpha)\bar{q} + 2F \neq 2(1 - \alpha)C(\bar{q}) \), that is, \( G(\bar{q}) \neq 0 \). If \( G(\bar{q}) < 0 \), then the firm with \( \bar{q} \) has the incentive to deviate from the online market to the physical market, which is a contradiction to \( \bar{q} \in Q \). If \( G(\bar{q}) > 0 \), then the continuity of \( G \) implies that there exists an \( \varepsilon > 0 \) such that for \( \bar{q} + \varepsilon < \bar{q} \), \( G(\bar{q} + \varepsilon) > 0 \), contradicting that \( \bar{q} + \varepsilon \notin Q \). Therefore, it must be that \( G(\bar{q}) = 0 \).

Last, suppose toward a contradiction that at \( \bar{q} \), \( p_{1}^{*} + (1 - 2\alpha)\bar{q} + 2F \neq 2(1 - \alpha)C(\bar{q}) \), that is, \( G(\bar{q}) \neq 0 \). If \( G(\bar{q}) < 0 \), then the firm with \( \bar{q} \) has the incentive to deviate from the online market to the physical market, which contradicts \( \bar{q} \in Q \). If \( G(\bar{q}) > 0 \), then the continuity of \( G \) implies that there exists an \( \varepsilon > 0 \) such that for \( \bar{q} = \bar{q} - \varepsilon > \bar{q} \), \( G(\bar{q}) > 0 \), contradicting that \( \bar{q} - \varepsilon \notin Q \). Therefore, it must be that \( G(\bar{q}) = 0 \). □

**References**


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