Optimal inter-regional transfers with endogenous local public good prices and variable local rent capture*

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Abstract

We examine a model where regions pre-commit to the provision of local public goods. On observing their policies a central agency chooses an inter-regional transfer to maximise social welfare. Mobile labour takes regional and central policies as given and then makes its location choice to maximize its welfare. Distinguishing features of the model are that regional and central policies change local public good prices and we allow for a continuum of possibilities with respect to local rent capture. We show first that provision of local public goods is distorted as regions manipulate policies to influence the agency’s transfer choice. We then show that the welfare maximizing transfer should take account of two new wage and local public good price externalities which account for differences in local public good costs arising from cost disability factors such as density/dispersion, geography and socio-demographic features. Last we develop a sufficient condition which explains the circumstances under which the transfer should favour regions with relatively high costs of public good provision arising from these factors. In this sense we provide an efficiency case for so-called expenditure equalization which is a feature of inter-regional transfers observed in many countries.

Key Words: federalism, intergovernmental relations, inter-governmental differentials and their effects, federal state relations.

JEL: H73, H77.

1 Introduction

An extensive and long standing theoretical literature provides an efficiency case for inter-regional redistributive transfers. Its foundations were established by James Buchanan (1950; 1952) and Buchanan and Goetz (1972) who argued that if there are region-specific economic rents and fiscal

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externalities related to the migration of labour, Tiebout-type decentralized equilibria are not generally efficient. In a later article, Flatters et al. (1974) adopted a multi-state model with fiscal externalities and economic rents, welfare maximizing states and free labour migration to satisfy an equal utility condition. By comparing the first order necessary conditions for a decentralized equilibrium with those from a central planner problem they showed that the decentralized outcome is inefficient, providing an efficiency case for inter-state transfers. Boadway and Flatters (1982) provided a synthesis of these results and showed that a decentralized (Nash) equilibrium is efficient if a central authority can mandate an inter-state transfer that corrects for the migration effects of location specific externalities and economic rents. They derive an expression for the optimal (welfare maximizing) inter-state transfer which is now well-known. The inter-regional redistributive transfer is, in essence, the sum of the fiscal and economic rent externalities that distort migration decisions.

Theorists have adopted this important idea in many different and interesting contexts which tell us much about efficient inter-regional transfers. While a review of the full literature is beyond the aims of this paper, the following works are noted. Myers (1990) has argued that with incentive equivalence a decentralised Nash outcome is still efficient if states can make voluntary transfers, obviating the need for a centrally mandated transfer. Mansoorian and Myers (1993), Mansoorian and Myers (1997) and Wellisch (1994) incorporate imperfect labour mobility arising from attachment to place into the standard model used in the literature. This yields an expression for the transfer which also includes a term capturing the effects of attachment. Caplan et al. (2000) focus on the timing of moves by the centre, regions and mobile labour. They construct a three stage game with imperfect mobility where regions are Stackelberg leaders who pre-commit to voluntary contributions to a pure national public good (e.g. environmental policy). The centre then makes its transfer choice after observing regional policies. Boadway et al. (2003) also consider the efficiency of decentralised equilibria in a three stage context with various orders of moves; for example, with the centre able to pre-commit as a first mover, followed by regions and mobile labour. They also look at other combinations of timing, including where the centre can pre-commit by moving first (see Bordignon and Tabellini (2001), Koethenbuerger (2008) and Boadway and Tremblay (2010) for further results). A conclusion to emerge here is that the efficiency of equilibria is sensitive to the order of moves. In particular, if regions move first they have the opportunity to distort their policies to manipulate the transfer subsequently chosen by the centre. As observed by Bordignon and Tabellini (2001) ‘....in equilibrium both regional governments have an incentive to under tax or overspend, depending on the circumstances, to gain from the redistribution scheme at the expense of the other region.’ Informational issues related to inter-state transfers have also been examined in, for example, Cornes and Silva (2000; 2002; 2003). Transfers between regions have also been considered within the context of tax competition; for example, see Koethenbuerger (2002), Bucovetsky and Smart (2006) and Hindriks et al. (2008), while Albouy (2012) examines interregional transfers in a federation with differences in amenities, public/private productivities, federal taxes, and residential land.

From our observations, the models adopted in the literature, no matter the application,
always assume that the prices of public and private goods are fixed from the point of view of regions and the centre. This is in contrast to input markets where regions and the centre indirectly influence the prices of mobile inputs through their policy choices. An implication is that the price of private relative to local public goods is fixed, as is the relative price of local public goods between regions. When mobile factors move in response to policies the focus is on income effects and relative price changes derived from input markets (e.g. labour and capital).

Fixing private good prices may be reasonable. Such goods are traded across regional and national boundaries and have a world supply price which, from the point of view of regional and central policy makers - at least those concerned with transfers - can be assumed as given. The assumption of fixed local public good prices is more difficult to sustain. They are not generally traded across regional or national boundaries (e.g. locally provided health and education services) and their regional price is likely to be affected by location choices and hence state and central policies. There may be economies or dis-economies of scale in the regional production of public goods so that their prices depend on population size and hence policies. Regions might also have different production technologies. In addition, given the nature of local services, one would expect the price of local public goods to depend on geographic features of regions, socio-demographic characteristics and density/dispersion.

Quite different assumptions are also adopted in the literature with respect to region-specific economic rent and where it is allocated. One is that a region’s rent is captured fully by local residents who then earn the region’s average product. This means that rents fully distort location choices and appear with maximum effect in the optimal transfer expression. At the other extreme, region-specific rents may not be captured locally at all - accruing fully to foreigners - or they may go to domestic residents but not on the basis of location. In these cases, rents do not distort location choices and are absent from the optimal transfer expression. Given the variety of assumptions, and the impact that each has in terms of the implications for efficiency and the transfer, it seems to us there is some benefit to a unified modelling approach to rent capture, with each of the possibilities dropping out as particular cases.

The literature has also not tackled the question of whether there is an efficiency case for inter-regional transfers based on cost differences, despite this being a feature of real-world transfer schemes. In this sense, there remains a gap between the theory and practice of inter-regional transfers. Indeed, the prevailing view is that such cost differences should not be taken into account by a corrective transfer. This sentiment is expressed by Albouy (2012) who notes that ‘....variations in these prices due either to factor costs or production efficiency are to be ignored. When prices are set efficiently, they represent the opportunity cost of scarce factors for producing tradeable output. Subsidising households to live in areas where providing local services costs more ignores these opportunity costs and leads to inefficient use of scarce factors’1.

This means that inter-regional transfer schemes which include cost parameters as a determinant of transfers are inefficient. Our contention is that this may be true when costs differ because of production technologies, factor input prices or scale. However, in many real-world

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1see Albouy (2012) page 827.
transfer schemes it is cost differences resulting from factors such as dispersion/density, geographical features of regions and socio-demographic considerations that are used as measures of inter-regional cost differences. As far as we know, no-one has ever examined whether cost differences arising from these factors should be compensable under an efficient inter-regional transfer scheme. The answer to this is important for theorists, since it would deepen understanding of the theory of transfers, but also policy makers looking to benchmark their cost based transfer schemes against efficiency norms.

Based on these comments, the aims of this paper are threefold. The first is to extend the model used in the theory of inter-regional transfers to allow local public good prices to change in response to location patterns and hence central and regional policies. We show this creates local public good price externalities that distort spatial location patterns. Efficiency is shown to require the inter-regional transfer to correct for these externalities resulting in a modified expression for the optimal transfer. A second aim is to introduce variable local rent capture allowing us to cater for a range of assumptions about how locals capture and disburse rents. We show that whenever there is less than full rent capture mobility creates a wage income externality which must be corrected by the optimal transfer. More generally we discover a trade-off between the rent and wage income externalities according to how diligently locals collect and disburse rents.

The remaining goal is to investigate whether there are any circumstances under which the optimal transfer should take account of inter-regional differences in local public good costs and if so what direction such transfers should take (high to low cost or vice versa). Here we show that efficient inter-regional transfers should take account of such cost differences but only when they arise from physical/environmental features of states, density/dispersion and socio-demographic characteristics of regional populations. Thus, we do not provide an efficiency case for transfers based on differences in production technologies, preferences or input prices. In this respect, we agree with the prevailing view that such differences should be reflected in spatial location decisions.

However, we do develop a plausible sufficient condition which tells us the circumstances under which the efficient transfer should redistribute income from states with relatively low costs due to these factors in favour of states with high costs as we see in the practice of transfers in many countries. The sufficient condition only requires some fairly general restrictions on the way in which social marginal benefits change across states in response to migration. If the sufficient condition does not hold - as might of course be the case - the efficient transfer may still be from low to high cost states or the opposite (high to low cost) - we cannot say. We note that which situation applies for any particular economy could be settled by further empirical work on wage and other elasticities. Hence, our theoretical results provide the basis for new empirical research on inter-state transfers.

To get our results we develop a model of a federation which differs from the standard one in that local public good prices are endogenous, and vary with migration. In other words, relative prices are allowed to change in our set up. From this, we develop a three stage game in which
welfare maximising regions pre-commit to local public good provision as Nash competitors in the first stage. A social welfare maximising central government chooses the inter-regional transfer in stage 2, conditional on regional choices, while mobile residents make their location decisions in the third stage for given regional and central policy choices. When deciding on local public good provision in stage 1 regions are assumed to take into account the impact of their choices on the inter-regional transfer as well as local public good prices. Regions and the centre correctly anticipate labour location choices in stage 3. The results above are developed directly from the sub game perfect equilibria to this game and a comparative static exercise which tells us how equilibria respond to changes in a parameter capturing the influence of the factors identified above on local public good costs.

The outline of the paper is as follows. Section 2 sets up the basic federal model while Section 3 introduces endogenous local public good prices. Section 4 solves the Pareto efficient problem as a benchmark and Section 5 constructs the policy game. Section 6 contains the key results while Section 7 concludes.

2 The federal economy

Consider a federal economy with two jurisdictions, \( i = 1, 2 \), which we call states from now on for convenience\(^2\). Each state has a welfare maximizing government that provides a local public good benefit, \( g_i \), for \( i = 1, 2 \). A central government chooses a self-financing lump sum inter state transfer, \( \rho \). The set of state choice variables is \( g = [g_1, g_2] \) while the set of choice variables for all decision-makers is \( s = [\rho, g] \). We assume that \( s = [\rho, g] \) is a non-empty, convex and compact subset of some Euclidean space \( \mathbb{R}^m \). The federation has a given supply, \( N \), of homogeneous and perfectly mobile households who each supply one unit of labour. This implies the following labour supply constraint\(^3\)

\[
N = n_1 + n_2 \tag{2.1}
\]

The production process in each state uses three inputs; (i) the mobile labour input with a shadow price, \( w_i \), equal to the marginal product of labour (also equal to the wage rate as we assume competitive labour markets); (ii) a fixed supply, \( k_i \), for \( i = 1, 2 \), of foreign-owned capital with a return equal to some given world return, \( r \); and (iii) an unpriced-priced fixed input, \( \nu_i \), which we think of as a natural resource\(^4\). Given this, define the vector of input prices as \( \omega_i = [w_i, r] \) and the vector of inputs as \( \upsilon_i = [n_i, k_i, \nu_i] \). A numeraire is produced in each state using the three inputs. Production of the numeraire is defined by

\[
f_i(\upsilon_i) \quad i = 1, 2 \tag{2.2}
\]

\(^2\)This could also be a decentralised unitary economy or a regional union of semi-independent nations, such as the European Union.

\(^3\)Total labour supply could vary in response to joint state and central policies, and hence social welfare within the economy, relative to the rest of the world but we do not pursue this possibility here.

\(^4\)The presence of natural resources in the production function, and differences in the endowment of this factor, matters in federations such as Canada and Australia, but is of less significance in federations with relatively small endowments of factors such oil, natural gas or other minerals.
We assume \( f_i(v_i) \), is continuous, increasing and quasi-concave, with \( f_i(0) = 0 \). Supposing the price of the numeraire is one, \( f_i(v_i) \) also defines the value of output. With diminishing returns to labour also define

\[
\frac{\partial f_i(v_i)}{\partial n_i} = w_i > 0, \quad \frac{\partial w_i}{\partial n_i} < 0 \quad i = 1, 2 \tag{2.3}
\]

where \( \partial w_i/\partial n_i \) is the change in the wage rate (marginal product) as labour supply varies.

The total economic rent created in state \( i \) from the use of the natural resource,

\[
R_i(v_i) = f_i(v_i) - w_i n_i - r k_i, \quad i = 1, 2 \tag{2.4}
\]

is the difference between the value of output and payments to mobile labour and foreign-owned capital\(^5\). Aggregate household income is the sum of wage income and some share, \( 0 \leq \beta_i \leq 1 \), of economic rent, for \( i = 1, 2 \). Formally, household income can be expressed as, \( I = w_i n_i + \beta_i R_i \) for \( i = 1, 2 \). Using (2.4), this becomes

\[
I_i(v_i) = (1 - \beta_i) w_i n_i + \beta_i (f_i(v_i) - r k_i) \quad i = 1, 2 \tag{2.5}
\]

If \( \beta_i = 0 \), all rent is collected by foreigners. This could arise if there are no resource rent taxes and household income is simply \( I_i(v_i) = w_i n_i \). Alternatively, locals might capture all available rent using a resource rent tax or other instrument. In this case, household income is \( I_i(v_i) = f_i(v_i) - r k_i \); that is, households are residual claimants. They receive, as income, total output of the state less a payment to foreigners for use of the fixed capital stock\(^6\). The specification at (2.5) encompasses all these alternatives as special cases.

An implication of Wildasin and Wilson (1998) is that one might expect full rent capture since the burden of rent taxes are likely borne by foreigners. Notwithstanding this, we still allow the rent capture parameter to vary from zero to one because, in practice, it seems that a portion of rent is likely to accrue to foreigners. Local policy instruments are unlikely to collect one hundred percent of the rent generated in a state. There is evidence, for example, that this is so in many of the world’s resource rich regions, including Alberta, Texas, Wyoming, Alaska, Brazil, Russia, sub-Saharan Africa and Australia. Rent capture by a state does not, of itself, necessarily imply distortions to migration decisions that need to be corrected with a transfer. For this to occur also requires that rents are distributed, via local budgets, to new migrants (not just existing residents) through services provided, cash transfers or tax cuts. There are many instances where states, and indeed countries, do try to exclude new migrants from accessing services. For the purposes of this paper, we assume that these returns are made available to migrants and that states make no attempt at exclusion.

\(^5\)There can be decreasing, constant or increasing returns in the production technology.
\(^6\)Another possibility frequently adopted in federalism models for simplicity is that there is no foreign-owned factor and all economic rents accrue to locals. In this case, per capita household income is simply equal to the average product of the state.
Households have a continuous, strictly quasi-concave utility function,

\[ u_i(x_i, g_i) \quad i = 1, 2 \]  

(2.6)

where \( x_i \) is per capita consumption of a pure private good. We suppose that \( x_i \) is a traded good and that its price is given (at the world supply price) and the same in each state. Furthermore, for modelling convenience, and without loss of generality, the price of the private good is assumed to be one\(^7\).

Mobility implies the equal utility condition

\[ u_1(x_1, g_1) = u_2(x_2, g_2) \]  

(2.7)

must also be satisfied in equilibrium. One can allow for migration costs, but if symmetric, as assumed here, they can be ignored. Attachment to place can also be incorporated, though this does not change our conclusions. We proceed without allowing for attachment to enable the results to be presented in the clearest fashion possible.

A lump sum, self-financing, inter-state transfer, \( \rho \), is chosen by a welfare maximizing central agency. If \( \rho > 0 \), the transfer is from state 1 to 2, and if \( \rho < 0 \), the transfer will be from state 2 to 1. This is ensured by the way in which the transfer is built into state budget constraints, as will be seen when we define per capita private good consumption at (4.4) below.

Total output in a state, \( \pi_i \), is the sum of produced output and the net transfer as follows:

\[ \pi_i(v_i, \rho) = I_i(v_i) \pm \rho \quad i = 1, 2 \]  

(2.8)

We use a ± sign on the right side of (2.8) since if i=1 the transfer, \( \rho \), is deducted from that state’s output, while if i=2 the transfer is added to that state’s output.

Recalling that \( k_i \) and \( \nu_i \) are fixed, a sufficient condition for \( \pi_i(v_i, \rho) \) to be continuous, increasing and quasi-concave in \( v_i \), is that \( \partial^2 w_i / \partial n_i^2 \leq 0 \). We assume this to be the case for the purposes of this paper and that \( \pi_i(v_i, \rho) \) is used, as an input, to produce a local public good, \( G_i \), for \( i = 1, 2 \). This allows us to define \( G_i = z_i(\pi_i(v_i, \rho)) \) as the production function for the local public good in state i for \( i = 1, 2 \). Assuming \( z_i \) is continuous, increasing and quasi-concave in \( \pi_i \), the production function for the local public good,

\[ G_i = h_i(v_i, \rho) \quad i = 1, 2 \]  

(2.9)

is continuous, increasing and quasi-concave in \( v_i \). Once produced, suppose the local public good is transformed into the benefit, \( g_i \), for \( i = 1, 2 \), enjoyed by mobile labour through the relationship, \( g_i n_i^\alpha = G_i \) for \( i = 1, 2 \) where \( 0 \leq \alpha \leq 1 \) is a congestion parameter which we assume to be the same across states. When \( \alpha = 0 \), \( G_i \) is a pure local public good (\( g_i = G_i \)). If \( \alpha = 1 \), \( G_i \) is a private good while for values of \( \alpha \) between zero and one \( G_i \) is impure. The relationship

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\(^7\)As we shall see below, we allow the price of public goods to be endogenous implying that the price of the private good relative to each public good can also change.
between output of the local public good and its benefit, together with (2.9) imply that

\[ g_i n_i^a = h_i(v_i, \rho) \quad i = 1, 2 \] (2.10)

must be satisfied.

3 Endogenous local public good costs

There exists a cost function, \( c_i(\omega_i, G) \equiv \min \{ \omega_i v_i \geq 0 | G_i \geq h_i(v_i, \rho), \forall v_i \gg 0 \} \) which, because the wage is only variable price in \( \omega_i \), can be defined as \( c_i = c_i(w_i, G_i) \) for \( i = 1, 2 \) where:

\[ p_i(w_i, G_i) = \frac{\partial c_i(w_i, G_i)}{\partial G_i} \quad i = 1, 2 \] (3.1)

is the price, or social marginal cost, of the local public good. This price differs across states due to production technologies, input prices and economies of scale. The actual price will vary for these reasons, but also because of differences in physical/environmental characteristics, socio-demographic composition of residents and population density (dispersion). As noted by Reschovsky (2007), which sources of actual cost difference should be included in real-world inter-state transfer schemes is a contentious and sometimes political choice - countries that undertake transfers on the basis of cost differences tend to adopt various mixes of factors.

Japan, Switzerland and Australia take account of cost differences arising from remoteness of populations from urban centres. Australia also takes account of inter-state differences in wages. In the United Kingdom, an econometric method is used to assess the workload of local governments adjusted for cost factors which include input prices (Porcelli and Vidoli (2013) and Dafflon (2012)). Methods of varying complexity are also used in Sweden, South Korea, France and Hungary to identify, and measure, differences in the inter-state cost of providing a given level of public service due to the factors identified above. The United States and Canada have such use/population parameters built into their specific purpose (tied) grants to states and provinces, even though they have no explicit equalization for cost differences, or, in the case of the United States, no explicit inter-state equalization at all.

We capture the effect on cost of these other factors in a very simple way using a cost disability parameter, \( \gamma_i \geq 1 \), for \( i = 1, 2 \). The parameter shifts the cost curve defined between least cost and public good output up and down. The actual cost of a given unit of public good output, \( \gamma_i c_i(w_i, G_i) \), exceeds, or is equal to, the underlying least cost, depending on the cost disability. Thus, for any given level of output, the actual cost of public good supply in a state has two parts; (i) the least cost which depends on output, input prices and production technologies; and (ii) an additional cost determined by the cost disability which captures the cost impact of the physical environment, socio-demographic composition and density/dispersion.

The parameter set for state \( i \) can now be defined as:

\[ \varphi_i = [N, k_i, \beta_i, r, \alpha, \gamma_i] \quad i = 1, 2 \] (3.2)
while the parameter set for the federation is $\varphi = \cup \varphi_i$. The budget constraint for state $i$ is:

$$n_i x_i + \gamma_i c_i = \pi_i(v_i, \rho) \quad i = 1, 2$$

(3.3)

From the budget constraint, and using (2.5) and (2.8), per capita private good consumption in state $i$ is:

$$x_i = \frac{1}{n_i} \{(1 - \beta_i)w_i n_i + \beta_i \{f_1(v_i) - r k_i\} \pm \rho - \gamma_i c_i\} \quad i = 1, 2$$

(3.4)

Using this, together with $n_2 = (N - n_1)$ to eliminate $n_2$, the equal utility condition is:

$$u_1 \left\{ \frac{1}{n_1} \{(1 - \beta_1)w_1 n_1 + \beta_1 \{f_1(n_1) - r k_1\} - \rho - \gamma_1 c_1\}, g_1 \right\} =$$

$$u_2 \left\{ \frac{1}{(N - n_1)} \{(1 - \beta_2)w_2(N - n_1) + \beta_2 \{f_2((N - n_1)) - r k_2\} + \rho - \gamma_2 c_2\}, g_2 \right\}$$

(3.5)

The equality implies $n_1$ is an implicit function of $s = (g, \rho)$ conditional on $\varphi_1$. We therefore define:

$$n_1 = n_1(s|\varphi_1)$$

(3.6)

Since $n_i$ is the only variable input in $\nu_i$, we can also define $\nu_i(n_i)$ for $i = 1, 2$. Using $n_2 = N - n_1$, the production function for the local public good in state $i$ is then:

$$G_i = h_i(v_i(n_i), \rho) \quad i = 1, 2$$

(3.7)

We also know that the wage, $w_i$, is a function of $n_i$ for $i = 1, 2$. Together with (3.7), and once again using $n_2 = N - n_1$, this means we can express the cost function for the local public good in state $i$ as:

$$c_i = c_i(w_i(n_i), h_i(v_i(n_1), \rho)) \quad i = 1, 2$$

(3.8)

where $n_1 = n_1(s|\varphi_1)$ is defined at (3.6). Defining the cost function in this way makes it clear that the minimum cost of providing each local good is a function of the policies of the states and the central government, conditional on $\varphi_1 = [N, k_1, \beta_1, r, \alpha, \gamma_1]$, the parameter set in state 1, which includes the cost disability parameter for that state, $\gamma_1$. Thus, states and the centre can, potentially, manipulate local public good prices, and hence relative prices, through their policy choices. The parameters in $\varphi_1$ also affect local public good prices.

4 Pareto efficiency

To provide a benchmark against which to compare the outcomes from decentralisation we suppose a mythical central planner can directly choose $x_1, g_1$ and $n_i$ for $i = 1, 2$ to maximise per capita utility in one state while holding per capita utility in the other state at some given level. Specifically, the planner maximises $u_1(x_1, g_1)$ subject to (i) $u_2(x_2, g_2) = \pi_2$; (ii) $n_1 x_1 + n_2 x_2 + \gamma_1 c_1 + \gamma_2 c_2 = I_1(v_1) + I_2(v_2)$; and (iii) $N = n_1 + n_2$. This is analogous to the formulation in Myers (1990) implying we do not restrict the planner to finding a solution consistent with
\( u_1(x_1, g_1) = u_2(x_2, g_2) \), the free mobility condition. If we were to impose free mobility on the planner’s problem we would replace the first constraint above with the stronger equal utility restriction. However, our formulation of the planner’s problem differs from the standard one in that the solution is constrained by endogenous local public good prices and the cost disability parameters which capture the impact on cost of density, geography and socio-demographic features. Using the definition of \( I(\nu_i) \) at (2.5), the Lagrange function is:

\[
Z = u_1(x_1, g_1) + \lambda[u_2(x_2, g_2) - \bar{u}_2] + \\
\phi(1 - \beta_1)w_1n_1 + \beta_1(f_1(\nu_1) - rk_1) + (1 - \beta_2)w_2n_2 + \beta_2(f_2(\nu_2) - rk_2) \\
- n_1x_1 - n_2x_2 - \gamma_1c_1 - \gamma_2c_2] + \lambda_1[N - n_1 - n_2] \\

\]  

(4.1)

From this the first order necessary conditions for \( x_i, g_i \) and \( n_i \) where \( i = 1, 2 \) become:

\[
n_i^{1-\alpha} \left( \frac{u_i g_i}{u_i x_i} \right) = \gamma_ip_i \quad i = 1, 2 \\

\]  

(4.2)

\[
\mu_1 = \mu_2 \\

\]  

(4.3)

where

\[
\mu_i = \left\{ w_i - x_i + (1 - \beta_i)\frac{\partial w_i}{\partial n_i}n_i - \gamma_i\frac{\partial c_i}{\partial n_i} \right\} \quad i = 1, 2 \\

\]  

(4.4)

is the social marginal benefit of adding a mobile unit of labour to state \( i \) for \( i = 1, 2 \).

Thus, for an outcome to be Pareto optimal - on the Utility Possibilities Frontier defined between a representative citizen in state 1 and her counterpart in state 2 - provision of local public goods must satisfy the Samuelson condition (4.2). In addition labour must be allocated across states to satisfy (4.3), namely, the social marginal benefit of adding a unit of labour to state 1 must be the same as the social marginal benefit of adding a unit of labour to state 2 (spatial efficiency).

Our first order necessary conditions differ from the standard expressions in two ways. First (4.2) can be thought of as a modified Samuelson condition because the marginal cost is equal to the underlying price based on least cost but is adjusted by the cost disability parameter which reflects the influence on local public good costs of socio-demographic, geographic and density influences. The impact of these factors on cost must be included in the social marginal cost of providing the public good as they result in the use of resources which have an opportunity cost. Second, the social marginal benefit expression has two new terms; \( (1 - \beta_i)(\partial w_i/\partial n_i)n_i \) and \( \gamma_i(\partial c_i/\partial n_i) \). The first of these captures the effect of a higher labour supply in a state on total wage income. This is present because we have allowed for less than full local economic rent capture. When \( \beta_i = 1 \) (full rent capture) this term disappears from the social marginal benefit condition. The other new term is present because a change in labour supply in a state now has an impact on the least cost of providing the local public good and for efficiency this effect must be a part of the social marginal benefit of an additional unit of labour.

Hence, the social marginal benefit of an additional migrant worker in a state consists of their impact on: (i) output as measured by \( w_i \) (marginal product); (ii) per capita consumption,
\( x_i \); (iii) total wage income, \((1 - \beta_i)(\partial w_i / \partial n_i)n_i\); and (iv) the disability adjusted cost of local public goods as captured by \( \gamma_i(\partial c_i / \partial n_i) \). For an outcome from the planner’s problem to be on the utility possibilities frontier all four effects of migration are part of social marginal benefit.

5 Decentralised public good provision

Let us suppose that local public good provision is decentralised to two states while a central government agency chooses a lump sum self financing inter-state transfer. Also suppose that these decision-making agents interact in a simple game with three stages. States are benevolent to the extent that they choose their local public good provision in stage 1 to maximise the per capita utility of a representative citizen within their political jurisdiction. They only care about the interests of citizens in other states indirectly, that is, to the extent that their own decisions affect migration. The central government agency is also assumed to be benevolent and chooses the inter-state transfer in stage 2 to maximise social welfare for the federation while taking state choices as given. As we shall see in doing so the central agency internalises migration externalities. Freely mobile labour makes its location choices in stage 3 taking as given levels of local public good provision and the inter-state transfer. When making their choices states take account of the effect of their decisions on the inter-state transfer and labour location choices. In choosing its transfer the central agency correctly anticipates labour location choices.\(^8\)

The order of moves here is based on the assumption that state decisions over local public good provision are long term relative to central decisions over inter-state transfers. This arises from our observation that in many federations inter-state transfers are decided on an annual basis using long run historical state spending and revenue data. Thus, we have:

**Stage 1**: States move simultaneously with Nash conjectures to choose \( g_1 \) and \( g_2 \) to maximise per capita utility for their own citizens subject to the free migration condition, state feasibility, the cost functions for local public goods and the total labour supply condition. States correctly anticipate the inter-state transfer as well as public good price and labour location responses to their choices.

**Stage 2**: The central agency chooses \( \rho \) to maximize social welfare for the federation subject to the equal utility condition, feasibility, local public good cost functions and the labour supply condition. The centre takes state local public good choices as given and correctly anticipates public good price and labour location responses to its choice of transfer.

**Stage 3**: Labour makes it location choices taking state and central policies as given.

We first solve the problem of the central government in stage 2 and then look at the solution to the states’ problems in stage 1.

---

\(^8\)The timing of decisions here is analogous to Caplan, Cornes and Silva (2000), namely, states can be thought of as Stackleberg leaders
5.1 The central agency’s transfer choice (Stage 2)

We interpret the equal utility condition \( u_1(x_1, g_1) = u_2(x_2, g_2) \) as the social welfare function for the federation. With the strong incentive equivalence imposed by this free mobility constraint social welfare is maximised by the central agency if it chooses \( \rho \) to maximise per capita utility in either state 1 or 2 subject to the equal utility condition, feasibility, the local public good cost functions and the implicit labour supply function. Given this we choose per capita utility in state 1 as the objective of the central agency though the results are identical if the agency chooses \( \rho \) to maximise per capita utility in state 2 subject to the same set of constraints. Hence, the central agency chooses \( \rho \), for given \( g_i \) where \( i = 1, 2 \) to maximise:

\[
\begin{align*}
  u_1 \left\{ \frac{1}{n_1} \left( (1 - \beta_1)w_1n_1 + \beta_1[f_1(v_1) - rk_1] - \rho - \gamma_1c_1 \right), g_1 \right\} 
\end{align*}
\]

subject to the following constraints:

\[
\begin{align*}
  (i) & \quad u_1 \left\{ \frac{1}{n_1} \left( (1 - \beta_1)w_1n_1 + \beta_1[f_1(v_1) - rk_1] - \rho - \gamma_1c_1 \right), g_1 \right\} = \\
  & \quad u_2 \left\{ \frac{1}{(N - n_1)} \left( (1 - \beta_2)w_2(N - n_1) + \beta_2[f_2(v_2) - rk_2] + \rho - \gamma_2c_2 \right), g_2 \right\} \\
  (ii) & \quad c_1 = c_1(w_1(n_1), h_1(v_1(n_1), \rho)) \\
  (iii) & \quad c_2 = c_2(w_2(n_1), h_2(v_2(n_1), \rho)) \\
  (iv) & \quad n_1 = n_1(s|\varphi_1)
\end{align*}
\]

The cost functions enter the constraint set because the transfer choice changes local public good prices through its effect on settlement patterns and hence the supply of labour to state 1. The solution proceeds by differentiating (5.1) with respect to \( \rho \) yielding:

\[
\begin{align*}
  \left\{ \frac{w_1 - x_1 + (1 - \beta_1) \frac{\partial w_1}{\partial n_1} n_1}{\frac{\partial n_1}{\partial \rho} - \gamma_1 \frac{\partial c_1}{\partial \rho}} \right\} = 1 
\end{align*}
\]

From constraint (ii) the local public good (least) cost response to a change in \( \rho \) is:

\[
\frac{\partial c_1}{\partial \rho} = \frac{\partial n_1}{\partial \rho} \left\{ \frac{\partial c_1}{\partial w_1} \frac{\partial w_1}{\partial n_1} + \alpha c_1 \frac{1}{n_1} \right\} 
\]

Using (5.3) in (5.2) and rearranging we get:

\[
\mu_1 \frac{\partial n_1}{\partial \rho} = 1 
\]

where \( \mu_1 \) is the social marginal benefit defined for the planner optimum. From the set of
constraints:
\[
\frac{\partial n_1}{\partial \rho} = \frac{A}{D} < 0 \tag{5.5}
\]
where

(i) \[ A = \left\{ \frac{u_{1,x_1}}{n_2} + \frac{u_{2,x_2}}{n_2} \right\} > 0 \]

(ii) \[ D = \left\{ \frac{u_{1,x_1}}{n_2} \mu_1 + \frac{u_{2,x_2}}{n_2} \mu_2 \right\} < 0 \]

It is well known that for this class of model that there is a potential problem for stability of equilibrium when the federation is under-populated, that is, social marginal benefits are positive in each state. For this reason we henceforth assume the federation is over-populated so that \( \mu_i < 0 \) for \( i = 1, 2 \). This in turn means that \( D < 0 \) and since \( A > 0 \) we know that \( \partial n_1/\partial \rho < 0 \). From the total labour supply constraint this means that \( \partial n_2/\partial \rho > 0 \). Using (5.5) in (5.4) and rearranging the first order necessary condition for the agency’s transfer is:

\[
\mu_1 = \mu_2 \tag{5.6}
\]

where \( \mu_2 \) is the social marginal benefit for state 2 from the solution to the central planner problem. Thus, the first order necessary condition is equivalent to the expression for an efficient spatial allocation of mobile labour from the Pareto optimal solution (see (4.3)).

For later use it is convenient to note that the social marginal benefit in state i can be defined as a function of the state’s labour supply, that is, \( \mu_i(n_i) \) for \( i = 1, 2 \). Furthermore, recalling that \( n_2 = N - n_1 \), the first order necessary condition for the transfer can be expressed as:

\[
F = \mu_1(n_1) - \mu_2(n_1) = 0 \tag{5.7}
\]

where \( n_1(s|\varphi_1) \) as previously defined.

### 5.2 State provision of local public goods (Stage 1)

In the first stage state 1 chooses \( g_1 \), for given \( g_2 \), to maximise

\[
u_1 \left\{ \frac{1}{n_1} (\beta_1 w_1 n_1 + \beta_1 [f_1(\nu_1) - r k_1] - \rho - \gamma_1 c_1), g_1 \right\} \tag{5.8}
\]

subject to the following constraints:

(i) \[ \nu_1 \left\{ \frac{1}{n_1} ((1 - \beta_1) w_1 n_1 + \beta_1 [f_1(\nu_1) - r k_1] - \rho - \gamma_1 c_1), g_1 \right\} = \nu_2 \left\{ \frac{1}{(N - n_1)} ((1 - \beta_2) w_2 (N - n_1) + \beta_2 [f_2(\nu_2) - r k_2] + \rho - \gamma_2 c_2), g_2 \right\} \]

(ii) \[ \mu_1(n_1) - \mu_2(n_1) = 0 \]

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(iii) \[ c_1 = c_1 (w_1(n_1), h_1(v_1(n_1), \rho)) \]

(iv) \[ c_2 = c_2 (w_2(n_1), h_2(v_2(n_1), \rho)) \]

(v) \[ n_1 = n_1(s|\varphi_1) \]

Notice that the local public good cost functions and the first order necessary condition for the inter-state transfer, chosen by the centre in stage 2, form part of the state’s constraint set. The state therefore takes account of the effect of its policies on local public good prices and the inter-state transfer when solving its maximisation problem. Differentiating the state’s objective with respect to \( g_1 \) yields:

\[ n_1^{1-\alpha} \left( \frac{u_1, g_1}{n_1, x_1} \right) = \gamma_1 p_1 - \frac{1}{n_1^{\alpha}} \left\{ \frac{\partial n_1}{\partial g_1} - \frac{\partial \rho}{\partial g_1} \right\} \]

where \( n_1^{1-\alpha}(u_1, g_1/u_1) \) is the marginal rate of substitution between \( g_1 \) and \( x_1 \) or the marginal benefit of \( g_1 \). This is equivalent to the Samuelson condition from the central planner optimum if it were not for the additional terms on the right side. They capture the effects of an incremental change in \( g_1 \) via changes in labour supply and the inter-state transfer. As with the transfer the impact on least cost of \( g_1 \) is captured within the social marginal benefit term, \( \mu_1 \), and does not appear separately in (5.9).

From the set of constraints to the state maximization problem we obtain two equations in the unknowns \( \frac{\partial n_1}{\partial g_1} \) and \( \frac{\partial \rho}{\partial g_1} \) as follows:

\[ \frac{\partial n_1}{\partial g_1} - A \frac{\partial \rho}{\partial g_1} = - \frac{u_1, g_1}{D} \]

\[ \frac{\partial \rho}{\partial g_1} \left\{ \frac{E}{D} A + 2B \right\} + E \frac{\partial n_1}{\partial g_1} = 0 \]

where we define:

(i) \[ B = \left\{ \frac{1}{n_1} + \frac{1}{n_2} \right\} > 0 \]

(ii) \[ E = \left\{ \frac{\partial \mu_1}{\partial n_1} - \frac{\partial \mu_2}{\partial n_2} \right\} = \left( 2 - \beta \right) \left\{ \frac{\partial w_1}{\partial n_1} + \frac{\partial w_2}{\partial n_2} \right\} + (1 - \beta) \left\{ \frac{\partial^2 w_1}{\partial n_1^2} + \frac{\partial^2 w_2}{\partial n_2^2} \right\} - \left\{ \frac{\mu_1}{n_1} + \frac{\mu_2}{n_2} \right\} - \left\{ \frac{\gamma_1}{n_1^2} + \frac{\gamma_2}{n_2^2} \right\} \]

Though \( A \) and \( B \) are positive, and \( D < 0 \) when the economy is over populated, \( E \) has indeterminate sign. We can see from the above that \( E \) captures the difference between the change in social marginal benefit in state 1 as we add one unit of labour and the change in social marginal benefit in state 2 as a unit of labour is added. In general, this difference can be positive, negative or zero. Its actual sign depends on the first and second derivatives of state wages with
with respect to labour supply, per capita social marginal benefits and the second derivative of the local public good cost functions with respect to labour supply. Thus, it depends in part on the concavity or convexity of the state wage and local public good cost functions in labour supply.

Solving (5.10) and (5.11) yields the following expressions for $\frac{\partial n_1}{\partial g_1}$ and $\frac{\partial \rho}{\partial g_1}$:

$$
\frac{\partial n_1}{\partial g_1} = \frac{-u_{1,g_1} \{ (EA)/D + 2B \}}{2D \{ (EA)/D + B \}}, \quad \frac{\partial \rho}{\partial g_1} = \frac{u_{1,g_1} E}{2D \{ (EA)/D + B \}} \tag{5.12}
$$

Since the sign of $E$ is indeterminate so too are the signs of the labour supply and transfer responses to state policies defined at (5.12).

Similarly, state 2 chooses $g_2$ for given $g_1$ to maximise

$$
\frac{1}{n_2} \left( (1 - \beta_2)w_2n_2 + \beta_2 [f_2(v_2) - rk_2] + \rho - \gamma_2 \tilde{c}_2 \right), g_2
$$

subject to the same set of constraints facing state 1, namely, (i) to (v) at (5.8). From (5.13):

$$
n_2^{-1-\alpha} \left( \frac{u_{2,g_2}}{u_{2,x_2}} \right) = \gamma_2 \rho_2 - \frac{1}{n_2^\alpha} \left\{ \mu_2 \frac{\partial n_2}{\partial g_2} + \frac{\partial \rho}{\partial g_2} \right\} \tag{5.14}
$$

From the constraint set and using the same methodology as for state 1 we obtain the following solutions for the migration and grant responses in state 2:

$$
\frac{\partial n_2}{\partial g_2} = \frac{-u_{2,g_2} \{ (EA)/D + 2B \}}{2D \{ (EA)/D + B \}}, \quad \frac{\partial \rho}{\partial g_2} = \frac{-u_{2,g_2} E}{2D \{ (EA)/D + B \}} \tag{5.15}
$$

Once again we cannot in general sign the labour supply and inter-state transfer responses to an incremental increase in $g_2$ due to the sign indeterminacy of $E$.

**Definition 1.** The first order necessary conditions for $g_1$ and $g_2$ and the expressions for the labour supply and inter-state transfer responses define best response functions $\hat{g}_1 = \hat{g}_1(g_2|\varphi)$ and $\hat{g}_2 = \hat{g}_2(g_1|\varphi)$. Assuming the second order sufficient condition for existence is satisfied\(^9\), a Nash equilibrium to stage 1 is a solution $g^* = (g^*_1, g^*_2)$ such that $g^*_1 = \hat{g}_1(g^*_2|\varphi)$ and $g^*_2 = \hat{g}_2(g^*_1|\varphi)$.

A perennial question in the study of decentralisation is whether states under or over-provide local public goods relative to the efficient level of provision consistent with (4.2). We now show that a Nash equilibrium to stage 1 involves sub-optimal local public good provision and that there is at least one plausible case in which local public goods are under-provided. We also argue that the sole source of the distortion is the presence of the centrally mandated transfer and the distortion to state policies this induces. We start the discussion with:

**Proposition 1.** A sufficient condition for $g_i$ (where $i=1,2$) to be under-provided relative to Pareto optimal levels in a Nash equilibrium to stage 1 is:

$$
\frac{\partial \mu_1}{\partial n_1} < \frac{\partial \mu_2}{\partial n_2} \tag{5.16}
$$

\(^9\)This boils down to a restriction that $x_i$ is concave in $g_i$ for $i=1,2$. Notes on this are available on request.
Proof. Given $A > 0$, $B > 0$ and $D < 0$ then if $E < 0$ from (5.12) and (5.15) we have:

$$
\frac{\partial n_i}{\partial g_i} > 0 \quad i = 1, 2; \quad \frac{\partial p}{\partial g_1} > 0, \quad \frac{\partial p}{\partial g_2} < 0
$$

(5.17)

Consider (5.9). Given the above, with $\mu_1 < 0$ because of the over-population assumption we have that $\mu_1(\partial n_1/\partial g_2)$ is negative while $\partial p/\partial g_1$ is positive. This implies the braces on the right side of (5.9) have a negative sign and that the marginal cost of $g_1$ exceeds $\gamma_1 p_1$ which is the marginal cost from the Pareto optimal solution at (4.2). Since the state equates the marginal cost of $g_1$ with the marginal benefit - the left side of (5.9) - the implication is that it under-provides $g_1$ relative to the Pareto optimal outcome. Similarly from the first order necessary condition for state 2 at (5.14) $\mu_2(\partial n_2/\partial g_2)$ is negative while $\partial p/\partial g_2$ is negative. This implies the braces on the right side of (5.14) are negative and the marginal cost of $g_2$ exceeds $\gamma_2 p_2$ which is the marginal cost for state 2 from the Pareto optimal solution. State 2 also under-provides $g_2$ relative to a Pareto optimal outcome, thus proving the result.

If the sufficient condition holds then as each state increases its local public good supply - for given provision in the other state - it will increase its own labour supply. With the over-population assumption this reduces its social marginal benefit and is perceived by each state to be a cost of higher provision (the state gets more unwanted labour). Furthermore, as a state raises its local public good provision it also reduces the transfer it receives - this too is perceived by the state as a cost of providing its local public good. As a result, the marginal cost of local public goods is perceived to be higher than the underlying cost, $\gamma_i p_i$, for $i=1,2$. Naturally, if the sufficient condition does not hold the transfer and migration response terms can be of any sign and we have no conclusive result about over or under-provision, though we still know public good provision is generally sub-optimal. Nevertheless, we have isolated in the above proposition one plausible case in which any equilibrium to stage 1 of the game entails under-provision.

From proposition 1 both the migration and grant responses appear to distort local public good provision. We now show that this is not the case. When there is no centrally mandated transfer the migration response together with the first order necessary condition collapse to the single Pareto optimal expression, (4.2). The migration response only stands as a separate distortion to marginal cost and provision when the transfer is present. In this sense, the transfer distorts local public good provision directly and indirectly through the migration response but the transfer is the sole of distortion. This is demonstrated in the following:

**Proposition 2.** If there is no centrally mandated inter-state transfer the provision of local public goods is Pareto optimal.

**Proof.** Suppose there is no centrally mandated inter-state transfer ($\rho = 0$) and the game is two-stage with states moving first (simultaneously) anticipating labour location responses. Labour chooses its settlement pattern in stage 2 conditional on state policies. In stage 1 state 1 chooses
\( g_1 \) for given \( g_2 \) to maximize:

\[
\begin{align*}
    u_1 & \left\{ \frac{1}{n_1} ( (1-\beta_1)w_1 n_1 + \beta_1 \left[ f_1(\nu_1) - r k_1 \right] - \gamma_1 c_1), g_1 \right\} \\
\end{align*}
\]

(5.18)

subject to the following constraints:

(i) \( u_1 \left\{ \frac{1}{n_1} ( (1-\beta_1)w_1 n_1 + \beta_1 \left[ f_1(\nu_1) - r k_1 \right] - \gamma_1 c_1), g_1 \right\} =
\]

\[ u_2 \left\{ \frac{1}{(N-n_1)} ( (1-\beta_2)w_2 (N-n_1) + \beta_2 \left[ f_2(\nu_2) - r k_2 \right] - \gamma_2 c_2), g_2 \right\} \]

(ii) \( c_1 = c_1 (w_1(n_1), h_1(\nu_1), \rho) \)

(iii) \( c_2 = c_2 (w_2(n_1), h_2(\nu_1), \rho) \)

(iv) \( n_1 = n_1 (g|\phi_1) \)

From (5.18):

\[
\begin{align*}
    n_1^{1-\alpha} \left( \frac{u_1.g_1}{u_1.x_1} \right) = \gamma_1 p_1 - \frac{\mu_1}{n_1} \frac{\partial n_1}{\partial g_1}
\end{align*}
\]

(5.19)

where from the set of constraints:

\[
\begin{align*}
    \frac{\partial n_1}{\partial g_1} &= -\frac{u_1.g_1}{D} > 0 \\
\end{align*}
\]

(5.20)

(5.19) and (5.20) imply:

\[
\begin{align*}
    n_1^{1-\alpha} \left( \frac{u_1.g_1}{u_1.x_1} \right) = \gamma_1 p_1
\end{align*}
\]

(5.21)

Comparing (5.21) with (4.2), the Pareto optimal first order necessary condition for \( g_1 \), we can see that \( g_1 \) is provided efficiently when there is no centrally mandated inter-state transfer. There is an analogous first order necessary condition for \( g_2 \) and hence provision of both local public goods is Pareto optimal in the absence of a transfer, thus proving the result.

\[ \square \]

5.3 Settlement patterns (Stage 3)

Mobile labour makes its location choices in stage 3 for given state policies and the central transfer, consistent with the equal utility condition. In making these choices mobile labour decides on the settlement pattern between the two states in the federation, for given state and central policies. The settlement pattern is the only thing that mobile labour decides in our model given its assumptions, although even then one can argue that since the labour supply to a state is a function of state and central polices it is really the states and the centre that choose this pattern given they know mobile labour allocates itself to equate per capita utility.
5.4 Efficiency

We can now see that in a sub-game perfect equilibrium the first order necessary condition for the inter-state transfer is efficient in the sense that it establishes an optimal spatial allocation of mobile labour across states as required for Pareto efficiency (see Section 3). However, since states’ manipulate provision of local public goods to influence their transfer the best responses for $g_1$ and $g_2$ are inconsistent with the Samuelson conditions which must also be satisfied for Pareto efficiency. Hence, a sub-game perfect equilibrium is inefficient despite the presence of an inter-state transfer chosen to maximize social welfare. For this reason, henceforth we refer to an equilibrium transfer chosen by the centre in stage 2 as constrained optimal; it establishes an efficient inter-state settlement pattern given sub-optimal local public good provision.

Inefficiency arising from state policy manipulation to favourably influence their transfer is a known feature of decentralized equilibria where states move first. This is explored in Bordignon and Tabellini (2001), Boadway et al. (2003), Koethenbuerger (2008) and Boadway and Tremblay (2010). In a game with the same order of moves as here, Boadway et al. (2003) also obtain the result that local public good provision is distorted though they do not derive under or over-provision results. An exception to the inefficiency result when states move first is Caplan et al. (2000) who obtain an efficient sub game perfect equilibrium in a game where states are first movers. However, their model differs from ours in several respects, including that states contribute to a single national public good and do not provide local public goods.

5.5 Discussion

Notwithstanding distorted local public good provision we conclude that extending the inter-state transfer model into a world where states and the centre can manipulate local public good prices through their policy choices and there is variable rent capture does not create inefficiency per se. This is because the local public good price responses to central and state policies and the consequences of variable local rent capture get incorporated into the social marginal benefit terms, as we know from the planner problem they should be for efficiency. This means that local public good provision is not distorted by endogenous prices or variable rent capture (though it is by state attempts to manipulate their transfer) while the transfer chosen by a welfare maximising centre in the second stage corrects for the additional migration externalities.

If we had instead assumed that all players select their policies simultaneously as Nash competitors while correctly anticipating labour location decisions it is possible to show that local public good provision is efficient. However, the first order condition for the inter-state transfer, our focus of interest, would be unchanged. The other possibility is that the centre pre-commits by moving first. In this case, the central government might be expected to take account of the impact of its transfer choice on state policies in the second stage, resulting in distortions to the transfer choice, though state provision of local public goods would be efficient as they could not manipulate their transfer. This would be an interesting avenue to explore; however, we retain the assumed timing above since this is how we believe decisions are made in practice. That is, states make longer term decisions over services and the transfer adjusts more
frequently taking state policies as given.

Finally, if \( \rho \) was given to either state 1 or 2 as a choice variable - with the implicit abolition of the central agency - and the state were to choose \( \rho \) as a second policy variable in a simultaneous game with its neighbour, it would choose the same constrained optimal transfer as the agency, but without the distortion to local public goods. This in essence is the result of Myers (1990). The implication is that central assignment of the transfer instrument is inefficient relative to decentralisation where centralisation involves states moving first and manipulating policies to influence their transfer.

In the event any of these alternative assumptions about the set up of the model were adopted the results to follow in the next section relating to the effect of endogenous prices, variable rent capture and the efficiency case for transfers to high cost disability states, would still apply. In this sense, our conclusions are independent of the particular details of the model structure and its underlying assumptions.

6 The constrained optimal inter-state transfer

We now explain how endogenous prices and variable local rent capture extend and modify the migration externalities corrected by the constrained optimal transfer. We then show that inter-state cost differences, as captured by the cost disability parameters, do matter from the point of view of spatial efficiency. Finally, we argue that under certain circumstances, which could be verified by further empirical work, the transfer should favour high cost states, as we observe in the practice of inter-regional transfers.

6.1 Wage income and public good price externalities

An implication of allowing local public good prices to be endogenous and for variable local rent capture is that there are two new migration externalities for the inter-state transfer to internalise. To show this, note that there is an alternative specification of the first order necessary condition for the transfer derived by using (3.4) and (4.4) in (5.6) to obtain, after rearrangement:

\[
\rho = \frac{-n_1(N - n_1)}{N} \left\{ (1 - \alpha) \left( \gamma_1 \frac{p_1(\omega_1, G_1)G_1}{n_1} - \gamma_2 \frac{p_2(\omega_2, G_2)G_2}{n_2} \right) - \left( \beta_1 \frac{R_1}{n_1} - \beta_2 \frac{R_2}{N - n_1} \right) + \left( (1 - \beta_1)n_1 \frac{\partial w_1}{\partial n_1} - (1 - \beta_2)n_2 \frac{\partial w_2}{\partial n_2} \right) - \left( \gamma_1 \frac{\partial c_1}{\partial w_1} \frac{\partial w_1}{\partial n_1} - \gamma_2 \frac{\partial c_2}{\partial w_2} \frac{\partial w_2}{\partial n_2} \right) \right\}
\]

(6.1)

This tells us the constrained optimal inter-state transfer is a function of four migration externalities. The first two are the well-known fiscal and rent externalities, though as we will see below, they are modified somewhat. The remaining two are the new wage income and local public good price externalities. The wage externalities arise from allowing variable local rent capture and the price externalities from assuming that prices are endogenous. The wage income
externalities are:

\[ WIE_1 = (1 - \beta_1)n_1 \frac{\partial w_1}{\partial n_1} < 0, \quad WIE_2 = (1 - \beta_2)n_2 \frac{\partial w_2}{\partial n_2} < 0 \]  \hspace{1cm} (6.2)

Suppose \( \beta_i < 0 \) for \( i = 1, 2 \) so there is less than full (including zero) rent capture in both states. Since \( \partial w_i / \partial n_i < 0 \), we know that \( WIE_1 \) and \( WIE_2 \) are negative - an increase in labour supply suppresses total wage income in a state via a lower wage, and the wage income externality is negative. This is an effect on existing residents’ income that a migrant would not consider when making private location choices. If \( \beta_i = 1 \) (full rent capture) in both states, then \( WIE_1 = WIE_2 = 0 \) and the wage income externality is zero. For \( \beta_i \) between zero and one, wage income and rent externalities exert separate influences on the transfer.

There is a trade-off between the wage income and rent externalities. When \( \beta_i = 0 \), for \( i = 1, 2 \) (no local rent capture), the rent externality drops out of the transfer equation and the wage income externality has its full effect on the transfer. At the other extreme, when \( \beta_i = 1 \) (full local rent capture), the wage externality drops out of the transfer expression and the entire regional rents influence the transfer. For intermediate values of \( \beta_i \), the higher the value of the parameter the less important is the wage income externality and the more important is the rent externality. This is a consequence of the assumption that households earn their wage income and some share, \( \beta_i \), of their state’s rent.

If we assume \( WIE_1 > WIE_2 \to WIE < 0 \), then the net effect of wage income externalities is to exert a positive influence on the transfer, \( \rho \). In other words, if the (negative) wage income effect is larger in state 1 than the (negative) wage income effect in state 2, the optimal transfer from state 1 to 2 is higher. Efficiency requires a transfer to encourage more people to locate in state 2 where the negative externality is lower at the margin. The opposite is the case when \( WIE_1 < WIE_2 \to WIE > 0 \).

The public good price externalities in states 1 and 2 are:

\[ PGPE_1 = \gamma_1 \frac{\partial c_1}{\partial w_1} \frac{\partial w_1}{\partial n_1}, \quad PGPE_2 = \gamma_2 \frac{\partial c_2}{\partial w_2} \frac{\partial w_2}{\partial n_2} \]  \hspace{1cm} (6.3)

We know from the properties of cost functions that \( \partial c_i / \partial w_i < 0 \) and we also know that \( \partial w_i / \partial n_i < 0 \) for \( i = 1, 2 \). Since the cost disability parameter is positive, this implies that a migrant to state \( i \) decreases the wage, and in so doing, the per unit (least) cost of producing a given amount of the local public good. This means that \( PGPE_i < 0 \) which can be interpreted as a positive externality since a decrease in the price of the local public good raises real state income. If \( PGPE_1 > PGPE_2 \to PGPE < 0 \), then the net effect of the public good price externalities on the inter-state transfer, \( \rho \), is a negative one. In other words, if the local public good price externality is higher in state 1 than state 2 the optimal transfer in favour of state 1 is higher than otherwise. The opposite is true if \( PGPE_1 < PGPE_2 \to PGPE > 0 \). Efficiency requires us to increase labour supply in the state with the relatively higher (positive) public good price externality.

We can also say something about how endogenous prices affect fiscal externalities. In the
standard model with fixed prices, fiscal externalities respond to migration through variations in output of the public good and labour supply in the denominator. In our model, there is an additional channel through which changes in the spatial allocation of labour affects fiscal externalities; namely, by changing local public service prices. Thus, our fiscal externalities capture all the potential influences migration has on fiscal externalities.

6.2 Cost disabilities and the constrained optimal transfer

We have shown that cost disabilities have a direct impact on the optimal inter-state transfer in two ways. First, an increase in the cost disability in state 1 increases its (positive) public good price externality. Based on the arguments above, this will encourage a larger transfer in favour of state 1, simply because the positive local public good price externality in that state has risen, ceteris paribus. Second, the fiscal externalities are pre-multiplied by the cost disability parameters. If $\gamma_1 > 1$, for $i = 1, 2$ the disabilities inflate the fiscal externalities, and to a different extent across states. A given increase in say, $\gamma_1$, raises the (positive) fiscal externality in state 1, thus encouraging a larger transfer in favour of state 1 (away from state ). In both cases, an increase in $\gamma_1$ raises the positive externalities generated by a marginal migrant to that state, and this requires a higher transfer in its favour. In other words, higher costs in a state due to density, geographic features and socio-demographic characteristics, imply a larger optimal transfer in its favour, at least through these two direct effects.

**Proposition 3.** The constrained optimal inter-state transfer should take account of the influence of dispersion/density, geography and socio-demographic features of state populations on the cost of provision of local public goods.

**Proof.** From $F = \mu_1(n_1) - \mu_2(n_1) = 0$ the optimal inter-state transfer is an implicit function of $\varphi_1$, the set of parameters in state 1. Since this set includes $\gamma_1$ we know the optimal transfer is an implicit function of $\gamma_1$ whose value reflects the impact of density/dispersion, geography and socio-demographic features of the state’s population on the cost of providing its local public good. Similarly, the optimal transfer can be expressed as a function of $\gamma_2$ and hence the impact of the disability factors in state 2 on its local public good costs. This proves the proposition.

The result here provides us with an efficiency case for transfers based on inter-state cost differences arising from the factors influencing the cost disability parameter. It is therefore efficient for the inter-state transfer to take cost differences arising from these factors into account, rather than allowing them to be reflected purely in labour location decisions and settlement patterns. The result does not of course provide an efficiency case for transfers based on differences in production technologies or input prices which are captured within $p_i(\omega_i, G_i)$, for $i = 1, 2$. In this sense, we agree with the prevailing view that cost differences from these sources should be reflected in settlement patterns.
6.3 Transfers to high cost disability states

The discussion above implied that a higher cost disability in a state would, through the public good price and fiscal externalities, encourage a larger transfer in its favour. To see if this is so when one considers all the effects of changing the disability parameters, consider the following proposition:

**Proposition 4.** A sufficient condition for the transfer received by state i for i=1,2 to be an increasing function of its disability parameter, and hence its local public good cost due to disability factors, is that the social marginal benefit, $\mu_i$, for i=1,2 is decreasing in $n_i$; that is

$$\frac{\partial \mu_i}{\partial n_i} < 0 \quad i = 1, 2$$ (6.4)

**Proof.** Consider state 1. From $F = \mu_1(n_1) - \mu_2(n_1)=0$ the implicit function theorem implies:

$$\frac{\partial \rho}{\partial \gamma_1} = -\frac{F_{\gamma_1}}{F_{\rho}}$$ (6.5)

where

$$F_{\gamma_1} = \frac{\partial \mu_1}{\partial \gamma_1} - \frac{\partial \mu_2}{\partial \gamma_1}, \quad F_{\rho} = \frac{\partial \mu_1}{\partial \rho} - \frac{\partial \mu_2}{\partial \rho}$$ (6.6)

Using (4.4), (3.4) and the cost function for $g_1$ we obtain:

$$\frac{\partial \mu_1}{\partial \gamma_1} = \frac{\partial \mu_1}{\partial n_1} \frac{\partial n_1}{\partial \gamma_1} + M_1$$ (6.7)

where

(i) $\frac{\partial \mu_1}{\partial n_1} = (1 - \beta_1) \frac{\partial^2 w_1}{\partial n_1^2} n_1 + \frac{\partial w_1}{\partial n_1} (2 - \beta_1) - \frac{\mu_1}{n_1} - \gamma_1 \frac{\partial^2 c}{\partial n_1^2}$ (6.8)

(ii) $\frac{\partial n_1}{\partial \gamma_1} = \frac{1}{D} \left( \frac{n_1 c_1}{u_{1x_1}} \right) < 0$ (6.9)

(iii) $M_1 = \frac{c_1}{n_1} (1 - \alpha) - \frac{\partial c_1}{\partial w_1} \frac{\partial w_1}{\partial n_1} > 0$ (6.10)

Similarly:

$$\frac{\partial \mu_2}{\partial \gamma_1} = -\frac{\partial \mu_2}{\partial n_2} \frac{\partial n_2}{\partial \gamma_1}$$ (6.11)

where

$$\frac{\partial \mu_2}{\partial n_2} = (1 - \beta_2) \frac{\partial^2 w_2}{\partial n_2^2} n_2 + \frac{\partial w_2}{\partial n_2} (2 - \beta_2) - \frac{\mu_2}{n_2} - \gamma_2 \frac{\partial^2 c_2}{\partial n_2^2}$$ (6.12)

These results enable us to define $F_{\gamma_1}$ as

$$F_{\gamma_1} = H \frac{\partial n_1}{\partial \gamma_1} + M_1$$ (6.13)

where

$$H = \left( \frac{\partial \mu_1}{\partial n_1} + \frac{\partial \mu_2}{\partial n_2} \right)$$ (6.14)
From (5.7) and (3.4):
\[
\frac{\partial \mu_1}{\partial \rho} = \frac{\partial \mu_1}{\partial n_1} \frac{\partial n_1}{\partial \rho} + \frac{1}{n_1}
\] (6.15)

Similarly,
\[
\frac{\partial \mu_2}{\partial \rho} = -\frac{\partial \mu_2}{\partial n_2} \frac{\partial n_2}{\partial \rho} - \frac{1}{n_2}
\] (6.16)

We can now express \( F_\rho \) as
\[
F_\rho = H \frac{\partial n_1}{\partial \rho} + B
\] (6.17)
where \( B > 0 \) is defined at (5.11). From (5.7):
\[
\frac{\partial n_1}{\partial \rho} = A \frac{D}{n_1} < 0
\] (6.18)
where \( A < 0 \) is also defined at (5.11). Using these results (6.4) becomes:
\[
\frac{\partial \rho}{\partial \gamma_1} = -\left\{ H \frac{\partial n_1}{\partial \gamma_1} + M_1 \right\} / \left\{ H \frac{\partial n_1}{\partial \rho} + B \right\}
\] (6.19)

Now consider state 2. Using the same logic as above we obtain:
\[
\frac{\partial \rho}{\partial \gamma_2} = \left\{ H \frac{\partial n_2}{\partial \gamma_2} + M_2 \right\} / \left\{ H \frac{\partial n_1}{\partial \rho} + B \right\}
\] (6.20)
where
\[
(i) \quad \frac{\partial n_2}{\partial \gamma_2} = \frac{1}{D} \left( \frac{n_2 c_2}{w_2 x_2} \right) < 0
\] (6.21)
\[
(ii) \quad M_2 = \frac{c_2}{n_2} (1 - \alpha) - \frac{\partial c_2}{\partial w_2} \frac{\partial w_2}{\partial n_2} > 0
\] (6.22)

If the numerator and denominator on the right side of (6.19) have the same sign, and similarly for (6.20), then \( \partial \rho/\partial \gamma_1 \) is negative, implying that the transfer received by state 1 is increasing in \( \lambda_1 \), and \( \partial \rho/\partial \gamma_2 \) is positive, implying that the transfer received by state 2 is increasing in \( \lambda_2 \). Given that \( m_i > 0 \) for \( i=1,2 \), \( \partial n_i/\partial \gamma_i < 0 \) for \( i=1,2 \), \( \partial n_1/\partial \rho < 0 \), \( D < 0 \) and \( B > 0 \), then if \( H < 0 \) the numerator and denominator of (6.19) will have the same sign and similarly for (6.20). A sufficient condition for \( H < 0 \) is that:
\[
\frac{\partial \mu_i}{\partial n_i} < 0 \quad i = 1,2
\] (6.23)
thus proving the result.

\[\square\]

6.4 Discussion

The sufficient condition is not especially restrictive. From (6.8) and (6.12) there is good reason to think there are cases where it would hold. The signs of the derivatives in the sufficient condition depend upon first and second order partials of state wage functions, the level of social marginal benefit per capita and changes in local public good costs resulting from variations in
labour supply. Of course the sufficient condition may not hold implying the constrained efficient transfer received by a state is either decreasing or increasing in the state’s disability parameter. Nevertheless, we have isolated a plausible case where the transfer favours states with relatively higher costs due to the disability factors.

There is no inconsistency between the sufficient condition for under-provision of local public goods (proposition 1) and the sufficient condition for the transfer a state receives to be increasing in its cost disability (proposition 3). Both sufficient conditions boil down to restrictions on the way in which social marginal benefits respond to changes in labour settlement patterns.

Finally, whether the sufficient condition in propositions 1 and 3 apply in practice will depend on the particular economic circumstances of a federation and the nature of its regional economies. This could be resolved for a specific country through further empirical work to estimate the various terms within (6.8) and (6.12). Thus, our theoretical results are a basis for further empirical research on the efficiency of inter-state transfers in federations and decentralized regional economies.

7 Conclusion

Our results tell us three new things about the theory of inter-regional transfers. First, when we allow state and central policies to change local public good prices and for variable local rent capture two new spatial externalities arise from labour mobility - one related to the price of local public goods and the other to wage income. Pareto efficiency requires both to be incorporated into the social marginal benefits created by mobile households together with the well known fiscal and rent externalities. We adopt a three stage decentralised federal set up where states move first and choose local public goods provision while a central agency moves second and chooses an inter-state transfer after observing state policies. In an equilibrium the optimal transfer chosen by the central agency does internalise both of the new externalities as required for efficiency. There is also a trade-off between the rent and wage income externalities depending on the degree of local rent capture. However, an equilibrium is inefficient because of state strategic behaviour over the transfer which leads to sub-optimal provision of local public goods. In general, we cannot tell whether there is over or under provision though we do provide a sufficient condition for under provision.

Second, the optimal inter-regional transfer should take account of the impact on local public good costs of factors such as density, socio-demographic features and geographic characteristics, as captured by the cost disability parameters applied to the state specific public good cost functions. Contrary to the conventional view the cost of providing local public goods does matter for the constrained optimal inter-state transfer. This result provides theoretical support for the inclusion of cost differences arising from these factors into inter-state transfer schemes as occurs in the practice of fiscal equalisation (e.g. expenditure equalisation). However, we agree with the prevailing view that differences in inter-state cost arising from input prices, technology or scale should be reflected in settlement patterns and not corrected for by an inter-state transfer.

Finally, through a sufficient condition we have shown there is at least one plausible case in
which the optimal transfer should redistribute in favour of states with high costs arising from factors affecting the cost disability parameter. The sufficient condition requires restrictions on how social marginal benefits change across states in response to migration. This result may also provide some comfort to those countries that include cost measures in their transfer schemes and redistribute in favour of high cost states. Of course, if our sufficient condition does not hold, as may very well be the case, the optimal transfer could still favour high cost states, but equally, it might require a transfer in favour of low cost states. The circumstances applying in any particular economy might be resolved with further empirical work to estimate how wages and local public good costs respond to migration.

References


