An asymmetric Melitz model of trade and growth

Takumi Naito*
Waseda University
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Abstract

To examine the effects of unilateral trade liberalization on growth and welfare of the liberalizing and partner countries through intraindustry reallocations, we formulate an asymmetric two-country Melitz model of trade and endogenous growth based on capital accumulation. We obtain two main results. First, each country’s mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its export productivity cutoff decreases. Second, compared with the old balanced growth path, a permanent fall in any import trade cost raises the growth rates of all countries for all periods, and welfare of all countries.

JEL classification: F13; F43

Keywords: Melitz model; Unilateral trade liberalization; Endogenous growth; Heterogeneous firms; Asymmetric countries

*Takumi Naito. Faculty of Political Science and Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan. E-mail: tnaito@waseda.jp.
1 Introduction

Trade liberalization in one country creates a greater export opportunity for more efficient firms in another country. This drives down the profits from domestic sales in the latter country, forcing the least efficient firms to exit. Such reallocation of resources improves the average productivity of surviving firms, which brings about additional gains from trade. This idea, known as the Melitz (2003) model of heterogeneous firms, has become one of the standard theories of international trade since the beginning of this century. However, since users of the Melitz model largely keep the assumption of symmetric countries, nothing is known about how the reallocation process caused by unilateral trade liberalization such as illustrated above affects the growth paths of the liberalizing and partner countries in asymmetric ways.\(^1\) It is important to consider unilateral trade liberalization because trade costs indeed vary across countries: developing countries, on average, have more room for liberalizing imports than developed countries.\(^2\)

To examine the effects of unilateral trade liberalization on growth and welfare of the liberalizing and partner countries through intraindustry reallocations, we formulate an asymmetric two-country Melitz model of trade and endogenous growth.

Some researchers combine the Melitz model with the R&D-based endogenous growth models synthesized by Grossman and Helpman (1991) to investigate the effects of symmetric trade liberalization on long-run growth (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011; Perla et al., 2014; Sampson, 2014). The literature starts from Baldwin and Robert-Nicoud (2008), who introduce heterogeneity in unit labor requirements for production in the standard variety expansion model. Trade liberalization affects each country’s expected R&D cost through two channels. On one hand, it makes it harder for potential entrants to survive in their domestic market (i.e., decreases the cutoff unit labor requirement for domestic sales), which increases the expected units of knowledge good required for successful entry. On the other hand, it lowers the price of knowledge good if and only if it increases the degree of international spillovers of knowledge stocks. Overall, trade liberalization raises or lowers the long-run growth rate, depending on whether the latter channel is stronger or weaker than the former. Dinopoulos and Unel (2011) and Perla et al. (2014) also report an ambiguous relationship between symmetric trade costs and long-run growth, whereas Haruyama and Zhao (2008) and Sampson (2014) find that the relationship is always negative.\(^3\) However, all of these works depend on the assumption of symmetric countries. This suggests that, at least at this point, the R&D-based endogenous growth models with heterogeneous firms are not useful for our purpose of studying the effects of unilateral trade liberalization on asymmetric countries.

To allow for asymmetric countries, we build on the multi-country AK model of Acemoglu and Ventura (2002). Motivated by their own empirical findings that a one percentage point rise in a country’s growth rate

\(^1\)Helpman et al. (2004) and Melitz and Ottaviano (2008) deal with asymmetric countries by adding a homogeneous good sector, whose constant marginal product of labor fixes each country’s wage. In contrast, Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013) allow wages to be endogenously determined in their asymmetric two-country models without a homogeneous good sector. However, all of these models are static. In their review of heterogeneous firm trade models, Melitz and Redding (2014) point out that: “(d)espite some work on dynamics, much of the literature on firm heterogeneity and trade remains static, and we have relatively little understanding of the processes through which large and successful firms emerge and the implications of these processes for the transitional dynamics of the economy’s response to trade liberalization.” (Melitz and Redding, 2014, p. 49)

\(^2\)In their comprehensive survey of trade costs, Anderson and van Wincoop (2004) conclude that: “(t)rade costs also vary widely across countries. On average, developing countries have significantly larger trade costs, by a factor of two or more in some important categories.” (Anderson and van Wincoop, 2004, p. 747)

\(^3\)Haruyama and Zhao (2008) employ the standard quality ladder model with international knowledge spillovers of the second-highest quality goods. Dinopoulos and Unel (2011) deal with heterogeneity in product quality (i.e., marginal utility) instead of productivity in the variety expansion model. Perla et al. (2014) consider heterogeneous firms’ choices between remaining in production and paying a cost to adopt a better technology from more productive domestic firms. Sampson (2014) assumes that productivity of each entrant is proportional to some summary statistic of the entire productivity distribution of domestic incumbents, which in turn shifts the productivity distribution of entrants to the right over time.
deteriorates its terms of trade by 0.6 percentage points, they construct a model where such terms of trade adjustments in the differentiated intermediate good sector lead the growth rates of capital in all countries to converge, so that the world income distribution is stable, in the long run. We incorporate the asymmetric Melitz framework with endogenous factor prices developed by Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013) into the intermediate good sector of the two-country Acemoglu-Ventura model. For each country’s capital to grow at a constant rate, the masses of domestic and imported varieties must be constant along a balanced growth path (BGP). This requires us to assume that the fixed entry and overhead costs are increasing in the GDP of the source countries as negative externalities. This assumption is consistent with Bollard et al. (2014), who find empirically that the number of firms per worker increases only modestly with output per worker over time and across countries, with elasticities of at most around 0.2, implying that the fixed costs rise with economic growth. Our simple model enables us to examine, in a completely analytical manner, how unilateral trade liberalization affects each country’s productivity cutoffs, extensive margin of exports (i.e., mass of exported varieties), openness (i.e., revenue share of exported varieties), and growth rate not only in the long run but also during the transition, and welfare.

We obtain two main results. First, each country’s mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its export productivity cutoff decreases. Although it sounds natural that a decrease in a country’s export productivity cutoff increases its mass and revenue share of exported varieties, it is far from trivial that such cutoff change always raises the country’s own growth rate because its growth equation does not include the cutoff directly. The nontrivial relationship can be understood by considering that a decrease in a country’s export productivity cutoff is followed by an increase in its domestic productivity cutoff from the free entry condition, and the resulting increase in its average productivity of surviving firms contributes to its faster growth. Just like a country’s domestic productivity cutoff is a sufficient statistic for its welfare in static and steady state heterogeneous firm trade models (e.g., Melitz and Redding, 2014), a country’s export productivity cutoff is a sufficient statistic for its growth rate as well as its mass and revenue share of exported varieties in our model. Second, compared with the old BGP, a permanent fall in any import trade cost raises the growth rates of all countries for all periods, and welfare of all countries. In the initial period, a permanent fall in a country’s import trade cost directly decreases the export productivity cutoff of the partner country, but it also indirectly decreases that of the liberalizing country itself through a fall in its relative rental rate of capital restoring its zero balance of trade. This raises the growth rates of both countries, with the relative supply of capital in one country to the other predetermined in that period. After the initial period, the relative supply of capital of either faster-growing country increases. Since this makes it more difficult for the intermediate good firms in that country to start exporting, its growth rate falls, whereas that of the other country rises for the opposite reason. Along the new BGP, both countries grow at the same rate, which is higher than the old BGP. One contribution of this result to the literature on heterogeneous firm models of trade and endogenous growth is that positive externalities from knowledge spillovers are unnecessary for trade liberalization to raise long-run growth. International spillovers are necessary for the case of growth-enhancing trade liberalization in Baldwin and Robert-Nicoud (2008), Haruyama and Zhao (2008), and Dinopoulos and Unel (2011), whereas domestic spillovers are the engine of endogenous growth in Perla et al. (2014) and Sampson (2014). Our model shows the positive growth effect of trade liberalization even under negative externalities. Another contribution is that even unilateral trade liberalization is sufficient for faster growth of all countries for all periods. In contrast to the literature, where cross-country symmetry needs to be imposed, our model is so flexible as to yield such a strong result.
The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 derives some basic properties. Section 4 examines the effects of unilateral trade liberalization. Section 5 concludes.

2 The model

Consider a two-country world economy. In country \(i (= 1, 2)\), there is a nontradable final good, which is used for consumption and investment. The final good is produced from a continuum of tradable differentiated intermediate goods under constant returns to scale and perfect competition. Each variety of intermediate good is produced from nontradable capital under increasing returns to scale and monopolistic competition. Capital is the only primary factor, whose growth rate is endogenously determined.

2.1 Households

The representative household in country \(i\) maximizes its overall utility \(U_i = \int_0^\infty \ln C_{it} \exp(-\rho_i t) dt\), subject to its budget constraint:

\[
p_{it}^Y (C_{it} + K_{it} + \delta_i K_{it}) = r_{it} K_{it}; K_{it} \equiv dK_{it}/dt,
\]

with \(\{p_{it}^Y, r_{it}\}_{t=0}^\infty\) and \(K_{i0}\) given, where \(t(\in [0, \infty))\) is time, \(C_i\) is consumption, \(\rho_i\) is the subjective discount rate, \(p_{it}^Y\) is the price of the final good, \(K_i\) is the supply of capital, \(\delta_i\) is the depreciation rate of capital, and \(r_i\) is the rental rate of capital. The time subscript is omitted whenever no confusion arises. Dynamic optimization under the logarithmic instantaneous utility implies that capital always grows at the same rate as consumption given by the Euler equation:

\[
\dot{K}_{it}/K_{it} = \dot{C}_{it}/C_{it} = r_{it}/p_{it}^Y - \delta_i - \rho_i \equiv \gamma_i \forall t.
\]

2.2 Final good firms

The representative final good firm in country \(i\) maximizes its profit \(\Pi_i^Y = p_{it}^Y Y_i - \int_{\Omega_i} p_i(\omega) x_i(\omega) d\omega\), subject to its production function \(Y_i = (\int_{\Omega_i} x_i(\omega) d\omega)^{1/\alpha}; \alpha \equiv (\sigma - 1)/\sigma \in (0, 1)\), with \(p_{it}^Y\) and \(\{p_i(\omega)\}_{\omega \in \Omega_i}\) given, where \(Y_i\) is the supply of the final good, \(\Omega_i\) is the set of available varieties, \(p_i(\omega)\) is the demand price of variety \(\omega\), \(x_i(\omega)\) is the demand for variety \(\omega\), and \(\sigma(> 1)\) is the elasticity of substitution between any two varieties. Cost minimization yields the conditional input demand function for variety \(\omega\):

\[
x_i(\omega) = p_i(\omega)^{-\sigma} P_i^\sigma Y_i; P_i \equiv (\int_{\Omega_i} p_i(\omega)^{1-\sigma} d\omega)^{1/(1-\sigma)},
\]

where \(P_i\) is the price index of the intermediate goods. Then the minimized expenditure for the intermediate goods is expressed as \(\int_{\Omega_i} p_i(\omega) x_i(\omega) d\omega = P_i Y_i \equiv E_i\). Finally, perfect competition drives the price of the final good to its unit cost:

\[
p_{it}^Y = P_i.
\]
2.3 Intermediate good firms

Our description of the intermediate good sector is based on Felbermayr et al. (2013), which is an asymmetric, two-country, and static version of Melitz (2003). After paying a sunk fixed entry cost, an intermediate good firm in country \( i \) draws its productivity \( \varphi \) from a country-specific cumulative distribution \( G_i(\varphi) \) (to be specified later) with its density \( g_i(\varphi) \). If a firm’s \( \varphi \) is sufficiently high that the resulting gross profit covers its fixed overhead cost which is specific to each source-destination pair \( (i, j), i, j = 1, 2 \), then it survives selling its variety to the destination country \( j \); otherwise, it exits from market \( j \) without having to pay the overhead cost. The fixed entry and overhead costs are specified as \( r_i K_i f_i^e \) and \( r_i K_i f_{ij} \), respectively, where \( f_i^e \) and \( f_{ij} \) are exogenous constants. This means that all fixed costs are proportional to the GDP of the source country \( i \) as negative externalities. This is not unrealistic: Bollard et al. (2014) estimate that the elasticity of the number of firms per worker with respect to the value added per worker is at most around 0.2 over time and across countries, suggesting that the fixed costs rise with development. Finally, we assume that all decisions of individual firms are static. More specifically, each variety has a product life of only one period, and so each intermediate good firm has to pay not only the overhead costs but also the entry cost in each period without having to pay the overhead costs of the year before.

An intermediate good firm in country \( i \) with productivity \( \varphi \) maximizes its profit in country \( j \) \( \pi_{ij}(\varphi) = p_{ij}^f(\varphi)y_{ij}(\varphi) - r_i k_i(\varphi), \) subject to its cost function (measured in terms of capital) \( k_i(\varphi) = K_i f_{ij} + y_{ij}(\varphi)/\varphi, \) the market-clearing condition for its variety \( y_{ij}(\varphi) = \pi_{ij} x_{ij}(\varphi), \) and the conditional input demand function for its variety \( x_{ij}(\varphi) = p_{ij}(\varphi)^{-\sigma} P_j^{\sigma-1} \) \( E_j \) from Eq. (3), with \( r_i, K_i, P_i, \) and \( E_j \) given, where \( p_{ij}^f(\varphi) \) is the FOB supply price of a variety, \( y_{ij}(\varphi) \) is the supply of a variety, \( k_i(\varphi) \) is the demand for capital as overhead and variable costs, and \( \tau_{ij}(\geq 1) \) is the iceberg trade cost factor of delivering one unit of a variety from country \( i \) to country \( j \), with \( \tau_{ii} = 1 \). Since \( \sigma \) is common across countries, each firm sets the same supply price for all destinations:

\[
(p_{ij}^f(\varphi) - r_i/\varphi)/p_{ij}^f(\varphi) = 1/\sigma \Leftrightarrow p_{ij}^f(\varphi) = r_i/(\alpha \varphi) \forall j. \tag{5}
\]

Using Eq. (5), the resulting revenue and profit are calculated as:

\[
\pi_{ij}(\varphi) = \alpha e_{ij}(\varphi)/\sigma - r_i K_i f_{ij} = (\tau_{ij} r_i)^{1-\sigma}(\alpha \varphi P_i)^{\sigma-1} E_j/\sigma - r_i K_i f_{ij}.
\]

Since \( \pi_{ij}(\varphi) \) is increasing in \( \varphi \), a firm in country \( i \) survives in country \( j \) if and only if \( \varphi \geq \varphi_{ij} \), where the productivity cutoff \( \varphi_{ij} \) is determined by:

\[
\pi_{ij}(\varphi_{ij}) = 0 \Leftrightarrow \varphi_{ij}(\varphi_{ij}) = (\tau_{ij} r_i)^{1-\sigma}(\alpha \varphi_{ij} P_i)^{\sigma-1} E_j = \tau_{ij} K_i f_{ij}. \tag{6}
\]

Dividing Eq. (6) by itself with \( j = i \), we obtain \( \varphi_{ij}/\varphi_{ii} = (P_i/P_j)(E_i/E_j)^{1/(\sigma-1)} \tau_{ij}(f_{ij}/f_{ii})^{1/(\sigma-1)}, i, j = 1, 2 \). In line with Melitz (2003), we assume that the variable and fixed export costs are sufficiently large that

\footnote{According to the Osiris database, in a total of 9,891 industrial companies, whose R&D expenses per operating revenue in 2010 is available, 8,497 firms pay no less than 0.1 percent of their revenue for R&D, of which 5,193 firms pay no less than 1 percent, for all available years during 2000-2010. This implies that the R&D cost, which can be regarded as the fixed entry cost in our model, is more likely to be paid recurrently than only one time.}
country i’s export productivity cutoff is larger than its domestic productivity cutoff:

$$\varphi_{ij}/\varphi_{ii} > 1, i, j = 1, 2, j \neq i.$$  

Let $$m_{ij} \equiv (1 - G_i(\varphi_{ij}))/\left(1 - G_i(\varphi_{ij})\right), i, j = 1, 2.$$ For $$j \neq i, m_{ij}(< 1)$$ represents the ex-ante probability of export survival conditional on domestic survival. We also define the density of $$\varphi$$ conditional on survival:

$$\mu_{ij}(\varphi|\varphi_{ij}) = \begin{cases} g_i(\varphi)/(1 - G_i(\varphi_{ij})) & \text{if } \varphi \geq \varphi_{ij}; \\ 0 & \text{otherwise.} \end{cases}$$

From now on, $$G_i(\varphi)$$ is specified as Pareto, the most common functional form in the literature: $$G_i(\varphi) = 1 - (b_i/\varphi)^{\theta} = 1 - b_i^{\theta} \varphi^{-\theta}; \theta > 0, \varphi \in [b_i, \infty).$$ The shape parameter $$\theta$$ is common, whereas the scale parameter $$b_i$$ can be different, across countries. Then it follows that $$g_i(\varphi) = \theta b_i^{\theta} \varphi^{\theta-1}, \mu_{ij}(\varphi|\varphi_{ij}) = \theta b_i^{\theta} \varphi^{\theta-1}$$ for $$\varphi \geq \varphi_{ij},$$ and $$m_{ij} = (\varphi_{ij}/\varphi_{ii})^{-\theta}.$$ The expected values of revenue and profit are calculated as:

$$\int_0^\infty e_{ij}(\varphi)g_i(\varphi)d\varphi = (1 - G_i(\varphi_{ij}))$$

$$\int_\varphi^{\varphi_{ij}} e_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi = b_i^{\theta} \varphi_{ij}^{-\theta}(\theta/\lambda)\sigma r_i K_i f_{ij},$$

$$\int_0^\infty \pi_{ij}(\varphi)g_i(\varphi)d\varphi = (1 - G_i(\varphi_{ij}))$$

$$\int_\varphi^{\varphi_{ij}} \pi_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi = b_i^{\theta} \varphi_{ij}^{-\theta}(\theta/\lambda - 1)r_i K_i f_{ij};$$

$$\lambda \equiv \theta - (\sigma - 1) > 0,$$

where we use $$e_{ij}(\varphi) = (\varphi/\varphi_{ij})^{\sigma-1}e_{ij}(\varphi_{ij}) = (\varphi/\varphi_{ij})^{\sigma-1}\sigma r_i K_i f_{ij}$$ from Eq. (6), and $$\int_\varphi^{\varphi_{ij}} \varphi^{\sigma-1}\mu_{ij}(\varphi|\varphi_{ij})d\varphi = (\theta/\lambda)\varphi_{ij}^{\sigma-1}.$$ The assumption that $$\lambda > 0$$ ensures that the average productivity of surviving firms $$\int_\varphi^{\varphi_{ij}} \varphi^{\sigma-1}\mu_{ij}(\varphi|\varphi_{ij})d\varphi$$ is finite, and $$\theta/\lambda(> 1)$$ is equal to the ratio of the average productivity to the cutoff productivity. Free entry implies that the total expected profit of an entrant in country $$i$$ is equal to its fixed entry cost, that is, $$\sum_j \int_0^\infty \pi_{ij}(\varphi)g_i(\varphi)d\varphi = r_i K_i f_i^e.$$ Noting that $$r_i K_i$$ appears in both sides of the free entry condition, it is simplified to:

$$f_i^e/[(\theta/\lambda - 1)b_i^{\theta}] = \sum_j \varphi_{ij}^{-\theta} f_{ij} = \varphi_{ii}^{-\theta} \sum_j m_{ij} f_{ij}.$$  

Finally, let $$M_i^e$$ denote the mass of entrants in country $$i.$$ Then the mass of entrants in country $$i$$ surviving in country $$j,$$ or the mass of varieties sold from country $$i$$ to country $$j,$$ is given by $$M_{ij} = M_i^e(1 - G_i(\varphi_{ij})).$$ This immediately implies that $$M_{ij}/M_{ii} = m_{ij}.$$  

2.4 Markets  

The market-clearing conditions for the final good, capital, and the intermediate goods are given by:

$$Y_i = C_i + K_i + \delta_K K_i, i = 1, 2, \quad (9)$$

$$K_i = \sum_j M_{ij} \int_{\varphi_{ij}}^{\varphi_j} k_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi + M_i^e K_i f_i^e, i = 1, 2, \quad (10)$$

$$y_{ij}(\varphi) = \pi_{ij}x_{ij}(\varphi), i, j = 1, 2.$$  

(11)
On the other hand, summing up the household budget constraint (1), the zero profit condition in the final good sector (4), and the free entry condition in the intermediate good sector (8), we obtain Walras’ law in country $i$:

$$0 = p_i^Y (C_i + \bar{K}_i + \delta_i K_i - Y_i) + r_i (\sum_j M_{ij} \int_{\phi_{ij}}^{\infty} k_{ij} (\varphi) \mu_{ij} (\varphi | \varphi_{ij}) d\varphi + M_i^{f} K_i f_i^e - K_i)$$

$$+ \sum_j M_{ji} \int_{\phi_{ji}}^{\infty} \tau_{ji} p_{ji}^f (\varphi) x_{ji} (\varphi) \mu_{ji} (\varphi | \varphi_{ji}) d\varphi - \sum_j M_{ij} \int_{\phi_{ij}}^{\infty} p_{ij}^f (\varphi) y_{ij} (\varphi) \mu_{ij} (\varphi | \varphi_{ij}) d\varphi.$$  

Summing this up for all countries, we can confirm Walras’ law in the world, implying that any one of the eight (i.e., (9) × 2, (10) × 2, (11) × 4) types of the market-clearing conditions is redundant, and that the price of any one of the eight types of goods and factors can be normalized to unity.

### 2.5 Dynamic system

From now on, we derive the dynamic system, regarding the iceberg trade costs $\tau_{21}$ and $\tau_{12}$ as the only policy variables. Let capital in country 2 be the numeraire: $r_2 \equiv 1$, and let $\kappa \equiv K_1 / K_2$ be the relative supply of capital in country 1 to country 2. From the zero cutoff profit condition (6) for all source-destination pairs, the ratio of the foreign export productivity cutoff to the domestic productivity cutoff in each destination is obtained as:

$$\varphi_{21} / \varphi_{11} = (r_1^\kappa - 1)^{1/\sigma_{-1}} \tau_{21} (f_{21} / f_{11})^{1/\sigma_{-1}}, \quad \tau_{21} = 1^{(14)}$$

$$\varphi_{12} / \varphi_{22} = (r_1^\kappa)^{1/\sigma_{-1}} \tau_{12} (f_{12} / f_{22})^{1/\sigma_{-1}}. \quad \tau_{12} = 1^{(15)}$$

A fall in a country’s import trade cost, and/or an increase in its relative rental rate and/or its relative supply of capital, makes more foreign firms survive relative to domestic firms. From Eqs. (8), (12), and (13), all productivity cutoffs can be solved as functions of $r_1^\kappa$, $\tau_{21}$, and $\tau_{12}$:

$$\varphi_{ij} = \varphi_{ij} (r_1^\kappa, \tau_{21}, \tau_{12}), i, j = 1, 2. \quad \varphi_{ij} = 1^{(14)}$$

Using Eqs. (7), (8), and $M_{ij} = m_{ij} M_{ii}$, the capital market-clearing condition (10) is solved for $M_{ii}$. Substituting the solution into $M_{ij} = (\varphi_{ij} / \varphi_{ii})^{-\theta} M_{ii}$, we obtain:

$$M_{ij} = (\alpha / \theta) b_1^\theta \varphi_{ij}^{-\theta} / f_i^e, i, j = 1, 2. \quad \varphi_{ij} = \varphi_{ij} (r_1^\kappa, \tau_{ij})$$

This means that the mass of varieties sold from country $i$ to country $j$ is decreasing in the corresponding productivity cutoff. Our specification of the fixed entry and overhead costs implies that $M_{ij}$ does not grow proportionately with the source country’s GDP, which is consistent with the empirical findings of Bollard et al. (2014).

Walras’ law in country $i$, together with Eqs. (9), (10), and (11), implies that its balance of trade is zero:

$$M_{ij} \int_{\phi_{ij}}^{\infty} e_{ij} (\varphi) \mu_{ij} (\varphi | \varphi_{ij}) d\varphi = M_{ji} \int_{\phi_{ji}}^{\infty} e_{ji} (\varphi) \mu_{ji} (\varphi | \varphi_{ji}) d\varphi, i, j = 1, 2, j \neq i. \quad \varphi_{ij} = \varphi_{ij} (r_1^\kappa, \tau_{ij})$$

Using Eqs. (7) and (15), this is rewritten as:

$$r_{12} b_1^\theta f_{12} / f_1^e = b_2^\theta f_{21} / f_2^e.$$  

$$r_{12} b_1^\theta f_{12} / f_1^e = b_2^\theta f_{21} / f_2^e.$$  

\begin{equation}
\tau_{12} b_1^\theta f_{12} / f_1^e = b_2^\theta f_{21} / f_2^e.
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\tau_{12} b_1^\theta f_{12} / f_1^e = b_2^\theta f_{21} / f_2^e.
\end{equation}
The left- and right-hand sides of Eqs. (16) correspond to country 1’s exports and imports, respectively. The intermediate good price index in Eq. (3) is rewritten as:

\[ P_i = (\alpha^d/\lambda)^{1/(1-\sigma)} \left[ \sum_j (B_j^d/f_j^d) \phi_j^{1-\lambda} (\tau_{ij} r_j)^{1-\sigma} \right]^{1/(1-\sigma)} \equiv Q_i(\varphi_{ii}, \varphi_{ji}, r_i, \tau_{ij} r_j), i, j = 1, 2, j \neq i. \]  

(17)

Country i’s simplified intermediate good price index \( Q_i(\varphi_{ii}, \varphi_{ji}, r_i, \tau_{ij} r_j) \) is increasing in the suppliers’ productivity cutoffs \((\varphi_{ji}, \varphi_{jj})\), whereas increasing and homogeneous of degree one in the suppliers’ factor and trade costs \((r_i, \tau_{ij} r_j)\). From Eqs. (2), (4), and (17), country i’s growth rate is expressed as:

\[ \gamma_i = 1/q_i(\varphi_{ii}, \varphi_{ji}, \tau_{ij} r_j/r_i) - \delta_i - \rho_i; q_i(\varphi_{ii}, \varphi_{ji}, \tau_{ij} r_j/r_i) \equiv Q_i(\varphi_{ii}, \varphi_{ji}, 1, \tau_{ij} r_j/r_i), i, j = 1, 2, j \neq i, \]  

(18)

where \( q_i(\varphi_{ii}, \varphi_{ji}, \tau_{ij} r_j/r_i) \) is country i’s simplified intermediate good price index divided by its rental rate, meaning that \( 1/q_i(\varphi_{ii}, \varphi_{ji}, \tau_{ij} r_j/r_i) \) is country i’s gross rate of return to capital. Country i’s growth rate is decreasing in \((\varphi_{ii}, \varphi_{ji})\) whereas decreasing in \(\tau_{ij} r_j/r_i\), country i’s import trade cost divided by its relative rental rate. From Eq. (18), the growth rate of \( \kappa \) is simply given by:

\[ \dot{\kappa}/\kappa = \gamma_1 - \gamma_2 = 1/q_1(\varphi_{11}, \varphi_{21}, \tau_{21}/r_1) - \delta_1 - \rho_1 - (1/q_2(\varphi_{22}, \varphi_{12}, \tau_{12} r_1) - \delta_2 - \rho_2). \]  

(19)

With the initial condition \( \kappa_0 = K_{10}/K_{20} \) and the trade costs \( \tau_{21} \) and \( \tau_{12} \) given, Eqs. (14), (16), and (19) characterize an equilibrium path \( \{\varphi_{11t}, \varphi_{12t}, \varphi_{21t}, \varphi_{22t}, r_{11t}, \kappa_t\}_{t=0}^{\infty} \).

Finally, using Eqs. (7) and (15), country i’s revenue share of exported varieties \( \beta_i = M_{ij} \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi)\mu_{ij}(\varphi)\varphi d\varphi/\sum_k M_{ik} \int_{\varphi_{ik}}^{\infty} e_{ik}(\varphi)\mu_{ik}(\varphi)\varphi d\varphi \) is rewritten as:

\[ \beta_i = \varphi_{ij}^{-\sigma} f_{ij}/\sum_k \varphi_{ik}^{-\sigma} f_{ik} = \varphi_{ij}^{-\sigma} f_{ij}/(\varphi_{ii}^{-\sigma} f_{ii} + \varphi_{ij}^{-\sigma} f_{ij}), i, j, k = 1, 2, j \neq i. \]  

(20)

Considering the zero balance of trade, this is equal to country i’s expenditure share of imported varieties \( \zeta_i = M_{ji} \int_{\varphi_{ij}}^{\infty} e_{ji}(\varphi)\mu_{ji}(\varphi)\varphi d\varphi/\sum_k M_{ki} \int_{\varphi_{ik}}^{\infty} e_{ik}(\varphi)\mu_{ki}(\varphi)\varphi d\varphi \). Eq. (20) serves as a measure of country i’s openness.

3 Basic properties

3.1 Productivity cutoffs, extensive margins, growth rates, and openness

First of all, logarithmically differentiating Eqs. (8), (12), and (13), and solving them for \( \varphi_{ij} \equiv d\ln \varphi_{ij} = d\varphi_{ij}/\varphi_{ij} \), the rate of change in \( \varphi_{ij} \), we obtain:

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5The intermediate good price index in Eq. (3) is given by \( P_i = (\sum_j M_{ij} \int \varphi_{ij}(\varphi)^{1-\sigma} g_j(\varphi) d\varphi)^{1/(1-\sigma)} \). Using \( M_{ij} = M_j^d(1 - G_j(\varphi_{ij})) \), \( \mu_j(\varphi) = g_j(\varphi)/(1 - G_j(\varphi_{ij})) \) for \( \varphi \geq \varphi_{ij} \), \( \int_{\varphi_{ij}}^{\infty} \varphi^{1-\sigma} \mu_j(\varphi) d\varphi = (\theta/\lambda)^{1-\sigma} \), and Eq. (5), this is rewritten as \( P_i = \{(\theta/\lambda)^{1-\sigma} \sum_j M_{ij} [\tau_{ij} r_j/(\tau_{ij} r_j)]^{1-\sigma}\}^{1/(1-\sigma)} \). Substituting Eq. (15) into this, we obtain Eq. (17).

6An increase in \( \varphi_{ij} \) tends to lower \( P_i \) through an increase in the average productivity of surviving firms, whereas it tends to raise \( P_i \) through a decrease in the mass of varieties. The latter is stronger than the former if and only if \( \lambda > 0 \).
\( \hat{\varphi}_{21} = \frac{1 - \beta_2}{(1 - \beta_1) - \beta_2} \left[ -\left( \sigma \hat{r}_1 + \hat{\kappa} \right) / (\sigma - 1) + (1 - \beta_1) \hat{r}_{21} - \beta_1 \hat{r}_{12} \right] \),

(21)

\( \hat{\varphi}_{12} = \left\{ \frac{1 - \beta_1}{(1 - \beta_1) - \beta_2} \right\} \left[ -\left( \sigma \hat{r}_2 + \hat{\kappa} \right) / (\sigma - 1) - \beta_2 \hat{r}_{21} + (1 - \beta_2) \hat{r}_{12} \right] ,

(22)

\( \hat{\varphi}_{11} = -\left[ \beta_1 / (1 - \beta_1) \right] \hat{\varphi}_{21} = -\left\{ \beta_1 / (1 - \beta_1) \hat{r}_{21} - (1 - \beta_2) \hat{r}_{12} \right\} \],

(23)

\( \hat{\varphi}_{22} = -\left[ \beta_2 / (1 - \beta_2) \right] \hat{\varphi}_{21} = -\left\{ \beta_2 / (1 - \beta_1) \hat{r}_{21} - (1 - \beta_2) \hat{r}_{12} \right\} \] ,

(24)

In Eq. (21), for example, \( \partial \ln \varphi_{21} / \partial \ln r_{21} > 0 \) and \( \partial \ln \varphi_{21} / \partial \ln (r_1^\mathcal{K} \kappa) < 0 \) are consistent with the discussion right after Eqs. (12) and (13). On the other hand, \( \partial \ln \varphi_{21} / \partial \ln \tau_{12} < 0 \) can be interpreted as follows. A fall in country 2’s import trade cost \( \tau_{12} \) allows more firms in country 1 to start exporting to country 2 (i.e., \( \varphi_{12} \) increases). For country 1’s free entry condition to be restored, some low-productivity firms in country 1 exit from their domestic market (i.e., \( \varphi_{11} \) increases). Due to tougher competition in market 1, some firms in country 2 cease exporting (i.e., \( \varphi_{21} \) increases).

The following proposition states important relationships among each country’s export productivity cutoff, extensive margin of exports, openness, and growth rate:

**Proposition 1** Each country’s mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its export productivity cutoff decreases.

**Proof.** See Appendix A. ■

Obviously \( M_{ij} \) depends only and negatively on \( \varphi_{ij} \) from Eq. (15). We can also easily see from Eq. (20) that \( \beta_i \) depends only and negatively on \( \varphi_{ij} \) because the former is directly decreasing in \( \varphi_{ij} \) whereas increasing in \( \varphi_{ij} \), and \( \varphi_{ij} \) is negatively related to \( \varphi_{ij} \) from Eq. (8). Moreover, even if Eq. (18) says that \( \gamma_i \) is a function of three variables, \( \varphi_{ij}, \varphi_{ji} \), and \( \tau_{1j} r_j / r_i \), their effects are summarized to only one variable \( \varphi_{ij} \): a decrease in \( \varphi_{ij} \) decreases \( q_i \), which raises \( \gamma_i \). An implication of this proposition is that looking at either one of the four endogenous variables is sufficient for predicting the other three. For example, if a country’s growth rate rises for whatever reason, then its export productivity cutoff decreases whereas its mass and revenue share of exported varieties increase.

### 3.2 Balanced growth path

As usual, a balanced growth path (BGP) is defined as a path along which all variables grow at constant rates. In the present model, a BGP is characterized by Eqs. (14), (16), and (19), with \( \dot{\kappa} / \kappa = 0 \):

\[ \varphi_{ij}^* = \varphi_{ij} \left( (r_{ij}^\mathcal{K} \kappa)^{\tau_{21}, \tau_{12}} \right), i, j = 1, 2, \]

\[ r_{ij}^* \kappa b_i(\varphi_{ij}^*)^{-\theta} f_{12} / f_i^* = b_i(\varphi_{ij}^*)^{-\theta} f_{21} / f_2^*, \]

\[ 1/q_i (\varphi_{i1}^*, \varphi_{i2}^*, \tau_{21} / r_{ij}^*) - \delta_1 - \rho_i = 1/q_2 (\varphi_{22}^*, \varphi_{i2}^*, \tau_{12} r_{ij}^*) - \delta_2 - \rho_2, \]

where an asterisk represents a BGP.

**Proposition 2** Suppose that \( \gamma_1 - \gamma_2 > 0 \) at \( \ln \kappa = \ln \kappa \) and \( \gamma_1 - \gamma_2 < 0 \) at \( \ln \kappa = \ln \kappa \), where \( \ln \kappa \) and \( \ln \kappa \) correspond to \( \ln (\varphi_{12}/\varphi_{11}) = 0 \) and \( \ln (\varphi_{21}/\varphi_{22}) = 0 \), respectively. Then there exists a unique \( \ln \kappa^* \in (\ln \kappa, \ln \kappa) \) which is globally stable.
The growth effects in the initial period, where a fall in $\tau$ both countries. At first sight Eq. (25) seems to contradict partly with Eq. (22): the latter suggests that the direct effect, $\gamma$, become relatively cheaper (cf. Eq. (A.2)). This indirectly decreases $t$ tends to create a trade deficit for country 1 (cf. Eq. (16)). For the deficit to be cleared, its capital should increase faster than country 2. Since this pushes $\kappa$ toward $\kappa^*$, country 1’s growth rate falls whereas country 2’s growth rate rises. This process continues until the two growth rates are equalized at point A.

4 Effects of unilateral trade liberalization

General expressions for the growth rates are derived as:

$$d\gamma_1 = -[(\beta_1/q_1)/(A)]\{(\sigma - 1)\hat{\kappa} + [\theta \sigma(1 - \beta_2) + (\sigma - 1)\beta_2]} \hat{\tau}_{21} + [\theta \sigma - (\sigma - 1)](1 - \beta_2)\hat{\tau}_{12}, \tag{25}$$

$$d\gamma_2 = -[(\beta_2/q_2)/(A)]\{-(\sigma - 1)\hat{\kappa} + [\theta \sigma - (\sigma - 1)](1 - \beta_1)\hat{\tau}_{21} + [\theta \sigma(1 - \beta_1) + (\sigma - 1)\beta_1] \hat{\tau}_{12}; \tag{26}$$

$$A \equiv \sigma \theta(2 - \beta_1 - \beta_2) - (\sigma - 1)(1 - \beta_1 - \beta_2) > 0.$$

Eqs. (25) and (26) mean that, with $\kappa$ given, a fall in any import trade cost raises the growth rates of both countries. At first sight Eq. (25) seems to contradict partly with Eq. (22): the latter suggests that a fall in $\tau_{21}$ directly increases $\varphi_{12}$. In fact, an increase in $\varphi_{12}$ and a decrease in $\varphi_{21}$ caused by a fall in $\tau_{21}$ tend to create a trade deficit for country 1 (cf. Eq. (16)). For the deficit to be cleared, its capital should become relatively cheaper (cf. Eq. (A.2)). This indirectly decreases $\varphi_{12}$. Since the indirect effect outweighs the direct effect, $\gamma_1$ rises as a result.

To examine the long-run growth effects, we substitute Eqs. (25) and (26) into $d\gamma_1 = d\gamma_2$ to solve for $\hat{\kappa}^*$:

$$\hat{\kappa}^* = [1/(\sigma - 1)][1/(\beta_1^* + \beta_2^*)]
\times \{-[(\beta_1^*/q_1^*)[\theta \sigma(1 - \beta_1^*) + (\sigma - 1)\beta_2^*] - (\beta_2^*/q_2^*)[\theta \sigma - (\sigma - 1)](1 - \beta_1^*)}] \hat{\tau}_{21}
+ [(\beta_2^*/q_2^*)[\theta \sigma(1 - \beta_1^*) + (\sigma - 1)\beta_1^*] - (\beta_1^*/q_1^*)[\theta \sigma - (\sigma - 1)](1 - \beta_2^*)}] \hat{\tau}_{12}. \tag{27}$$

Substituting Eq. (27) back into either Eq. (25) or (26), we obtain:

$$d\gamma_1^* = d\gamma_2^* = -[(\beta_1^* + \beta_2^*)/(\beta_1^*/q_1^* + \beta_2^*/q_2^*)][(\hat{\tau}_{21} + \hat{\tau}_{12})]. \tag{28}$$

This implies that a fall in any import trade cost raises the balanced growth rate.

To consider the short-run growth effects of unilateral trade liberalization, e.g., a fall in $\tau_{21}$, we first see its growth effects in the initial period, where $\hat{\kappa}_0 = 0$. Eqs. (25) and (26) immediately give:

---

7They are obtained by just substituting Eq. (A.3) into Eqs. (21) and (22), and substituting them into Eq. (A.7).
\[
\frac{\partial \gamma_{10}}{\partial \ln \tau_{21}} = -[(\beta_1^*/q_1^*)/A^*] \theta \sigma (1 - \beta_1^*) + (\sigma - 1) \beta_1^* < 0,
\frac{\partial \gamma_{20}}{\partial \ln \tau_{21}} = -[(\beta_2^*/q_2^*)/A^*] \theta \sigma - (\sigma - 1)(1 - \beta_2^*) < 0,
\]

where all endogenous variables are evaluated at the old BGP.

Fig. 1 illustrates the case where \((\beta_1^*/q_1^*)[\theta \sigma (1 - \beta_1^*) + (\sigma - 1) \beta_1^*] > (\beta_2^*/q_2^*)[\theta \sigma - (\sigma - 1)(1 - \beta_2^*)]\). Suppose that we are originally at point A, the old BGP, and that \(\tau_{21}\) falls permanently to \(\tau'_{21}\) (< \(\tau_{21}\)). Then both dashed curve \(\gamma_1(\kappa; \tau_{21}, \tau_{12})\) and curve \(\gamma_2(\kappa; \tau_{21}, \tau_{12})\) shift up to solid curve \(\gamma_1(\kappa; \tau'_{21}, \tau_{12})\) and curve \(\gamma_2(\kappa; \tau'_{21}, \tau_{12})\), respectively, but the former shift is larger than the latter. The growth rates of country 1 and country 2 in the initial period are given by point B: \((\kappa^*, \gamma'_{10})\) and point C: \((\kappa^*, \gamma'_{20})\), respectively. Since country 1 starts to grow faster than country 2, \(\kappa\) starts to increase. This continues to pull down country 1’s growth rate whereas pushes up country 2’s growth rate along curve \(\gamma_1(\kappa; \tau'_{21}, \tau_{12})\) and curve \(\gamma_2(\kappa; \tau'_{21}, \tau_{12})\), respectively. The growth rates are equalized at point D: \((\kappa^{**}, \gamma^{**}_{10})\), the new BGP, which is to the northeast of point A. We observe that, compared with the old BGP, a permanent fall in \(\tau_{21}\) raises the growth rates of all countries for all periods. Moreover, from Proposition 1, this is also true for masses and revenue shares of exported varieties.

In the other case where \((\beta_1^*/q_1^*)[\theta \sigma (1 - \beta_1^*) + (\sigma - 1) \beta_1^*] < (\beta_2^*/q_2^*)[\theta \sigma - (\sigma - 1)(1 - \beta_2^*)]\), which is possible if \(\beta_1^*\) is small, the upward shift of curve \(\gamma_2(\kappa; \tau_{21}, \tau_{12})\) is now larger than curve \(\gamma_1(\kappa; \tau'_{21}, \tau_{12})\), so point C is now higher than point B. Since \(\kappa\) starts to decrease, country 1’s growth rate goes up whereas country 2’s growth rate goes down until they are equalized at point D, which is now to the northwest of point A. Even in this case, compared with the old BGP, a permanent fall in \(\tau_{21}\) raises the growth rates as well as masses and revenue shares of exported varieties of all countries for all periods.

Finally, considering that country \(i\)’s consumption in period \(t\) is given by \(C_{it} = C_{i0} \exp(\int_0^t \gamma_{is} ds) = \rho_i K_{i0} \exp(\int_0^t \gamma_{is} ds)\) under the logarithmic instantaneous utility, the fact that \(\gamma'_{is} > \gamma_{is}\) for all \(t\) implies that \(C'_{it} > C_{it}\) for all \(t\), and hence \(U'_{it} > U_{it}\). The following proposition summarizes our results:

**Proposition 3** Compared with the old BGP, a permanent fall in any import trade cost raises the growth rates of all countries for all periods, and welfare of all countries.

This result, together with Proposition 1, shows a striking coincidence with Naito (2012, Propositions 2 to 4), who embeds a continuum-good Ricardian framework with perfect competition in the intermediate good sector of the two-country Acemoglu-Ventura model. However, the mechanism behind the result is quite different. In Naito (2012), a fall in a country’s import trade cost directly raises its own growth rate, but the former indirectly lowers the latter through a fall in its relative rental rate due to the decreased demand for its capital. Since the direct effect outweighs the indirect effect, the growth rate of the liberalizing country goes up in total. On the other hand, the growth rate of the partner country also goes up because of a rise in its relative rental rate. In contrast, in the present model, a fall in a country’s import trade cost directly lowers its own growth rate whereas raises that of the partner country through changes in export productivity cutoffs. In addition, such trade liberalization lowers the relative rental rate of the liberalizing country satisfying its zero balance of trade, which has counteracting effects on countries’ growth rates. The indirect effect is stronger than the direct effect for the liberalizing country, whereas the opposite is true for the partner country.

The robust positive growth effect of (even unilateral) trade liberalization is theoretically remarkable because it is obtained even with imperfect competition and without positive externalities from knowledge.
spillovers. In the literature, trade liberalization can raise long-run growth only under international spillovers (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011) or domestic spillovers (e.g., Perla et al., 2014; Sampson, 2014). This tempts us to suppose that freer trade cannot foster economic growth in the Melitz-type models of trade and growth without positive externalities. Our result shows that the conjecture is not true.

5 Concluding remarks

This paper makes both theoretical and policy contributions. On the theoretical side, we first provide an asymmetric Melitz model of trade and growth. In the literature on heterogeneous firm models of trade and growth (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011; Perla et al., 2014; Sampson, 2014), all papers consider R&D as the engine of endogenous growth, but they cannot drop the assumption of symmetric countries. By replacing R&D with capital accumulation, we successfully formulate an asymmetric Melitz model, where a possible difference in countries’ growth rates itself creates asymmetric adjustments of their export productivity cutoffs and hence their growth rates toward convergence. On the policy side, our model supports unilateral trade liberalization as a way of promoting growth globally. Naito (2012) obtains a similar result in his two-country Acemoglu-Ventura model with a perfectly competitive intermediate good sector. This paper complements his work by showing that the growth-enhancing effect of unilateral trade liberalization is valid even under imperfect competition.

Because of its flexibility, our theoretical framework can be applied to problems involving asymmetric countries. For example, by replacing the iceberg import trade costs with the revenue-generating import tariffs, we can study the optimal tariff of each country. Compared with the static optimal tariff model of Felbermayr et al. (2013), the growth effect will pull down a country’s optimal tariff in our model. Another example is to increase the number of countries to more than two. By doing this, we can see the effects of preferential trade liberalization on both member and nonmember countries. Even the nonmember countries will partly gain from preferential trade liberalization due to the growth effect. Although these extensions might cause some technical difficulties, it is worth trying them.

Appendix A. Proof of Proposition 1

First of all, logarithmic differentiation of Eq. (15) immediately gives:

$$\hat{M}_{ij} = -\theta \hat{\phi}_{ij}, i, j = 1, 2.$$  \hspace{1cm} (A.1)

Next, logarithmically differentiating Eq. (16), and using Eqs. (21) and (22), $\hat{r}_1$ is solved as:

$$\hat{r}_1 = (1/A)\{-B\hat{\kappa} + \theta(\sigma - 1)[(1 - \beta_1)\hat{\tau}_{21} - (1 - \beta_2)\hat{\tau}_{12}]; \hspace{1cm} (A.2)$$

$$A \equiv \sigma\theta(2 - \beta_1 - \beta_2) - (\sigma - 1)(1 - \beta_1 - \beta_2) > 0,$n

$$B \equiv \theta(2 - \beta_1 - \beta_2) - (\sigma - 1)(1 - \beta_1 - \beta_2) \in (0, A).$$

Substituting Eq. (A.2) into $\sigma \hat{r}_1 + \hat{\kappa}$, and noting that $A - \sigma B = (\sigma - 1)^2(1 - \beta_1 - \beta_2)$, we obtain:
\[
(\sigma \hat{r}_1 + \hat{r})/(\sigma - 1) = (1/A)\{(\sigma - 1)(1 - \beta_1 - \beta_2)\hat{r} + \theta\sigma[(1 - \beta_1)\hat{r}_{21} - (1 - \beta_2)\hat{r}_{12}]\}. \quad \text{(A.3)}
\]

Logarithmically differentiating Eq. (17) divided by \(r_i\), \(\hat{q}_i\) is calculated as:

\[
\hat{q}_i = \sum_j[(b_j^0/f_j^0)\varphi_{ij}^{-\lambda}(\tau_{ji}r_j/r_i)^{1-\sigma}/\sum_k(b_k^0/f_k^0)\varphi_{ki}^{-\lambda}(\tau_{ki}r_k/r_i)^{1-\sigma}][-(\lambda/(1 - \sigma))\hat{\varphi}_{ji} + \hat{r}_{ji} + \hat{r}_j - \hat{r}_i].
\]

On the other hand, using Eqs. (12), (13), and (16), Eq. (20) is rewritten as:

\[
\begin{align*}
\beta_1 &= (b_2^0/f_2^0)\varphi_{21}^{-\lambda}(\tau_{21}/r_1)^{1-\sigma}/[(b_1^0/f_1^0)\varphi_{11}^{-\lambda} + (b_2^0/f_2^0)\varphi_{21}^{-\lambda}(\tau_{21}/r_1)^{1-\sigma}], \\
\beta_2 &= (b_1^0/f_1^0)\varphi_{12}^{-\lambda}(\tau_{12}r_2)^{1-\sigma}/[(b_2^0/f_2^0)\varphi_{22}^{-\lambda} + (b_1^0/f_1^0)\varphi_{12}^{-\lambda}(\tau_{12}r_2)^{1-\sigma}].
\end{align*}
\]

Using these expressions, \(\hat{q}_i\) is simplified to:

\[
\hat{q}_i = [\lambda/(\sigma - 1)][(1 - \beta_1)\hat{\varphi}_{ii} + \beta_2\hat{\varphi}_{ji}] + \beta_1(\hat{r}_{21} - \hat{r}_1). \quad \text{(A.4)}
\]

For \(i = 1\), substituting Eqs. (21) and (23) into Eq. (A.4) gives:

\[
\begin{align*}
\hat{q}_1 &= [\lambda/(\sigma - 1)][(1 - \beta_1)/(1 - \beta_1 - \beta_2)][-(2 - \beta_1 - \beta_2)(\sigma\hat{r}_1 + \hat{r})]/(\sigma - 1) + (1 - \beta_1)\hat{r}_{21} - (1 - \beta_2)\hat{r}_{12} \\
&+ \beta_1(\hat{r}_{21} - \hat{r}_1).
\end{align*}
\]

Compared with Eqs. (21) to (24), this expression involves \(\hat{r}_1\) as well as \((\sigma\hat{r}_1 + \hat{r})/(\sigma - 1), \hat{r}_{21},\) and \(\hat{r}_{12}\), so we express \(\hat{r}_1\) in terms of the other three. Solving Eq. (A.2) for \(\hat{\varphi}\), and substituting it into Eq. (A.3), \(\hat{r}_1\) is expressed as:

\[
\hat{r}_1 = \{1/[(\sigma - 1)(1 - \beta_1 - \beta_2)]\}[-B(\sigma\hat{r}_1 + \hat{r})/(\sigma - 1) + \theta(1 - \beta_1)\hat{r}_{21} - (1 - \beta_2)\hat{r}_{12}]. \quad \text{(A.5)}
\]

Substituting Eq. (A.5) into the expression for \(\hat{q}_1\), we obtain:

\[
\hat{q}_1 = [\beta_1/(1 - \beta_1 - \beta_2)][(\sigma\hat{r}_1 + \hat{r})/(\sigma - 1) - \beta_2\hat{r}_{21} + (1 - \beta_2)\hat{r}_{12}].
\]

Comparing this with Eq. (22), we have \(\hat{q}_1 = [\beta_1/(1 - \beta_1)]\hat{\varphi}_{12}\).

Similarly, for \(i = 2\), from Eqs. (22), (24), (A.4), and (A.5), we can show that:

\[
\hat{q}_2 = [\beta_2/(1 - \beta_1 - \beta_2)][-(\sigma\hat{r}_1 + \hat{r})/(\sigma - 1) + (1 - \beta_1)\hat{r}_{21} - \beta_1\hat{r}_{12}].
\]

From this and Eq. (21), we have \(\hat{q}_2 = [\beta_2/(1 - \beta_2)]\hat{\varphi}_{21}\). Therefore:

\[
\hat{q}_i = [\beta_i/(1 - \beta_i)]\hat{\varphi}_{ij}, i, j = 1, 2, j \neq i. \quad \text{(A.6)}
\]

Differentiating Eq. (18), and using Eq. (A.6), the amount of change in \(\gamma_i\) is simply given by:

\[
d\gamma_i = (1/q_i)(\hat{q}_i) = -(1/q_i)[\beta_i/(1 - \beta_i)]\hat{\varphi}_{ij}, i, j = 1, 2, j \neq i. \quad \text{(A.7)}
\]
Turning to $\beta_i$, logarithmically differentiating Eq. (20) gives $\hat{\beta}_i = -\theta(1 - \beta_i)(\hat{\varphi}_{ij} - \hat{\varphi}_{ii})$. Since $0 = (1 - \beta_i)\hat{\varphi}_{ii} + \beta_i\hat{\varphi}_{ij}$, or $\hat{\varphi}_{ij} = (1 - \beta_i)(\hat{\varphi}_{ij} - \hat{\varphi}_{ii})$ from Eq. (8), we obtain:

$$\hat{\beta}_i = -\theta\hat{\varphi}_{ij} = \hat{\kappa}_{ij}, \ i, j = 1, 2, j \neq i, \ (A.8)$$

where the last equality follows from Eq. (A.1). Eqs. (A.1), (A.7), and (A.8) imply Proposition 1.

Appendix B. Proof of Proposition 2

With $\hat{\tau}_{21} = \hat{\tau}_{12} = 0$, substituting Eq. (A.3) into Eqs. (21) and (22), and substituting them into Eq. (A.7), $d\gamma_i$ is expressed as:

$$d\gamma_1 = -(\beta_1/q_1)[(\sigma - 1)/\kappa]\hat{\kappa},$$

$$d\gamma_2 = (\beta_2/q_2)[(\sigma - 1)/\kappa]\hat{\kappa}.$$

Substituting them into the differentiated form of Eq. (19), and noting that $\dot{\kappa}/\kappa = d\ln\kappa/dt$ and $\hat{\kappa} = d\ln\kappa$, we obtain:

$$d(d\ln\kappa/dt)/d\ln\kappa = -(\beta_1/q_1 + \beta_2/q_2)[(\sigma - 1)/\kappa] < 0.$$

This implies that $\ln\kappa^*$ determined by $d\ln\kappa/dt = 0$ is unique if exists. Also, since $d\ln\kappa/dt > 0$ if and only if $\ln\kappa < \ln\kappa^*$, a BGP is globally stable.

To ensure existence, we first find the lower and upper bounds of $\ln\kappa$. From Eqs. (21) to (24) and (A.3), $\ln(\varphi_{21}/\varphi_{11})$ is increasing, whereas $\ln(\varphi_{21}/\varphi_{22})$ is decreasing, in $\ln\kappa$. Considering the assumption that $\varphi_{12}/\varphi_{11} > 1$ and $\varphi_{21}/\varphi_{22} > 1$, $\ln\kappa^*$, the lower bound of $\ln\kappa$, is determined by $\varphi_{12}/\varphi_{11} = 1$, or $\ln(\varphi_{12}/\varphi_{11}) = 0$. Similarly, $\ln\bar{\kappa}$, the upper bound of $\ln\kappa$, is determined by $\varphi_{21}/\varphi_{22} = 1$, or $\ln(\varphi_{21}/\varphi_{22}) = 0$. Then there exists $\ln\kappa^* \in (\ln\kappa, \ln\bar{\kappa})$ if $d\ln\kappa/dt = \gamma_1 - \gamma_2 > 0$ at $\ln\kappa = \ln\kappa$ and $d\ln\kappa/dt = \gamma_1 - \gamma_2 < 0$ at $\ln\kappa = \ln\bar{\kappa}$. 

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References


Fig. 1. Growth effects of a permanent fall in country 1’s import trade cost $\tau_{21}$.

Note: $i$’s growth rate $\gamma_i$ moves in the same direction as its:
- mass of exported varieties $M_{ij}$
- revenue share of exported varieties $\beta_i$