Multi-Product Firms, Quality Differentiation
and Comparative Advantage

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(Very preliminary)

Abstract

This paper introduces a new theoretical model to explore how the behavior of heterogeneous intermediate suppliers affects the product quality choice of heterogeneous final good producers. It explains the choices of quality, inputs and the export prices of a multi-product firm in response to different market characteristics. We found that a more productive firm exports a larger and higher quality range of products as well as sources their inputs from more productive suppliers. Under greater competition, a firm’s top quality- and most expensive-products would be the best-performing ones due to their lowest quality-adjusted prices. The export behavior and export product range of a firm is also affected by the endowment of its sector and country, where it is always easier to start exporting, with a larger export product range in the capital-intensive sector than in the labour-intensive sector.

JEL codes: F11, F12, L1

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1. Introduction

Recent empirical works provide more stylized facts about the differences between exporters and non-exporters as well as across exporters to different market destinations: they are not only different in “who they are” but also very different in “what they sell” and “what they buy”. It is well established that exporters often come from the comparative advantageous sectors and among the mass of heterogeneous firms in a sector, successful exporters are those more productive ones (Melitz, 2003; Bernard, Redding and Schott, 2007). For the second question, it has been found that exporters either sell fewer products (Mayer, Melitz and Ottaviano, 2014; Eckel & Neary, 2010; Bernard, Redding & Schott, 2011) or sell higher quality products (Verhoogen, 2008; Hallak & Sivadasan, 2009; Kugler and Verhoogen, 2012) compared to non-exporters. With regard to the last question, empirical studies are revealing interesting facts that exporters vary the quality of their products across market destinations by sourcing inputs of different quality levels, different prices and different origins (Manova & Zhang, 2012). Despite becoming an increasingly important focus in the empirical literature, this heterogeneity in firms’ input sourcing behaviour remains neglected theoretically.

Existing models on export product choice are unanimously relying on a powerful assumption of labour as a single production factor, and hence, must either assume away the product quality differentiation in analysing multi-product firms or assume away the common multi-product dimension of firms in analysing their product quality. As a result, the literature on export product choice is split between vertical differentiation and horizontal differentiation. Little has been known about the quality differentiation across multiple products produced by a same firm as well as the variation in the product quality range, output prices and input prices of firms operating in different sectors or countries. Furthermore, this split in the literature even exposes contradicting predictions about firms’ export pricing strategies. Specifically, the horizontal product differentiation literature predicts a lower price strategy (Mayer, Melitz and Ottaviano, 2014), whereas the vertical product differentiation literature (Hallak & Sivadasan, 2009; Kugler and Verhoogen, 2012) predicts a higher price-higher quality strategy for successful exporters.
In solving these gaps, we introduce a model to analyse the behaviour of heterogeneous final good producers and intermediate good suppliers simultaneously. Our paper attempts to show that introducing a heterogeneous intermediate sector to the standard heterogeneous firm model would provide trade theorists with a more efficient instrument to analyse firms’ behaviour from more aspects simultaneously and to explore deeper into within-firm decisions. In this model, we explore three levels of heterogeneity: across firms, across products within a firm, and across inputs within a product. Particularly, the model examines the product choice of a firm that produces multiple vertically differentiated products and sources its vertically differentiated inputs from heterogeneous intermediate suppliers.

The heterogeneity in the ability of the suppliers is translated to the heterogeneity in the aggregate prices of intermediate goods of different quality level. Particularly, only more productive intermediate suppliers can cover the higher investment to produce higher quality intermediate goods. This determines the mass of sufficiently productive suppliers that can produce at a certain quality level in the market, where there are increasingly fewer suppliers of higher quality intermediate goods. As a result, higher quality intermediate goods are scarcer and more expensive, and as a pass-through effect, higher quality final products have higher production costs and higher prices.

A firm’s product range is, therefore, endogenously determined by the interaction between its productivity, and the competition level in both the intermediate good market and the final good market. A more productive firm would be able to cover the higher production costs to produce higher quality varieties, as well as lower the production cost to be profitable even with a lower quality variety. And hence, a more productive firm exports a larger range of products of both higher and lower quality with a larger range of export prices as well as sources its inputs from more suppliers.

Therefore, the first contribution of our paper is to examine the choices of products, both horizontally and vertically, and the variation in prices across products for each exporter, which have either been forgone in existing literature. In this attempt, the paper is closely connected with a recently emerging body of literature which addresses the mentioned export
price puzzle, i.e., whether a firm exports its most expensive high-end products or its cheapest products (Eckel, Iacovone, Javorcik & Neary, 2015; Manova & Zhang, 2013 and Feenstra & Romalis, 2014). It is found in this literature that for certain sectors where the goods are rather homogenous, the competition on prices would be more efficient; while for more differentiated goods sectors, exporters better succeed with quality competition and higher prices. Our model, on the other hand, suggests that the price pattern is the same in any sector, where the top expensive high-end varieties would be the best-performing ones for an exporter due to their lower quality-adjusted prices. The association between price and the perceived quality level is, however, less pronounced in the labour-intensive sector, which further clarifies the long and short quality ladder across sectors found in Khandewal (2010).

Second, to our best knowledge, this paper is the first to explore the heterogeneity in the input sourcing behaviour across exporters of different ability and origins. It provides explanations to the recently found stylized facts in international trade that a firm that exports more offers a wider range of export prices and pays a wider range of input prices as well as source inputs from more origins (Manova & Zhang, 2012).

Third, given the different scarcity of intermediate goods in each country and the intensive use of intermediate goods in each sector, we are able to draw an insight about how country and sector characteristics would affect the product scope and quality of exporters. In this way, the paper contributes to the analysis on comparative advantage and heterogeneous firms’ behaviour initiated by Bernard, Redding and Schott (2007). In our model, the distribution of firm productivity in a country determines the abundance of productive intermediate good suppliers, and thus, the abundance of quality intermediate goods and their prices. On the other hand, the intensive use of intermediate goods in relative to labour in a final good production determines the extent of impact that the intermediate good cost has on the price and profit of a final good producer. Therefore, the endowments of a country and of a sector do not only determine the mass of exporters but also determine the quality level and the scope of products that a firm exports.
The rest of the paper is organized as follows. Section 2 presents the basic model structure and describes the problems facing the consumers, the final good producers and the intermediate good suppliers. Section 3 derives the market aggregation and solves for the equilibrium solutions. Based on these derived solutions, section 4 analyses the choices of product scope, quality, inputs and prices of each exporter. Section 5 explores how the above choices are affected by the competition levels in different sectors and section 6 provides a numerical exercise to illustrate the major predictions. Finally, section 7 concludes the paper and maps out potential extensions to the model.

2. Model

The model considers two symmetric countries, each has two production sectors, a final good sector and an intermediate good sector. In each sector, there is a continuum of firms differentiated by their labour productivity \( \varphi \), in which each firm produces a continuum of quality-differentiated varieties of either a final good or an intermediate good. Consistent with the standard heterogeneous firm model, both final good producers (hereinafter final producers, indexed by \( c \)) and intermediate good suppliers (hereinafter suppliers, indexed by \( i \)) are assumed to have their productivities randomly drawn from a same nation-wide Pareto productivity distribution \( G(\varphi) \), and its probability density function:

\[
g(\varphi) = \begin{cases} 
  k \varphi^{k-1}, & \varphi \geq \varphi_0, \\
  0, & \text{otherwise}, 
\end{cases}
\]  

[1]

where the shape parameter \( k \) describes the dispersion of firms’ productivity distribution and \( \varphi_0 \) is the minimum value of \( \varphi \). It should be noted that subscripts \( c \) and \( i \), which are convenient notations for firms in each sector, is analogous to \( \varphi \).

A variety is defined by its perceived quality \( \lambda \), and its producer (a.k.a. brand). This perceived quality is solely determined by the quality of its employed intermediate goods, where the intermediate good can be understood as a material to produce the final good or a technical design embodying skills and brain value of the final good. The highest quality of a variety in
a country, $\lambda_M$, is assumed to be given as it is bounded by that country’s national technology frontier, i.e., $\lambda \in (0, \lambda_M]$.

The final good can be traded between the two countries with a “melting-iceberg” transportation cost of $\tau$ per unit, $\tau \geq 1$, i.e., $\tau$ units have to be shipped for one unit to arrive at the foreign country. Intermediate goods are, whereas, assumed to be non-tradable, so the final producer can only source its intermediate goods from its local suppliers. The model assumes away the FDI option, and the only access to a foreign market is via exporting.

A final producer is required to invest a fixed cost $f_\tau > 0$ to enter the foreign market. This fixed cost comprises the firm’s spending on marketing activities to make its product recognizable to the customers and its distribution setup costs in the new market.

2.1. Final consumer

Consumers face a continuum of varieties of the final good which are differentiated by their quality and brands. The preference of a representative consumer over a continuum of final good varieties has a standard Dixit-Stiglitz constant elasticity of substitution (CES) function:

$$ U = \int_0^{\lambda_M} \left( \frac{\theta-1}{\theta} Q(\lambda)^{\frac{\theta}{\theta-1}} \right) \frac{\theta}{\theta-1} d\lambda, $$

where $Q(\lambda)$ is a consumption index, which also takes a CES form,

$$ Q(\lambda) = \left[ \int_0^{n_\lambda} q_c(\lambda) \frac{\sigma-1}{\sigma} dc + \int_0^{n_\lambda} q_c^e(\lambda) \frac{\sigma-1}{\sigma} dc \right]^{\frac{\sigma}{\sigma-1}}. $$

In this expression, $q_c(\lambda)$ is the consumption of a domestic variety and $q_c^e(\lambda)$ is the consumption of an imported variety of quality $\lambda$ and brand $c$; $n_\lambda$ and $n_\lambda^e$ are the (endogenous) number of final good domestic brands and foreign brands available at quality level $\lambda$ in the market, $\theta > 1$ is the elasticity of substitution between two quality-differentiated varieties.
from a same brand, and $\sigma > 1$ is the elasticity of substitution between two brand-differentiated varieties of the same quality.

A representative consumer maximizes her utility subject to the budget constraint:

$$\max_{q_c(\lambda), q_c^r(\lambda)} \quad U(Q(\lambda))$$

$$\text{s.t.} \quad \int_0^{\lambda_M} \left[ \int_0^n q_c(\lambda)p_c(\lambda)dc + \int_0^n q_c^r(\lambda)p_c^r(\lambda)dc \right] d\lambda = Y,$$

where $Y$ is the national income, $p_c(\lambda)$ and $p_c^r(\lambda)$ are respectively the domestic price and export price of a final good variety of quality $\lambda$ and brand $c$. This is a two-steps optimization problem. First, to minimize her spending, subject to the target consumption index $Q(\lambda)$ at each quality level, the representative consumer will choose the following consumption of a domestic variety

$$q_c(\lambda) = Q(\lambda)P(\lambda)^{\sigma}p_c(\lambda)^{-\sigma},$$

and of an imported final good variety

$$q_c^r(\lambda) = Q(\lambda)P(\lambda)^{\sigma}p_c^r(\lambda)^{-\sigma}.$$

In the above solution, $P(\lambda)$ is the price index of the final good varieties of quality $\lambda$:

$$P(\lambda) = \left[ \int_0^n p_c(\lambda)_{1-\sigma} dc + \int_0^n p_c^r(\lambda)_{1-\sigma} dc \right]^{\frac{1}{1-\sigma}}. \quad [2]$$

Second, knowing the prices of the final good at different quality levels, the representative consumer chooses her consumption index at each quality level, $Q(\lambda)$, to maximize the utility, subject to her budget constraint, and the optimal solution is given by

$$Q(\lambda) = YP^{\theta-1}P(\lambda)^{-\theta} \lambda^{\theta-1},$$

where $P$ is the (quality-adjusted) aggregate price of all the final good varieties:
In order to simplify the derivation further below, while maintaining the dual heterogeneity (in terms of quality and ability) in the supply side, which is the major focus of this model, the two elasticities of substitution $\theta$ and $\sigma$ are assumed to be equal.\footnote{Without this assumption, the final equilibrium solutions, which determine a firm’s choice of products, will depend on the interaction among three elasticities of substitution, including two elasticities across final good varieties, and one elasticity across intermediate good varieties, instead of depending on two elasticities as in this version. While this assumption helps to simplify the derivation, it does not change the model’s major results. On the other hand, it is common in reality that consumers often view brand as a cue for quality. Since high quality goods are, most of the time, produced by good firms, there is a strongly perceived association between good brands and good products. In other words, the variation in consumer’s preferences across brands may originate from the variation in consumer’s expectation about the quality of goods from those brands, which supports the plausibility of this assumption.} This assumption implies that, taking into account a same change in the relative price, there would be a same change in the relative consumption of any two varieties, where they can be either two quality-differentiated varieties of a same brand, or two brand-differentiated varieties of the same quality.

The final demand for each domestic final good variety and each imported variety are, therefore, given by

$$q_c(\lambda) = Y p^{\theta-1} \lambda^{-\theta} \lambda^{\theta-1},$$  \hspace{1cm} [4]

and

$$q^*(\lambda) = Y p^{\theta-1} \lambda^{-\theta} \lambda^{\theta-1}.$$  \hspace{1cm} [5]

\subsection*{2.2. Final producer}

Each final producer $c$ produces a continuum of quality-differentiated varieties of the final good $\lambda_c \in \Lambda_c$, where $\Lambda_c$ is the set of varieties produced by the firm (hereinafter a firm’s product range) and endogenous to its productivity, $\varphi_c$. As assumed, the national technology frontier, which determines the highest quality possible in a country, is exogenous from an
individual firm’s decision, so each firm’s product range is bounded, \( \Lambda_c \in (0, \lambda_{\max}] \). At each quality level, there is an endogenous continuum of firms producing horizontally differentiated varieties of the same quality, and hence, monopolistic competition takes place in the market. Firms are assumed to live for one period, i.e., they need to pay all the relevant fixed costs in the beginning of any new period.

### 2.2.1. Production technology

A final good variety of a certain quality is produced using homogenous labour and a composite intermediate good of the corresponding quality. By employing a composite intermediate good of quality \( \lambda \) in the production, a final producer can produce a final good variety with the perceived quality \( \lambda \). This composite intermediate good is a composition of horizontally differentiated intermediate good varieties of the same quality, which can be sourced from various suppliers.\(^3\)

The production of a final good variety is, therefore, expressed via the following function

\[
q_c(\lambda) = \varphi_c L_c(\lambda)^{1-\beta} I_c(\lambda)^\beta, \tag{6}
\]

where \( L_c(\lambda) \) is the employed labour, parameter \( \beta \) measures the intermediate good intensity of the product, \( \beta \in (0,1) \), and the composite intermediate good \( I_c(\lambda) \) has a CES form

\[
I_c(\lambda) = \left( \int_0^{n_{i\lambda}} q_c(\lambda_i) \frac{\varepsilon-1}{\varepsilon} d\lambda \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{7}
\]

In this function, \( q_c(\lambda_i) \) is the quantity of each intermediate good variety of quality \( \lambda \) that a final producer \( c \) purchases from a supplier \( i \), \( n_{i\lambda} \) is the endogenous mass of suppliers that produce \( \lambda \)-quality intermediate good varieties, and \( \varepsilon > 1 \) is the constant elasticity of substitution between any two intermediate good varieties of quality \( \lambda \) from different brands.

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\( ^3 \) This is common in reality, where in producing a quality good, all of its inputs have to reach a same standard. This can also apply to a multi-type intermediate input case, where intermediate goods of different quality levels are incompatible to each other in the final production.
In this model, labour is homogenous and the size of the local labour market is assumed to be sufficiently large that labour can be hired from the local market at a constant wage rate $w$.

In order to produce a variety with quality $\lambda$, each final producer needs to invest a fixed cost $f_c(\lambda) = \lambda^r$, where $r > 0$, and is industry-specific. This quality investment can be understood as an R&D investment to have a production capacity compatible with the targeted quality of the firm’s product, and is increasing in the targeted product quality $\lambda$.\(^4\) It does not affect the productivity of the firm, nor is it affected by the productivity of the firm.

### 2.2.2. Total profit maximization problem

A final producer’s problem is to choose its domestic and export product ranges $\Lambda_c$ and $\Lambda_{cx}$, as well as the output and price of each variety within its product ranges to maximize its total profits, subject to its production technology and the consumer’s demand. The total profit for each final producer from both the domestic and export markets is expressed by:

$$\pi(\Lambda_c + \Lambda_{cx}) = \int_{\lambda \in \Lambda_c} \pi_c(\lambda) d\lambda + \int_{\lambda \in \Lambda_{cx}} \pi_{c^x}(\lambda) d\lambda,$$

where $\pi_c(\lambda)$ and $\pi_{c^x}(\lambda)$ are, respectively, its profit from selling each variety of quality $\lambda$ to the domestic market and the foreign market.

Since there is no constraint other than firm’s productivity on firm’s ability, the production decision of each variety can be made independently from each other. The total profits maximization problem facing each final producer is, thus, reduced to two problems: (i) to maximize the profit from each variety in each market, $\pi_c(\lambda)$ and $\pi_{c^x}(\lambda)$, and (ii) to maximize its product ranges by producing any variety which yields non-negative optimal profit.

It is at a firm’s discretion to choose both its inputs and outputs to minimize its production cost and maximize its profit of each of its variety.

\(^4\) This assumption of increasing investment in quality is in the same spirit with Sutton (1991, 1998), Hallak & Sivadasan (2009), and Kugler & Verhoogen (2012). Among these papers, Hallak & Sivadasan (2009) differ from my model in their assumption that more productive firms are able to lower the quality investment cost.
2.2.3. Production cost minimization decision

Since a final producer can only source its intermediate inputs from local suppliers, the cost minimization problem for a firm, either an exporter or a non-exporter is the exactly the same. Each final producer chooses the amount of each input, in terms of labour and intermediate goods to minimize its production cost subject to its production technology, where the production cost for each variety comprises labour cost, intermediate good costs and the fixed quality investment,

\[ TC_c(\lambda) = L_c(\lambda)w + \int_0^{n_j \lambda} q_c(\lambda_i) p_i(\lambda) \, di + f_c(\lambda), \tag{9} \]

and \( p_i(\lambda) \) is the price of each intermediate good variety with quality \( \lambda \) and brand \( i \).

This is a two-steps cost minimization, where each firm chooses the amount of each intermediate good variety \( q_c(\lambda_i) \) to minimize its total spending on the composite intermediate goods \( I_c(\lambda) \):

\[
\min_{q_c(\lambda_i)} \int_0^{n_j \lambda} q_c(\lambda_i) p_i(\lambda) \, di \\
\text{subject to:} \quad \left( \int_0^{n_j \lambda} q_c(\lambda_i) \frac{\varepsilon^{-1}}{\varepsilon} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} = I_c(\lambda). 
\]

Solving this cost minimization problem yields the following amount of intermediate goods that a final producer \( c \) sources from each intermediate supplier \( i \)

\[ q_c(\lambda_i) = I_c(\lambda) P_i(\lambda)^{\frac{\varepsilon-1}{\varepsilon}} p_i(\lambda)^{-\frac{1}{\varepsilon}}, \tag{10} \]

where \( P_i(\lambda) \) is the price index of all \( \lambda \)-quality intermediate good varieties, and is given by

\[ P_i(\lambda) = \left[ \int_0^{n_j \lambda} p_i(\lambda)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}. \tag{11} \]

Knowing the price of the intermediate goods, the firm then chooses its relative use of intermediate goods \( I_c(\lambda) \) and labour \( L_c(\lambda) \) to minimize its total production cost, subject to its technology:
A final producer will choose to employ the two factors, labour and intermediate good, such that

\[ L_c(\lambda) = \frac{1 - \beta}{\beta} \frac{P_t(\lambda)}{w} I_c(\lambda). \]  

By substituting [13] into the production function of the final good [6], we can write the firm’s demand for intermediate goods as a function of its productivity and its output quantity:

\[ I_c(\lambda) = \frac{q_c(\lambda)}{\varphi_c} \left( \frac{\beta w}{(1 - \beta) P_t(\lambda)} \right)^{1 - \beta}. \]  

Substituting \( L_c(\lambda) \) from [13] and \( I_c(\lambda) \) from [14] into the total cost function [12], then taking its derivative with respect to \( q_c(\lambda) \) give us the marginal cost of production, which is constant with regard to \( q_c(\lambda) \):

\[ C_c(\lambda) = \frac{1}{\varphi_c} \left( \frac{w}{1 - \beta} \right)^{1 - \beta} \left( \frac{P_t(\lambda)}{\beta} \right)^{\beta}. \]  

### 2.2.3.1. Profit maximization problem

Because the domestic market and the export market are symmetric, and a firm pays neither the transportation cost nor the export entry cost in serving the domestic market, a rational firm would never export a certain variety alone without selling the same variety in the domestic market, and hence, the fixed quality investment for each variety incurs only once in the domestic market.
Hence, in facing the final consumer’s for each variety, given by [4] and [5], a firm will choose the price and quantity of each variety to maximize its domestic profit:

\[ \pi_c(\lambda) = p_c(\lambda)q_c(\lambda) - C_c(\lambda)q_c(\lambda) - f_c(\lambda), \]

and its export profit:

\[ \pi^e_c(\lambda) = p^e_c(\lambda)q^e_c(\lambda) - \tau C_c(\lambda)q^e_c(\lambda) - f_x. \]

Given the iso-elastic demand in each market, a firm will charge an optimal price equal to a constant markup of its marginal cost for a domestic variety,

\[ p_c(\lambda) = \frac{\theta}{\theta - 1} C_c(\lambda) = \frac{B}{\phi_c} P_l(\lambda)^\beta, \]  

\[ \text{[16]} \]

and a constant markup of its marginal cost and the iceberg transportation cost for an export variety,

\[ p^e_c(\lambda) = \frac{\theta}{\theta - 1} \tau C_c(\lambda) = \frac{\tau B}{\phi_c} P_l(\lambda)^\beta, \]  

\[ \text{[17]} \]

where for notation simplification:

\[ B = \frac{\theta}{\theta - 1} \left( \frac{w}{1 - \beta} \right) 1^{-\beta} \left( \frac{1}{\beta} \right)^\beta, B > 0. \]

\[ \text{[18]} \]

It can be seen that the final good prices are increasing in the marginal cost of production, and hence, decreasing in the firm’s productivity and increasing in the intermediate good price index. The optimal profits from selling a variety of quality \( \lambda \) in the domestic market and the export market are as follows:

\[ \pi_c(\lambda) = \frac{Y}{\theta} \left( \frac{\lambda \phi_c}{BP_l(\lambda)^\beta} \right)^{\theta - 1} - \lambda', \]

\[ \text{[19]} \]
\[ \pi_c^X(\lambda) = \frac{Y}{\theta} \left[ \lambda \phi_c \frac{P}{\tau BP_1(\lambda)^\beta} \right]^{\theta-1} - f_X. \]  

2.2.4. The export benchmark quality and export productivity cutoff

Firm \( c \) will produce a variety of quality \( \lambda \) if \( \pi_c(\lambda) \geq 0 \), and it will export this variety if both \( \pi_c(\lambda) \geq 0 \) and \( \pi_c^X(\lambda) \geq 0 \). As shown in [19] and [20], both domestic profit and export profit are increasing in firm productivity. Denote \( \phi(\lambda) \) and \( \phi^X(\lambda) \) as the zero-profit productivity cutoffs to supply a \( \lambda \)-quality variety in the domestic market and the export market, respectively, their specifications are given by:

\[ \phi(\lambda) = \frac{\tau - \theta + 1}{\lambda - \theta - 1} P_1(\lambda)^\beta B P^{-1} \left( \frac{\theta}{Y} \right)^{\frac{1}{\theta - 1}}, \]  

\[ \phi^X(\lambda) = \frac{\tau}{\lambda} P_1(\lambda)^\beta B P^{-1} \left( \frac{\theta f_X}{Y} \right)^{\frac{1}{\theta - 1}}. \]

Let’s consider a benchmark quality \( \lambda_H \) such that \( \phi(\lambda_H) = \phi^X(\lambda_H) \), it follows that

\[ \lambda_H = \left( \frac{\tau^{\theta-1} f_X}{Y} \right)^{\frac{1}{\theta - 1}}. \]

It can be easily checked that:

\[ \left\{ \begin{array}{l} \phi^X(\lambda) > \phi(\lambda) \quad \forall \lambda < \lambda_H \\ \phi^X(\lambda) \leq \phi(\lambda) \quad \forall \lambda \geq \lambda_H. \end{array} \right. \]

It means that for varieties in the low quality range, \( \lambda < \lambda_H \), the productivity cutoff to export is higher than the productivity cutoff to sell to the domestic market, so firm has to be more productive to export. Specifically, only more productive firms with \( \phi_c \geq \phi^X(\lambda) \) can export a \( \lambda \)-quality variety, while those firms with \( \phi^X(\lambda) > \phi_c \geq \phi(\lambda) \) can sell this \( \lambda \)-quality variety to the domestic market only.
For varieties in the higher quality range, $\lambda \geq \lambda_H$, the productivity cutoff to export is lower than the productivity cutoff to serve the domestic market. It means that if a firm can make zero-profit in the domestic market with a $\lambda$-quality variety, it will also be profitable in exporting this variety. In this case, the domestic productivity cutoff becomes the export productivity cutoff for these high quality varieties.

Denote $\varphi^X$ as the productivity cutoff to produce a $\lambda_H$-quality variety domestically. It follows from the definition of $\lambda_H$ that $\varphi^X = \varphi(\lambda_H) = \varphi^x(\lambda_H)$, where $\varphi(\lambda)$ is given by [21], hence, $\varphi^X$ is specified as follows:

$$
\varphi^X = \lambda_H \frac{r-\theta+1}{\theta-1} P_l(\lambda_H)^{\beta} B P^{-1} \left(\frac{\theta}{Y}\right)\frac{1}{\sigma-1}.
$$

[24]

**Proposition 1. The minimum export productivity cutoff**

(1.1) $\lambda_H = \left(\frac{r-\theta+1}{\theta-1} f_x\right)^{\frac{1}{\theta}}$ is the benchmark quality level to export and $\varphi^X$ is the productivity cutoff to succeed with at least one variety in the export market, i.e., the minimum export productivity cutoff.

(1.2) The minimum export productivity cutoff $\varphi^X$ is increasing in the variable export cost $r$ and the fixed export cost $f_x$.

Since $\varphi^X$ is the zero-profit productivity cutoff to export a $\lambda_H$-quality variety, and the export profit $\pi^x_\tau(\lambda)$ given by [20] is increasing in productivity, any firm with a productivity higher or equal to $\varphi^X$ is profitable in exporting a $\lambda_H$-quality variety. Hence, all firms with $\varphi_c \geq \varphi^X$ will export.

A low productive firm (with a productivity lower than $\varphi^X$), whereas, can only produce low quality varieties $\lambda < \lambda_H$. Due to the firm’s low productiveness, these low quality varieties are only available at high production costs and prices, and hence, fail to be exported. Therefore, all firms with $\varphi_c < \varphi^X$ do not export (See Appendix 1 for proof).
It can be seen that $\lambda_H$ is increasing in both the fixed trade cost $f_x$ and the variable trade cost $\tau$. Therefore, under a reduction of either $f_x$ or $\tau$, the quality benchmark to export will be lower. Consequently, the minimum export productivity cutoff will also be lower.

Trade would be prohibited between the two countries if the export quality benchmark exceeds the maximum quality level a country can produce, i.e., $\lambda_H = \left(\frac{\tau^{\theta-1}f_x}{\theta} \right)^{\frac{1}{\theta}} > \lambda_M$. That case occurs when either the transportation cost between the two countries or the fixed entry cost to export is extremely high. For trade between the two countries to occur, it is assumed that the common technology frontier is such high that:

$$\lambda_H = \left(\frac{\tau^{\theta-1}f_x}{\theta} \right)^{\frac{1}{\theta}} \leq \lambda_M.$$ 

At this stage, our productivity cutoffs depends on the intermediate goods price index $P_l(\lambda)$ and the final good price index $P$. As shown in [11], $P_l(\lambda)$ is determined by the mass of suppliers at each quality level and the price of each intermediate good variety, and hence, it is dependent on the behaviour of the supplier in the intermediate goods market.

### 2.3. Intermediate good supplier

Similar to the final producer, each supplier has its labour productivity $\varphi_i$ randomly drawn from the nationwide productivity distribution $G(\varphi)$. Each supplier produces a continuum of quality-differentiated varieties of the intermediate good, $\lambda_i \in \Lambda_i$, where the product range $\Lambda_i$ is endogenously determined by its productivity and $\Lambda_i \in (0, \lambda_M]$.

#### 2.3.1. Production technology

An intermediate good variety is produced using only labour via the following production function

$$q_i(\lambda) = \varphi_i L_i(\lambda).$$  \[25\]
It is assumed that each intermediate good variety from the same supplier is produced with the same amount of labour per unit, depending on the supplier’s ability \( \varphi_i \), i.e., \( L_i(\lambda) = L_i \) and is constant for each supplier. The quality of an intermediate good variety within a supplier’s product range is generated from the quality investment for that variety. Particularly, similar to the final producer case, a supplier needs to invest a fixed cost \( f_i(\lambda) = \lambda^r \) to produce a variety with the respective quality \( \lambda \). This fixed cost does not affect the productivity of the firm, nor is it affected by the productivity of the firm.

Parameter \( r \) is interpreted as the difficulty level to generate quality goods, equivalently \( \frac{1}{r} \) is the scope for quality differentiation. As common in the product quality literature (Verhoogen & Kugler, 2012), this scope for quality differentiation characterizes the effectiveness of R&D spending in improving the technical dimensions of quality or the effectiveness of advertising expenditures in raising the perceived quality of the firm’s output.

Given this production technology, the marginal cost of an intermediate good variety is given by:

\[
C_i(\lambda) = \frac{\partial TC_i(\lambda)}{\partial q_i(\lambda)} = \frac{w}{\varphi_i}. \tag{[26]}
\]

### 2.3.2. Profit maximizing decision

Similarly to the final producer, a supplier’s total profits come from every single-variety profit over its product range, where the profit maximization of each single-variety can be solved separately. In maximizing its profit from each variety, a supplier takes the demand for its output from each final producer derived in [10] and [27] as given. Particularly, it faces the following demand for its output variety \( \lambda_i \):

\[
q_i(\lambda) = \int_0^{n_\lambda} q_c(\lambda_i)dc + \int_0^{n_\lambda} q^*_c(\lambda_i)dc = P_i(\lambda)^e p_i(\lambda) - \epsilon \left[ \int_0^{n_\lambda} I_c(\lambda)dc + \int_0^{n_\lambda} I^*_c(\lambda)dc \right],
\]
where, as defined earlier, \( n_\lambda \) and \( n_\lambda^x \) are the mass of domestic final producers and exporting final producers that purchase \( \lambda \)-quality intermediate goods. \( q_\lambda^x(\lambda_i) \) is the demand for an intermediate good variety of an exporting firm, which has a similar specification to [10]:

\[
q_\lambda^x(\lambda_i) = I_\lambda^x(\lambda)P_i(\lambda)^\epsilon p_i(\lambda)^{-\epsilon},
\]

where \( I_\lambda^x(\lambda) \) is firm \( c \)'s total demand for \( \lambda \)-quality intermediate goods with the same specification as [14]:

\[
I_\lambda^x(\lambda) = q_\lambda^x(\lambda) \frac{1}{q_c(1 - \beta)P_i(\lambda)}^{1-\beta}.
\]

Let

\[
D_\ell(\lambda) = \int_0^{n_\lambda} I_\ell(\lambda) d\lambda + \int_0^{n_\lambda^x} I_\ell^x(\lambda) d\lambda,
\]

where it can be seen that \( D_\ell(\lambda) \) is the aggregate demand for intermediate good varieties of quality \( \lambda \) faced by a supplier.

A supplier will choose its price and output to maximize its profit from each intermediate good variety:

\[
\max_{p_i(\lambda), q_i(\lambda)} \quad \pi_i(\lambda) = q_i(\lambda)p_i(\lambda) - q_i(\lambda)C_i(\lambda) - f_i(\lambda) \quad [30]
\]

s.t. \( q_i(\lambda) = P_i(\lambda)^\epsilon p_i(\lambda)^{-\epsilon}D_i(\lambda) \).

Given the constant marginal cost [26] and the iso-elasticity demand function, by the first order condition, a supplier \( i \) would maximize its profit by setting the following price:

\[
p_i(\lambda) = \frac{\epsilon}{\epsilon - 1} \frac{w}{\varphi_i},
\]

where note that this price depends on the firm’s productivity and does not depend on the quality \( \lambda \).
At this optimal price, the supplier’s optimal profit from each intermediate good variety is as follows

$$\pi_i(\lambda) = \frac{1}{\varepsilon} D_i(\lambda) P_i(\lambda)^\varepsilon \left( \frac{\varepsilon - 1}{\varepsilon W} \right)^{\varepsilon-1} \phi_i^{\varepsilon-1} - \lambda^\varepsilon. \tag{33}$$

Since $\pi_i(\lambda)$ is increasing in $\phi_i$, it follows that the zero-profit productivity cutoff for a supplier to produce a $\lambda$-quality intermediate good variety is:

$$\phi_i(\lambda)^{\varepsilon-1} = \frac{\varepsilon \lambda^\varepsilon}{D_i(\lambda) P_i(\lambda)^\varepsilon \left( \frac{\varepsilon W}{\varepsilon - 1} \right)^{\varepsilon-1}}. \tag{34}$$

This supplier’s productivity cutoff is decreasing in the price index of intermediate good varieties $P_i(\lambda)$, and the aggregate demand for intermediate goods $D_i(\lambda)$, which is, in turn, determined by the aggregate behaviour of the final producers.

3. Market aggregation and equilibrium

3.1. Market aggregation

Market equilibrium in the open economy is characterized by the three productivity cutoffs $\phi(\lambda)$, given by [21], $\phi^x(\lambda)$, given by [22] for the final good market, and $\phi_i(\lambda)$, given by [34] for the intermediate good market. Together, these productivity cutoffs determine the aggregate variables, which will be derived below, including the aggregate demand for intermediate goods and the price index in each market. The quality benchmark $\lambda_H$ defined in [23] is important to derive these aggregate variables because it separates two types of final producers, exporting firms and domestic-oriented firms.

Is should be noted that, our aggregate variables are integrated over firms, which are denoted as $c$ or $i$ in each respective market, while firms in each market follows the cumulative distribution function $G(\phi)$. Therefore, in integrating a certain variable over firms, the differentiation with regard to firm, either $dc$ or $di$, is equivalent to $dG(\phi)$, where $dG(\phi) = g(\phi) d\phi$ and $g(\phi) = k \phi^k \phi^{-(k+1)}$ as defined in [1].
3.1.1. Aggregate demand for intermediate goods by quality

Substituting \( I_c(\lambda) \) from [14] and \( I_c^x(\lambda) \) from [28] into the specification of \( D_I(\lambda) \) in [29] gives

\[
D_I(\lambda) = \left[ \frac{\beta w}{(1 - \beta) P_I(\lambda)} \right]^{1 - \beta} \left[ \int_0^{n_\lambda} \frac{1}{\varphi_c} q^e(\lambda) dc + \int_0^{n_{\lambda}^x} \frac{1}{\varphi_c} q^x(\lambda) dc \right].
\]

Using the optimal outputs, \( q_c(\lambda) \) in [4] and \( q^x(\lambda) \) in [5], and the optimal prices in each market, \( p_c(\lambda) \) in [16] and \( p^x(\lambda) \) in [17], the aggregate demand for intermediate goods can be rewritten as follows:

\[
D_I(\lambda) = YP^{\beta - 1} P_I(\lambda)^{\beta - \beta - 1} \lambda^{\beta - 1} \int_0^{n_\lambda} \frac{1}{\varphi_c} q^e(\lambda) dc + \int_0^{n_{\lambda}^x} \frac{1}{\varphi_c} q^x(\lambda) dc.
\]

Note that the mass of firms supplying domestically \( n_\lambda \) and exporting \( n_{\lambda}^x \) have been now identified by the two productivity cutoffs \( \varphi(\lambda) \) and \( \varphi^x(\lambda) \). According to Proposition 1, all firms that produce a high quality variety \( (\lambda \geq \lambda_H) \) also export this variety, i.e., \( \varphi^x(\lambda) = \varphi(\lambda) \) and \( n_{\lambda}^x = n_\lambda \). For the low quality varieties \( (\lambda < \lambda_H) \), however, only more productive firms export, i.e., \( \varphi^x(\lambda) > \varphi(\lambda) \), thus, \( n_{\lambda}^x < n_\lambda \).

i) **For high quality range \( \lambda \geq \lambda_H \),** the aggregate demand for intermediate goods is an aggregation of intermediate good demand from firms that both export and sell to the domestic market:

\[
D_I(\lambda) = (1 + \tau^{-\theta}) YP^{\beta - 1} P_I(\lambda)^{\beta - \beta - 1} \lambda^{\beta - 1} \int_0^{\varphi(\lambda)} \phi^{\beta - 1} g(\phi) d\phi.
\]

ii) **For low quality range \( \lambda < \lambda_H \),** the aggregate demand for intermediate goods is an aggregation of input demand from two types of firm, exporters and non-exporters:

\[
D_I(\lambda) = YP^{\beta - 1} P_I(\lambda)^{\beta - \beta - 1} \lambda^{\beta - 1} \left[ \int_0^{\varphi(\lambda)} \phi^{\beta - 1} g(\phi) d\phi + \tau^{-\theta} \int_{\varphi^x(\lambda)}^{\infty} \phi^{\beta - 1} g(\phi) d\phi \right].
\]
3.1.2. Intermediate good price index by quality

$P_I(\lambda)$ is the aggregation of $p_I(\lambda)$ across intermediate good varieties of the same quality $\lambda$ from surviving suppliers, i.e., supplier with $\varphi_i \geq \varphi_I(\lambda)$. Given the specification of $p_I(\lambda)$ in [32], this aggregate price is expressed as follows:

$$P_I(\lambda) = \left(\frac{\epsilon W}{\epsilon - 1}\right) \left[\int_{\varphi_I(\lambda)}^{\infty} \varphi^{\epsilon-1} g(\varphi) d\varphi\right]^{\frac{1}{1-\epsilon}}. \quad [37]$$

3.1.3. Final good price indices

**Final good price index by quality $P(\lambda)$**

$P(\lambda)$, given by [2], is an aggregation of variety price $p_c(\lambda)$ over all the domestic final producers that produce quality $\lambda$ and $p_c^x(\lambda)$ over all the foreign producers that export $\lambda$-quality final good to this market. Substituting the optimal domestic price $p_c(\lambda)$ and export price $p_c^x(\lambda)$ from [16] and [17] to [2] gives the following final good price index by quality

$$P(\lambda) = P_I(\lambda)\beta B \left[\int_0^{n_\lambda} \varphi_c^{\theta-1} dc + \tau^{1-\theta} \int_0^{n_\lambda^x} \varphi_c^{\theta-1} dc\right]^{\frac{1}{1-\theta}}. \quad [38]$$

(i) For the high quality range ($\lambda \geq \lambda_H$):

All firms that produce a $\lambda$-quality final good variety will export that variety as well, i.e., $\varphi^x(\lambda) = \varphi(\lambda)$ and $n_\lambda^x = n_\lambda$, so the specification of $P(\lambda)$ for the high quality range is as follows:

$$P(\lambda) = \left(1 + \tau^{1-\theta}\right)^{\frac{1}{1-\theta}} P_I(\lambda)\beta B \left[\int_{\varphi(\lambda)}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi\right]^{\frac{1}{1-\theta}} \quad \forall \lambda \geq \lambda_H. \quad [38]$$

(ii) For the low quality range ($\lambda < \lambda_H$):

As discussed, only more productive firms can export a low quality variety, i.e., $\varphi^x(\lambda) > \varphi(\lambda)$, so the final good price index for a low quality level is:
\[ P(\lambda) = P_1(\lambda)^\beta \left\{ \int_0^{\phi(\lambda)} \phi^{\theta-1} g(\varphi) d\varphi + \tau^{1-\theta} \int_{\phi(\lambda)}^{\phi^x(\lambda)} \phi^{\theta-1} g(\varphi) d\varphi \right\}^{\frac{1}{1-\theta}} \forall \lambda < \lambda_H. \] 

**Final good price index \( P \)**

From its specification in [3], the final good price index \( P \) is an aggregation of the quality-adjusted price index \( \frac{P(\lambda)}{\lambda} \) over all levels of quality \( \lambda \in (0, \lambda_{M}] \). Therefore, given the above price index by quality, the aggregate price index of all the final good varieties can be written as:

\[ P = B \left\{ \int_0^{\lambda_H} \frac{1}{\lambda} P_1(\lambda)^\beta \left[ \int_0^{\phi(\lambda)} \phi^{\theta-1} g(\varphi) d\varphi + \tau^{1-\theta} \int_{\phi(\lambda)}^{\phi^x(\lambda)} \phi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{1-\theta}} d\lambda \right\}^{\frac{1}{\frac{1}{1-\theta}}} + 
\]

\[ (1 + \tau^{1-\theta}) \int_{\lambda_H}^{\lambda_M} \frac{1}{\lambda} P_1(\lambda)^\beta \left[ \int_0^{\phi(\lambda)} \phi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{1-\theta}} d\lambda \right\}^{\frac{1}{1-\theta}}. \]

### 3.2. Equilibrium

#### 3.2.1. Equilibrium solution

The market equilibrium is determined by:

(i) the mass of final producers in the domestic final good market, which is determined by the domestic productivity cutoff \( \phi(\lambda) \), given by [21],

(ii) the mass of exporters in the final good market, which is determined by the export productivity cutoff \( \phi^x(\lambda) \) given by [22],

(iii) the mass of the suppliers in the intermediate good market, which is determined by the productivity cutoff of the suppliers \( \varphi(\lambda) \) given by [34],

These productivity cutoffs in each market depend on the aggregate variables derived above, including the aggregate demand for intermediate goods, given by [35] and [36], and the price indices in each market, given by [38], and [40]. Hence, the equilibrium is characterized by the following set of six simultaneous equations, with six endogenous variables:
\[ \varphi(\lambda) = \frac{r - \theta + 1}{\lambda^{\theta}} BP^{-1} P_l(\lambda) Y \left( \frac{\theta}{Y} \right)^{\frac{1}{\beta - 1}}, \]

\[ \varphi^x(\lambda) = \frac{T}{\lambda} BP^{-1} P_l(\lambda)^{\beta} \left( \frac{\theta^x Y}{Y} \right)^{\frac{1}{\beta - 1}}, \]

\[ \varphi_l(\lambda)^{\varepsilon - 1} = \frac{\ell Y^\varepsilon}{D_l(\lambda) P_l(\lambda)^{\varepsilon-1}} \left( \frac{\ell W}{\varepsilon - 1} \right)^{\varepsilon - 1}, \]

\[
D_l(\lambda) = \begin{cases} 
YP^{\theta - 1} P_l(\lambda)^{\beta - \beta - 1} \lambda^{\theta - 1} \frac{\beta (\theta - 1)}{\beta} \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi, & \lambda \geq \lambda_H \\
YP^{\theta - 1} P_l(\lambda)^{\beta - \beta - 1} \lambda^{\theta - 1} \frac{\beta (\theta - 1)}{\beta} \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi + \tau^{\theta - 1} \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi, & \lambda < \lambda_H. 
\end{cases}
\]

\[ P = B \int_{\lambda_H}^{\lambda_M} \left[ \frac{1}{\lambda} P_l(\lambda)^{\theta - 1} \left[ \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi + \tau^{1 - \theta} \int_{\varphi^x(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi \right]^{\frac{1}{1-\theta}} d\lambda \right]^{1-\theta} \]

Note that in order to have definite solutions to the integral terms, the following conditions, which are popular in the heterogeneous firm literature, must hold:5

\[
\begin{cases} 
\ell > \theta - 1 \\
\ell > \varepsilon - 1. 
\end{cases} \tag{41}
\]

Solving the above simultaneous equations by the substitution method gives solutions to the six endogenous variables, namely \( \varphi(\lambda), \varphi^x(\lambda), \varphi_l(\lambda), D_l(\lambda), P_l(\lambda) \) and \( P \) as functions of exogenous parameters.

---

5 Given the specification of \( g(\varphi) \) from [1], the integrals \( \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi \) and \( \int_{\varphi^x(\lambda)}^\infty \varphi^{\theta - 1} g(\varphi) d\varphi \) can be rewritten as \( k^{\frac{\theta}{k - \theta + 1}} \int_{\varphi(\lambda)}^\infty \varphi^{\theta - 1 - (k + 1)} d\varphi \) and \( k^{\frac{\theta}{k - \varepsilon + 1}} \int_{\varphi^x(\lambda)}^\infty \varphi^{\theta - 1 - (k + 1)} d\varphi \), which can be simplified to \( \frac{k^{\frac{\theta}{k - \theta + 1}} \varphi(\lambda)^{\theta - 1 - k}}{k^{\frac{\theta}{k - \theta + 1}} \varphi(\lambda)^{\theta - 1 - k}} \) and \( \frac{k^{\frac{\theta}{k - \varepsilon + 1}} \varphi^x(\lambda)^{\theta - 1 - k}}{k^{\frac{\theta}{k - \varepsilon + 1}} \varphi^x(\lambda)^{\theta - 1 - k}} \) under condition [41].
In deriving these solutions, it is useful to denote the following terms:

\[ Z(\tau) = \left[ \frac{\kappa \phi k}{k - \theta + 1} \frac{\theta M(\tau)}{Y} \right] \frac{1}{\tau^2}, \]  

where

\[ M(\tau) = \int_0^{\lambda_H} \lambda^{r - \theta + 1}(\theta - 1)(\epsilon - 1) d\lambda + \int_{\lambda_H}^{\lambda} \frac{\beta (k - \epsilon + 1)}{\beta (k - \epsilon + 1)} d\lambda, \]  

and

\[ A = \left[ \frac{\epsilon W (k - \epsilon + 1) k^2}{\epsilon - 1} \right] \left[ \frac{\epsilon (k - \theta + 1)}{\beta (\theta - 1)} \right] \frac{k - \epsilon + 1}{k^2 (\epsilon - 1)}, A > 0. \]

\( M(\tau) \) can be understood as the mass of available varieties across all quality levels in a country, and \( Z(\tau) \) is an indication of the country’s competition level, which will be elaborated further below.

The solution to the price index of the intermediate goods of quality \( \lambda \) can be written as

\[ P_1(\lambda) = \begin{cases} A \lambda^{-(r - \theta + 1)(k - \epsilon + 1)} \left[ \frac{Z(\tau)^{k - \epsilon + 1}}{k \phi k} \right]^{\frac{1}{\tau^2}} \left[ 1 + \tau^{-\theta} \right] \frac{\beta (k - \epsilon + 1)}{\beta (k - \epsilon + 1)} & \forall \lambda \geq \lambda_H \ [44] \\
A \lambda^{-(r - \theta + 1)(k - \epsilon + 1)} \left[ \frac{Z(\tau)^{k - \epsilon + 1}}{k \phi k} \right]^{\frac{1}{\tau^2}} \left[ 1 + \tau^{-\theta} \left( \frac{\lambda}{\lambda_H} \right)^{\frac{r(k - \theta + 1)}{\theta - 1}} \right] \frac{\beta (k - \epsilon + 1)}{\beta (k - \epsilon + 1)} & \forall \lambda < \lambda_H, [45] \end{cases} \]

The domestic productivity cutoff to produce a variety of quality \( \lambda \) is given by:

\[ \varphi(\lambda) = \begin{cases} Z(\tau) \lambda^{-(r - \theta + 1)(\epsilon - 1)} \left[ 1 + \tau^{-\theta} \right] \frac{\beta (k - \epsilon + 1)}{k (\epsilon - 1) - \beta (k - \epsilon + 1)} & \forall \lambda \geq \lambda_H [46] \\
Z(\tau) \lambda^{-(r - \theta + 1)(\epsilon - 1)} \left[ 1 + \tau^{-\theta} \left( \frac{\lambda}{\lambda_H} \right)^{\frac{r(k - \theta + 1)}{\theta - 1}} \right] \frac{\beta (k - \epsilon + 1)}{k (\epsilon - 1) - \beta (k - \epsilon + 1)} & \forall \lambda < \lambda_H, [47] \end{cases} \]
The productivity cutoff to export each quality level is given by:

\[
\varphi^x(\lambda) = \begin{cases} 
Z(\tau, \lambda, \theta, \eta, \mu, \kappa) \frac{r}{Z(\tau, \lambda, \theta, \eta, \mu, \kappa)} \frac{1}{Z(\tau, \lambda, \theta, \eta, \mu, \kappa)} \left[ 1 + \theta \left( \frac{\lambda}{\lambda_H} \right)^{\frac{r}{\kappa}} \right]^{-1} \\
\forall \lambda \geq \lambda_H [48]
\end{cases}
\]

Note that, for the high quality range, the two productivity cutoffs \(\varphi^x(\lambda)\) and \(\varphi^x(\lambda)\) are the same given the discussion about the quality benchmark in section 2.2.4.

The productivity cutoff for suppliers to produce an intermediate good of a certain quality is solved as

\[
\varphi^x(\lambda) = \begin{cases} 
\varphi(\lambda) \left[ \frac{\varepsilon(k - \varepsilon + 1)(k - \theta + 1)}{\beta(\theta - 1)} \right] \frac{1}{Z(\tau, \lambda, \theta, \eta, \mu, \kappa)} \left[ 1 + \theta \left( \frac{\lambda}{\lambda_H} \right)^{\frac{r}{\kappa}} \right]^{-1} \\
\forall \lambda \geq \lambda_H [50]
\end{cases}
\]

Substituting the above market-specific solutions into a firm’s specific functions, particularly the firm profit functions \([19], [20]\) in the domestic market and export market, and the export price function in \([17]\), yields the following results.

The domestic profit and export profit from a variety are as follows

\[
\pi_c(\lambda) = \frac{R_c(\lambda)}{\theta} - \lambda^r, \tag{52}
\]

and

\[
\pi_c(\lambda) = \tau^{1-\theta} \frac{R_c(\lambda)}{\theta} - f_x, \tag{53}
\]

where the domestic revenue from a variety \(R_c(\lambda)\) is given by
This domestic profit function and the following export profit function are the key equations for the analysis about a firm’s export product choice in the next section. Finally, one of our focused variables, the export price of the final good variety is given by

\[ R_c(\lambda) = \begin{cases} 
\theta \lambda \left(\frac{(\theta - 1)(\epsilon - 1) - \beta(k - \epsilon + 1)}{\epsilon - 1} \right)^{\theta - 1} \left[ \frac{\varphi_c}{Z(\tau)} \right]^{\theta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \beta(k - \epsilon + 1)(\theta - 1) \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}}, \\
\theta \lambda \left(\frac{(\theta - 1)(\epsilon - 1) - \beta(k - \epsilon + 1)}{\epsilon - 1} \right)^{\theta - 1} \left[ \frac{\varphi_c}{Z(\tau)} \right]^{\theta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \beta(k - \epsilon + 1)(\theta - 1) \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}} \right] \right] \forall \lambda \geq \lambda_H \quad [54]
\end{cases} \]

\[ p^* (\lambda) = \begin{cases} 
\frac{\tau B A^{\beta}}{\varphi_c} \left[ \frac{\beta(k - \epsilon + 1)(r - \theta + 1)}{\theta - 1} \right] \left[ \frac{Z(\tau)}{k \varphi^k} \right]^{\beta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \frac{\beta(k - \epsilon + 1)}{\theta - 1} \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}}, \\
\frac{\tau B A^{\beta}}{\varphi_c} \left[ \frac{\beta(k - \epsilon + 1)(r - \theta + 1)}{\theta - 1} \right] \left[ \frac{Z(\tau)}{k \varphi^k} \right]^{\beta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \frac{\beta(k - \epsilon + 1)}{\theta - 1} \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}} \right] \right] \forall \lambda \geq \lambda_H \quad [56]
\end{cases} \]

\[ p^* (\lambda) = \begin{cases} 
\frac{\tau B A^{\beta}}{\varphi_c} \left[ \frac{\beta(k - \epsilon + 1)(r - \theta + 1)}{\theta - 1} \right] \left[ \frac{Z(\tau)}{k \varphi^k} \right]^{\beta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \frac{\beta(k - \epsilon + 1)}{\theta - 1} \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}}, \\
\frac{\tau B A^{\beta}}{\varphi_c} \left[ \frac{\beta(k - \epsilon + 1)(r - \theta + 1)}{\theta - 1} \right] \left[ \frac{Z(\tau)}{k \varphi^k} \right]^{\beta - 1} \left[ 1 + \tau^{-\theta} \right] \left[ \frac{\beta(k - \epsilon + 1)}{\theta - 1} \right]\epsilon - 1 \right]^{\theta - 1})^{-\theta}} \right] \right] \forall \lambda < \lambda_H \quad [57]
\end{cases} \]

3.2.2. Equilibrium Conditions

As well documented in the empirical literature, the price of a product is increasing in its quality level. Hence, for the equilibrium solution to yield an interpretation that is meaningful and consistent with this stylized fact, the final good price should be an increasing function of the product’s perceived quality \( \lambda \). The condition for \( p^* (\lambda) \), given by [56], to be increasing in \( \lambda \) is:

\[ \frac{\beta(k - \epsilon + 1)(r - \theta + 1)}{(\theta - 1)((\epsilon - 1) - \beta(k - \epsilon + 1))} > 0, \]

or equivalently either of the following conditions:

\[ A: \quad \epsilon - 1 < k < (\epsilon - 1) \left[ 1 + \frac{1}{\beta} \right] \text{ and } r > \theta - 1 \quad [58] \]

\[ B: \quad k > (\epsilon - 1) \left[ 1 + \frac{1}{\beta} \right] \text{ and } r < \theta - 1. \]
Recent empirical studies have provided reliable estimations of $k$ across industries and countries. Spearot (2014) found that the average shape parameter is 2 and the median shape estimate is 1.47 across all manufacturing industries across 56 countries. The countries with the lowest shape parameter are Belgium – Luxembourg, New Zealand and Poland with the shape parameter of 1.07, 0.54 and 1.02 respectively. The countries with the highest shape parameters are Uruguay, Quata, Chile and Bolivia with the values of 3.57, 3.69, 3.09 and 3.08. On the other hand, the mean estimated value of the elasticity of substitution $\theta$ across all SITC-3 sectors, as shown in Mohler (2009) is from 2.8 to 48, with median value ranging from 2.1 to 4.8. Broda & Weinstein (2008) also provides a mean estimated elasticity of substitution of 4 and a median estimation of 2.2. Or as shown in the calibration works of Melitz and Redding (2013) and Head, Mayer and Thoenig (2013), the elasticity of substitution between varieties is set at 4, which is taken from the estimation of Bernard, Eaton, Jensen and Kortum (2003) with plant-level U.S. manufacturing data.

The empirical evidence of a low $k$ value range and relatively high elasticity of substitution (note that the elasticity of substitution between substitutable intermediate good varieties of the same quality, $\varepsilon$ would be even higher than $\theta$), therefore, provides good supports for condition [58.A]. Henceforth, condition [58.A] is assumed to be satisfied.

On the other hand, let's consider conditions for an industry to exist in a country given the equilibrium solutions. For simplicity, consider the final producer’s profit function in [52] in the autarky case, i.e., $\tau$ goes to infinity, its derivative with respect to quality $\lambda$ is given by:

$$
\frac{\partial \pi_c(\lambda)_{\text{autarky}}}{\partial \lambda} = \left[ \frac{\varphi_c}{\tilde{z}_{\text{autarky}}} \right]^{\theta-1} \frac{(\theta - 1)(\varepsilon - 1) - r\beta(k - \varepsilon + 1)}{(\varepsilon - 1) - \beta(k - \varepsilon + 1)} \lambda^{(\theta-1)(\varepsilon-1)-r\beta(k-\varepsilon+1)} - r\lambda^{r-1},
$$

where $z_{\text{autarky}} = \left[ \frac{k\varphi k}{k-\theta+1} \int_0^{\lambda_M} \lambda^{r-\frac{k(\varepsilon-\theta+1)(\varepsilon-1)}{(\varepsilon-1)-\beta(k-\varepsilon+1)}} d\lambda \right].$

It can be seen that $\frac{\partial \pi_c(\lambda)_{\text{autarky}}}{\partial \lambda} < 0 \quad \forall \lambda, \varphi_c > 0$, if $(\theta-1)(\varepsilon-1)-r\beta(k-\varepsilon+1) \leq 0$.

Meanwhile, $\pi_c(0) = 0$, so in that case, $\pi_c(\lambda)_{\text{autarky}} < 0 \quad \forall \lambda > 0$. 

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This is a situation, where no firm would ever choose any positive level of product quality and the industry fails to exist. Therefore, for an industry to exist in a country, the following condition must hold:

\[
\frac{(\theta - 1)(\varepsilon - 1) - r\beta(k - \varepsilon + 1)}{(\varepsilon - 1) - \beta(k - \varepsilon + 1)} > 0,
\]

or equivalently

\[
r < \frac{(\theta - 1)(\varepsilon - 1)}{\beta(k - \varepsilon + 1)}. \tag{59}
\]

This condition suggests that for an industry to exist in a country, the quality investment needs to be sufficiently efficient in raising the product quality \((r)\) should be sufficiently low). Note that the RHS of inequality [59] is decreasing in \(k\) and \(\beta\). Hence, as \(k\) increases (ceteris paribus), it can only tolerate a lower value of \(r\). In other words, a sector which requires high quality investment would not exist in a low productive country. On the other hand, as \(\beta\) decreases, the RHS of inequality [59] increases and it can tolerate higher values of \(r\). In other words, a sector which requires very high quality investment would exist in a low productive country if only it is more labour-intensive.

Combining conditions [41], [58.A] and [59], the necessary conditions/ assumptions for the equilibrium solutions can be summarised in the following set of final conditions, which will be applied to all analysis henceforth:

\[
\begin{align*}
\varepsilon > 1, \theta > 1 \\
1 > \beta > 0 \\
\varepsilon - 1 < k < (\varepsilon - 1)\left[1 + \frac{1}{\beta}\right] \\
\theta - 1 < r < \frac{(\theta - 1)(\varepsilon - 1)}{\beta(k - \varepsilon + 1)}
\end{align*} \tag{60}
\]
4. Firm’s export product choice

Using the solutions to a firm’s profit from each variety in the domestic market and the export market above, the second important decision of the firm, which is to maximize its product range in each market, can now be discussed.

4.1. The export productivity cutoff by quality

Note that there are two export productivity cutoffs in this model, the minimum export productivity cutoff \( \varphi^X \) to export at least one variety as indicated in Proposition 1, and \( \varphi^x(\lambda) \), which is the productivity cutoff to export a variety of a certain quality (or, equivalently, to export a certain range of varieties) and is discussed below.

**Proposition 2. The export productivity cutoff by quality**

2.1. The productivity cutoff to produce a high quality intermediate good variety is increasing in the quality of that variety: \( \varphi_1(\lambda) \), given by [50], is increasing in \( \lambda \), \( \forall \lambda \geq \lambda_H \).

2.2. The productivity cutoff to produce and export a high quality final good variety is increasing in the quality of that variety: \( \varphi^x(\lambda) \), given by [48], is increasing in \( \lambda \), \( \forall \lambda \geq \lambda_H \).

2.3. The productivity cutoff to export a low quality final good variety is decreasing in the quality of that variety: \( \varphi^x(\lambda) \), given by [49], is decreasing in \( \lambda \), \( \forall \lambda < \lambda_H \).

See Appendix 2 for proof.

Proposition 2.1 shows that it requires more productive suppliers to produce higher quality intermediate goods. Intuitively, only the more productive suppliers are able to cover the respective higher quality investment in producing high quality intermediate goods. Hence, as quality rises, the intermediate good market becomes more concentrated in a smaller mass of sufficiently productive suppliers, and hence, the intermediate good price index increases.

Proposition 2.2 states that it requires more productive firms to produce and export higher quality final goods. The intuition is that only a more productive firm can pay the high
production cost to produce a high quality variety. Firstly, the high quality final good variety requires higher quality investment fixed cost. Secondly, to produce high quality final good varieties, it requires high quality intermediate goods, which are only available at a higher cost, as indicated from Proposition 2.1.

On the other hand, Proposition 2.3 states that it requires more productive firms to export a lower quality variety. Intuitively, lower quality varieties do not raise enough demand and revenue to cover the fixed entry cost to export. Only those higher productive firms, which can produce each variety at a lower cost, are able to be profitable with exporting a lower quality variety.

Our results are consistent with Verhoogen & Kugler’s (2012) results that higher ability firms produce higher quality outputs and use higher quality inputs. Note that Verhoogen & Kugler (2012) discusses this relationship in a single product firm framework, while our model formalizes the result in a multi-product firm framework and offers an additional result as shown in Proposition 2.3.

**Figure 1 Export productivity cutoff and export product range**
Figure 1 provides a graphical illustration of the productivity cutoff to export a variety across different quality levels. In the diagram with quality $\lambda$ on the horizontal axis and firms’ productivity $\varphi$ on the vertical axis, the upward dotted curve represents the domestic productivity cutoff by quality, and the V-shaped smooth curve represents the export productivity cutoff corresponding to each quality level.

It can also be seen in Figure 1 that the horizontal distance between the two branches of the V-shaped export productivity cutoff curve is the export quality range corresponding to each level of firm’s productivity. The specification of the two quality bounds $\lambda_{c}\lambda$ and $\lambda_{c}$ that define each firm’s export product range will be shown below.

### 4.2. The export quality range

This section focuses on the product choice of an exporting firm only, i.e., a firm with productivity $\varphi_{c} \geq \varphi^{X}$.

#### 4.2.1. The upper bound of the export quality range

As argued in section 2.2.4, if a firm succeeds with a variety $\lambda \geq \lambda_{H}$ domestically, it will export this variety as well, so the upper bound of the domestic quality range and the export quality range are the same. As a direct result from Proposition 2.2, an exporter with productivity $\varphi_{c} > \varphi^{X}$ is able to produce varieties of higher quality, $\lambda > \lambda_{H}$, so this quality bound lies in the high quality range, and thus, can be derived from the domestic profit function in [52]. This domestic profit function can be rearranged as:

$$\pi_{c}(\lambda) = \lambda \left\{ \frac{\beta(k-\varepsilon+1)}{(\varepsilon-1)^{\beta(k-\varepsilon+1)}} \left( \frac{\varphi_{c}}{Z(\tau)} \right) \Gamma(\varepsilon-1)^{-\beta(k-\varepsilon+1)} \right\}^{\theta-1} \left(1 - \frac{r-\theta+1}{\Gamma(\varepsilon-1)^{-\beta(k-\varepsilon+1)}} \right),$$

for all $\lambda > \lambda_{H}$, where the power terms of $\lambda$ are positive under condition [60].

Denote
\[
\overline{\lambda}_c = \left[ \frac{\varphi_c}{Z(\tau)} (1 + \tau^{-\theta}) \beta(k-\varepsilon+1) \right] \frac{(\theta-1)[(\varepsilon-1)-\beta(k-\varepsilon+1)]}{(\theta-1)[(\varepsilon-1)-\tau\beta(k-\varepsilon+1)]}. [61]
\]

It can be easily checked that:

\[
\begin{cases}
\pi_c(\lambda) < 0 & \forall \lambda > \overline{\lambda}_c \\
\pi_c(\lambda) \geq 0 & \forall 0 \leq \lambda \leq \overline{\lambda}_c.
\end{cases}
\]

Hence, a firm will produce any quality level \( \lambda \in (0, \overline{\lambda}_c] \) (note that, the productivity cutoff is increasing in the quality level of the variety, so if a firm is able to produce variety \( \lambda_H \), it will be able to produce all varieties in the range \( \lambda \in (0, \lambda_H]) \). Therefore, \( \overline{\lambda}_c \) is the highest quality variety that firm \( c \) can produce and export.

**4.2.2. The lower bound of the export quality range**

Let \( \overline{\lambda}_{cX} \) be the lower bound of the export quality range, i.e., the lowest quality variety that would still generate non-negative profit for firm \( c \) in the export market. A firm with the minimum export productivity cutoff \( \varphi^X \) will export variety \( \lambda_H \) only, i.e., \( \overline{\lambda}_{cX} = \overline{\lambda}_c = \lambda_H \).

While, as suggested from Proposition 2.3, a higher productive firm with productivity \( \varphi_c > \varphi^X \) can export a lower quality variety, i.e., \( \lambda_{cX} < \lambda_H \), and thus, \( \overline{\lambda}_{cX} \) can be specified by the profit function for the low quality range.

Since \( \pi_c^X(\lambda) \), given by [53], is strictly increasing in \( \lambda \), \( \forall \lambda > 0 \), the zero-profit quality is the lowest quality variety \( \lambda_{cX} \) a firm can export. It should be noted that a closed form solution for \( \lambda_{cX} \) cannot be obtained given the rather complex function of \( \pi_c^X(\lambda) \) in the low quality range.

Instead, the zero-profit quality level \( \lambda'_{cX} \), derived from the profit specification in the high quality range, can serve as a lower bounded level for \( \lambda_{cX} \) (see Appendix 3), i.e.,

\[
\lambda'_{cX} = \left[ \frac{Z(\tau)}{\varphi_c} \lambda_H \beta^{-1} \right] \frac{\beta(k-\varepsilon+1)}{\beta(k-\varepsilon+1)} \frac{(\theta-1)[(\varepsilon-1)-\beta(k-\varepsilon+1)]}{(\theta-1)[(\varepsilon-1)-\tau\beta(k-\varepsilon+1)]}. [62]
\]
To summarize, the export product range of a sufficiently productive firm can be defined by \( \left( \lambda'_{cX}, \overline{\lambda}_c \right) \), where the two quality bounds \( \overline{\lambda}_c \) and \( \lambda'_{cX} \) are defined in [61] and [62].

One direct result from this specification of the export product range is that firm drops its low quality varieties in exporting. Specifically, given its domestic quality range \( (0, \overline{\lambda}_c] \), firm will only export varieties in the quality range \( (\lambda'_{cX}, \overline{\lambda}_c] \), while dropping all the varieties in the low quality range \( (0, \lambda'_{cX}] \). Intuitively, only the higher quality variety can generate enough demand and revenue to cover the fixed entry cost to export, while the least quality varieties fail to do so.

This result resembles Bernard et al.’s (2011) in the sense that firms will drop their low quality varieties in exporting, though as discussed, Bernard et al. (2011) did not discuss the source of quality differentiation across within-firm products.

### 4.3. Firm’s productivity and its export quality range

This section explores the correlation between a firm’s productivity and its export quality range as well as its demand for corresponding intermediate goods.

**Proposition 3. Firm’s productivity and its export quality range**

3.1.  A more productive firm is able to export a larger range of quality-differentiated varieties. Specifically, the lower quality bound \( \lambda'_{cX} \), given by [62] is decreasing in \( \varphi_c \), and the upper quality bound \( \overline{\lambda}_c \), given by [61], is increasing in \( \varphi_c \).

3.2.  A more productive firm demands for a larger variety of intermediate goods and sources them from more suppliers.

The solutions to \( \lambda'_{cX} \) and \( \overline{\lambda}_c \) in [61] and [62] are the power functions of \( \varphi_c \) and it can be easily checked that
Hence, as the firm’s productivity increases, $\lambda'_{cX}$ decreases and $\lambda_c$ increases, resulting in a larger export product range $\left(\lambda'_{cX}, \lambda_c\right)$, i.e., the firm is able to export both higher quality varieties as well as lower quality varieties. Since each export variety has a different export price depending on its quality level, proposition 3.1 suggests that a more productive firm owns a wider range of export prices.

Proposition 3.2 follows directly from proposition 3.1. Since a more productive firm produces a greater continuum of quality-differentiated varieties, of both higher quality range and lower quality range, it consequently demands for a greater variety of intermediate goods from various suppliers.

This result lends good supports to the recent empirical findings by Manova and Zhang (2012) that firms that export more would charge higher export prices as well as set a wider range of export prices, and firms that export more and offer a wider range of export prices pay a wider range of input prices and source inputs from more origins.

## 4.4. Revisiting the export price puzzle: Better quality or cheaper cost?

As discussed in section 3.2.2, under condition [60], the final good price is increasing in its quality level. This higher price comes from the higher price to purchase higher quality intermediate goods, $P_1(\lambda)$, as suggested from Proposition 2.1. Since firms drop their lower quality varieties in exporting, it follows that firms export those higher quality, and hence higher priced varieties. Therefore, this model supports the empirical findings by Manova & Zhang (2013) that exporting firms succeed with the higher quality varieties in the foreign market. We will show below that these higher quality varieties generate higher export sales and profits for firms, while are accompanied by lower quality-adjusted selling prices, which validate the assumption of firms’ core competency in the lowest cost varieties.
Proposition 4. A firm's core products and pricing decision

4.1. A higher quality variety generates higher revenues and export profit for the firm, i.e., \( R_c(\lambda), R^x_c(\lambda) \) and \( \pi^x_c(\lambda) \) are increasing in \( \lambda \forall \lambda > 0 \).

4.2. A higher quality variety has a lower quality-adjusted export price. Let \( \hat{p}^x_c(\lambda) = \frac{1}{\lambda} p^x_c(\lambda) \) be the quality-adjusted export price of a \( \lambda \)-quality variety, \( \hat{p}^x_c(\lambda) \) is increasing in \( \lambda \; \forall \lambda > 0 \).

4.3. The positive correlation between quality and the final good price is more pronounced in the intermediate good-intensive sector.

It can be easily checked that the domestic revenue \( R_c(\lambda) \), given by [54] and [55], is increasing in \( \lambda \forall \lambda > 0 \) under condition [60]. So do the export revenue given by \( R^x_c(\lambda) = \tau^{-\theta} R_c(\lambda) \), and the export profit given by [52]. Therefore, higher quality varieties are sold better in both the domestic market and the export market. As shown in [62], the export quality threshold is increasing in the fixed entry cost, hence, only these top quality varieties of the firm would be successful in those tougher export markets. Therefore, a firm’s core products are its best quality, and at the same time, its most costly and highest priced varieties.

On the other hand, the quality-adjusted price \( \hat{p}^x_c(\lambda) \), which is derived by dividing the price functions in [56] and [57] by \( \lambda \), is decreasing in \( \lambda \) under the equilibrium condition [60] (see Appendix 4). This suggests that the perceived quality of a variety rises faster than its production cost and its selling price, or each unit of quality can be generated with a decreasing marginal cost. In that spirit, the common assumption that a firm’s core products are its cheapest ones employed in the earlier non-quality-differentiation firm models (Mayer et al., 2014; Eckle & Neary, 2010) can be validated and this model’s result can be interpreted in a consistent way to their established results, i.e., each firm specializes in its lowest quality-adjusted priced products in exporting to tougher markets.

This prediction also provides a strong theoretical support to the established empirical evidence that export prices and revenues are positively correlated across products within a
manufacturer, i.e., a firm’s top-selling varieties tend to be their most expensive articles (Manova and Zhang, 2013).

Given the assumption of homogenous labour input, the higher price of a high quality variety comes from the higher cost of its high quality intermediate goods. This pass-through effect is, therefore, larger in an intermediate good-intensive sector, where the intermediate goods cost accounts for a major part of the production cost (i.e., those sectors with a higher $\beta$). Particularly, as shown in the export price function [17]:

$$p_i^x(\lambda) = \frac{\tau B}{\varphi_c} P_t(\lambda)^\beta,$$

the higher the value of $\beta$, the higher the effect of a change in the intermediate good price index on the final good price will be.

This result suggests that the positive correlation between the firm’s best-selling product and its price, which is often regarded as a good empirical evidence for the quality-based competition strategy by successful exporters, is more pronounced in the intermediate good-intensive sector than in the labour-intensive sector. This prediction finds a strong empirical support from Khandewal (2010), who found significant evidence in American firms that there is a “long quality ladder” in the capital-intensive sectors, where prices vary considerably and price variation well reflects the variation in product quality. In the more labour-intensive sectors, however, a short quality ladder is found, where prices vary less across products, and hence, is not a good measurement of quality.

Since the product quality is differentiated by the intermediate goods and labour is homogenous, the intermediate good-intensive sector corresponds to the differentiated good sector while the labour-intensive sector would correspond more to the non-differentiated good sector. In that sense, this prediction also provides a better lens to see why the empirical evidence of a firm’s top selling varieties as its most expensive ones is very significant in the differentiated good sector but insignificant in the non-differentiated good sector as documented in Eckel et al. (2015) or Fan, Li and Yeaple (2014).
It should be also noted that, while a firm always exports its top quality varieties, the quality level of its top varieties is affected by the market competition level as will be discussed in section 5. For example, for an existing exporter, under higher competition, its quality upper bound is lower, i.e., it will drop the previously top quality variety in its production range. It also means that a firm’s current best quality and core product may be its previously second-best quality variety. Hence, in comparing firm’s product range before and after a shock in the competition level, we may observe a quality decrease in firm’s best-selling variety, which corresponds to a decrease in the price of the best-selling variety.

5. Market competition and a firm’s product range

5.1. Specification of the market competition level

The productivity cut-off to produce and export a certain high quality variety \( \varphi^x(\lambda) \), given by [46], comprises three components corresponding to its determining factors,

\[
\varphi(\lambda) = Z(\tau) \lambda^{(\theta - 1)(\varepsilon - 1) - \beta(k - \varepsilon + 1)} \left[ 1 + \tau^{-\theta} \right] k((\varepsilon - 1) - \beta(k - \varepsilon + 1))^{-1}.
\]

(i) The first component \( Z(\tau) \) represents the demand-pull factor, i.e., the market competition effect, which will be the focus of the analysis below. Specifically, a higher competition in the market depresses the demand share and profit for each firm, hence, pushes up the productivity cutoff.

(ii) The second component \( \lambda^{(\theta - 1)(\varepsilon - 1) - \beta(k - \varepsilon + 1)} \) represents the cost-push factor, i.e., the productivity cutoff is increasing in the intermediate good price index \( P_t(\lambda) \). Specifically, it requires higher quality, thus, more expensive intermediate goods to produce a higher quality variety, and a firm needs to be sufficiently productive to cover for that higher cost.

(iii) And the third component \( \left[ 1 + \tau^{-\theta} \right] k((\varepsilon - 1) - \beta(k - \varepsilon + 1))^{-1} \) indicates the effect of trade on the intermediate good cost, and thus, on the productivity cutoff. Particularly, this term is equal to 1 in the autarky, and increasing in the trade cost \( \tau \).
First, as specified in [42], the competition effect

\[
Z(\tau) = \left[ \frac{kp^k}{k - \theta + 1} \frac{\theta M(\tau)}{Y} \right]^{\frac{1}{k}}
\]

plays an important role in analysing exporters’ behaviour and their product ranges, as it appears in all the equilibrium solutions shown in section 3.2.

In this specification, \( M(\tau) \), given by [43], is an integral over all levels of quality in a country, and hence, it indicates the mass of available varieties in the market. The larger the mass of available varieties, the more competitive the market is. Particularly, from the demand side, each variety from each firm has a smaller market share given a larger mass of varieties in the market.

Other determinants of this competition effect \( Z(\tau) \) includes the market size \( Y \), and the country’s productiveness characterized by \( \varphi \) and \( k \). In a country with a higher shape parameter \( k \), i.e., having a larger share of low productive firms, the competition level is lower. The reason is that in the country with many low productive firms, each firm produces fewer varieties, and the mass of available varieties across all quality levels is consequently smaller. The same logic is applied to the case where the minimum productivity level \( \varphi \) in the country is lower. On the other hand, at any given mass of varieties, a larger market size \( Y \), i.e., a larger national spending on the final goods, would alleviate this competition effect.

Intuitively, a high competition level will depress the market price index, hence, decrease the market share and profit for each variety/firm in the market. Consequently, only more productive firms, which have lower production costs, will survive. Alternatively, it needs to sell a higher quality variety to generate sufficient demand to compensate for the lower price. In other words, under higher market competition, the productivity cutoff to produce a variety at any quality level will increase, so does the quality threshold for a certain firm to survive, and as a result, the product range of a certain firm will shrink. On the contrary, in a market of low competition level, the price and the profit of each firm are higher, resulting in a lower productivity cutoff to survive and a larger product quality range for a firm.
Second, regarding to the cost-push effect on the productivity cutoff, under bilateral trade liberalization, firms have more chance to export, and hence, create an extra demand for intermediate goods in the domestic market. Higher demand for intermediate goods at any quality level within the export quality range would allow each supplier to earn higher profit and more suppliers to survive, which in turn lower the intermediate good price index. For a final producer, the lower intermediate good price index helps to lower its production cost and raise its profit. For this reason, under trade liberalization, the productivity cutoff to produce and export is lower, which is reflected through the decrease in component 
\[1 + \tau^{-\theta} \frac{-\beta(k-\varepsilon+1)}{k[(\varepsilon-1)-\beta(k-\varepsilon+1)]}\] in [46].

5.2. Sector’s endowment and exporters’ behaviour

This section discusses how the export productivity cutoff and a firm’s export quality range is affected by the sector’s intensive use of intermediate goods, where the intermediate good is also interpreted as a capital input, and a higher parameter \(\beta\) indicates a more capital-intensive sector. It should be noted that, the discussion in this section is applied to varieties in the high quality range, i.e., \(\lambda \geq \lambda_H\), where \(\lambda_H\) is given by [23] and \(\lambda \geq \lambda_H \geq 1\).

Since \(Z(\tau)[1 + \tau^{-\theta}] \frac{-\beta(k-\varepsilon+1)}{k[(\varepsilon-1)-\beta(k-\varepsilon+1)]}\) is decreasing in \(\beta\) (see Appendix 5 for proof), it means that the competition is higher in a labour-intensive sector than in an intermediate good-intensive sector. Intuitively, this can be explained by the abundance of homogenous labour input and the relative scarcity of intermediate goods, which is costly to produce and requires sufficiently productive suppliers. Therefore, it is cheaper to produce a labour-intensive good, and there will be a larger mass of labour-intensive varieties in the market and a greater competition level in the labour-intensive sector compared to the intermediate good-intensive sector.

On the other hand, the cost-push factor \(\lambda \frac{-(\varepsilon-1)}{\beta(k-\varepsilon+1)}\) is increasing in \(\beta\), i.e., the cost-push factor is higher in the intermediate good-intensive sector. The reason is simply because
firms in the intermediate good-intensive sector spend a larger share of its production cost on
the intermediate goods and is more sensitive to the changes in the intermediate goods price as
indicated by Proposition 4.2.

It can be seen that an intermediate good-intensive firm faces lower competition in the output
market but higher competition in the input market, while it is the other way round for a
labour-intensive firm. Higher output revenue may be offset by the higher input costs,
therefore, a firm’s exporting decision and its product choice depend on the net effect of the
two mentioned forces.

**Proposition 5. Intermediate good intensity and exporter's behaviour**

5.1. For a relatively low quality variety, the export productivity cutoff is higher in the
labour-intensive sector than in the intermediate good-intensive sector, i.e., \( \varphi(\lambda) \) is
decreasing in \( \beta \).

5.2. For a sufficiently high quality variety, the export productivity cutoff is lower in the
labour-intensive sector than in the intermediate good-intensive sector, i.e., \( \varphi(\lambda) \) is
increasing in \( \beta \).

5.3. The export quality range in the labour-intensive sector is smaller than in the
intermediate good-intensive sector for relatively less productive exporters.

For relatively low values of \( \lambda \), there is an abundance of suppliers, and hence less competition
to obtain corresponding intermediate goods for each firm. Hence, the cost-push effect is
minimal in either sector, and is dominated by the competition effect in the output market.
Given a higher competition level in the labour-intensive sector, which depresses market share
and prices, it is more difficult to produce and export compared to an intermediate good-
intensive firm.

On the other hand, for sufficiently high quality varieties, given the scarcity of sufficiently
productive suppliers, there is a fierce competition for intermediate goods in the input market.
The cost-push effect is, therefore, significant, and would dominate the competition effect in
the output market. As we know, the cost-push effect is more serious for the intermediate good-intensive firms, it drives up the production cost, and hence, reduces the firm’s profit, and makes it harder for a firm to produce and export.

Therefore, there would be more exporters generally, but fewer exporters of top quality varieties from the intermediate good-intensive sector compared to the labour-intensive sector.

And finally, Proposition 5.3 is a direct result of Proposition 5.1 and 5.2. Particularly, the low productive exporters can only produce the relatively low quality varieties. Given the higher export productivity cutoff for this quality range in the labour-intensive sector compared to the intermediate good-intensive sector, a relatively low productive firm produces and exports a smaller range of varieties in the labour-intensive sector. On the other hand, for highly productive firms, who are able to produce both the very high quality varieties and the low quality varieties, such comparison of the export quality range across sectors is less clear.

These results are seemingly distinguished from Bernard et al. (2007). In that paper, it is predicted that the exporters’ behaviour pattern across sector would change according to the country’s endowment, i.e., it is more difficult to export from a comparative disadvantaged sector, and hence, there would be more capital-intensive exporters from a capital-abundant country, and more labour-intensive exporters from a capital-scarce country. Our model, on the other hand, predicts that the exporters’ behaviour pattern, indicated by Proposition 5.1 and 5.2, is the same for all countries.

For relatively low trade costs (including transportation cost and fixed entry cost), the export quality benchmark \( \lambda_H \), given by [23], lies in the lower export quality range indicated by Proposition 5.1. Hence, it is always easier to start exporting in the intermediate good-intensive sector compared to the labour-intensive sector. This result explains for the fact that exporters are also more capital-and skill-intensive even in developing countries, which are plausibly abundant in unskilled labor (Alvarez & Lopez, 2005).

Only in the case of very high trade costs, i.e., when the two countries are very distant from each other both geographically and culturally, does the export quality benchmark \( \lambda_H \) lies in
the higher export quality range indicated by Proposition 5.2, and it would be easier to export from the labour-intensive sector in that case.

On one hand, it should also be noted that, the country symmetry assumption in this paper (which aims to simplify the equilibrium solutions) makes it less justified to compare the results with Bernard et al. (2007), which considers a model of two asymmetric sectors in two asymmetric countries. On the other hand, the earlier analyses can also be applied to the asymmetric countries case, which then produces a consistent prediction to Bernard et al. (2007) conditional on quality. Particularly, let’s compare two countries, a labour-abundant country (lacking in productive suppliers) and an intermediate good-abundant country (abundant in productive suppliers). Intermediate good intensive firms from the labour-abundant country face higher competition in the home intermediate good market, which pushes up their production cost, as well as face higher competition in the final good market, which depresses prices. In such case, intermediate good-intensive firms in the labour-abundant country face a higher the export productivity cutoff at any quality level compared to firms in the other country. As a result, there would be fewer capital-intensive exporters from the labour-abundant country as well as each firm exports a lower quality and smaller range of products.

6. Numerical illustration

While the major theoretical results have been motivated and found good supporting evidences from the recent empirical studies, the simulation in this part is useful to illustrate these new theoretical results in a systematic way.

6.1. Major parameters and calibration

The elasticity of substitution for both the final good varieties and intermediate good varieties are set at: $\theta = \varepsilon = 4$, which, as discussed in equilibrium condition in section 3.2.2, are consistent with the estimates using plant-level U.S. manufacturing data in Bernard, Eaton, Jensen and Kortum (2003), and has been commonly applied in other calibrating works, including Melitz & Redding (2013) or Head, Mayer & Thoenig (2013). In fact, the size of
these elasticities of substitution does not affect our qualitative predictions, as long as it satisfies the equilibrium condition [60].

Given the above values of $\theta$ and $\varepsilon$, to satisfy the equilibrium condition [60.3], i.e.,

$$\varepsilon - 1 < k < (\varepsilon - 1) \left[ 1 + \frac{1}{\beta} \right],$$

the acceptable range of the shape parameter $k$ is: $3 < k < 3 \left[ 1 + \frac{1}{\beta} \right]$.

Therefore, with $k = 5$, this condition will always be satisfied for any value of $\beta \in (0,1)$. This give us the full freedom in choosing $\beta$, our interested labour intensity parameter. Other values of $k$ will also be considered in the robustness check part.

On the other hand, parameter $r$ is subject to condition [60.4], i.e.,

$$\theta - 1 < r < \frac{(\theta - 1)(\varepsilon - 1)}{\beta(k - \varepsilon + 1)}$$

so the acceptable range of $r$ is: $3 < r < \frac{4.5}{\beta}$. Therefore, $r$ is set at 4.5, so the above condition is always satisfied with a full value range of $\beta$.

Table 1. Calibration of the fixed parameters

<table>
<thead>
<tr>
<th>Type of parameters</th>
<th>Parameters</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>World/Country-specific</td>
<td>World quality frontier</td>
<td>$\lambda_M = 30$</td>
</tr>
<tr>
<td></td>
<td>National income</td>
<td>$Y=200$</td>
</tr>
<tr>
<td></td>
<td>Productivity Pareto distribution’s</td>
<td></td>
</tr>
<tr>
<td>World/Country-specific</td>
<td>scale parameter, and shaper parameter</td>
<td>$\varphi = 1$</td>
</tr>
<tr>
<td></td>
<td>Wage</td>
<td>$k = 5$</td>
</tr>
<tr>
<td></td>
<td>Fixed entry cost to export</td>
<td>$f_x = 10$</td>
</tr>
<tr>
<td></td>
<td>Iceberg transportation cost</td>
<td>$\tau = 2$</td>
</tr>
<tr>
<td>Sector-specific</td>
<td>Elasticities of substitution</td>
<td>$\theta = \varepsilon = 4$</td>
</tr>
<tr>
<td></td>
<td>Difficulty to raise quality</td>
<td>$r = 4.5$</td>
</tr>
</tbody>
</table>
The fixed entry cost to the foreign market is set arbitrarily at 10 and the world technology frontier is arbitrarily assumed to be 30 across all simulations. Specifically, the calibration for all the fixed parameters are described in Table 1 above.

For other parameters, i.e., the float parameters, the detailed calibration is as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameters</th>
<th>Applied simulation</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector-specific</td>
<td>$\beta$</td>
<td>Simulations A, B</td>
<td>$\beta$ runs from 0.1 to 0.9</td>
</tr>
<tr>
<td>Firm-specific</td>
<td>$\varphi_c$</td>
<td>Simulations B</td>
<td>$\varphi_c$ runs from 0 to $+\infty$</td>
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<td></td>
<td></td>
<td>Simulations A</td>
<td>$\varphi_c = 7$</td>
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<tr>
<td>Product-specific</td>
<td>$\lambda$</td>
<td>Simulations A, B</td>
<td>$\lambda$ runs from 1 to $\lambda_M = 30$</td>
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<tr>
<td></td>
<td></td>
<td>Robustness check</td>
<td>$\beta$ runs from 0.1 to 0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k \in (3.3 \left[ 1 + \frac{1}{\beta} \right])$</td>
</tr>
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### 6.2. Results

**Simulation A: Pricing decision of a firm**

This section provides the numerical simulation to support Proposition 4 about a firm’s export prices across sector.

Figure 2 graphs the export prices of varieties $p^s_x(\lambda)$ (equations [57] and [56]) within the same perceived quality range produced by a firm with productivity $\varphi_c = 7$ in three scenarios. The blue line, red line and green line respectively represent the export prices of varieties when this firm switches from a more labour intensive sector ($\beta = 0.4$) to more intermediate good-intensive sectors ($\beta = 0.5$ and $\beta = 0.6$). In all sectors, the positive correlation between the quality of the varieties and their export prices is revealed.
Particulars and Pricing

Simulation B: Exporters’ behaviour in different sectors

This section provides the numerical simulation to support Proposition 1 about the minimum productivity to export, and Proposition 2 about the export productivity cutoff at each level of product quality, as well as Proposition 3 about the correlation between a firm’s ability and its product range, and examine how these export productivity cutoffs and product range change across sectors of different labour intensity as predicted in Proposition 5.

At the trade cost $\tau = 2$ and $f_x = 10$, the export quality benchmark is $\lambda_H = 2.65$, and the minimum productivity cutoff $\varphi^X$ is 3.5 when $\beta = 0.7$, 4.7 when $\beta = 0.5$, and 6.7 when $\beta = 0.2$. In Figure 3, the horizontal and vertical axes represent the quality level and firms’ productivity. The smooth curves represent the export productivity cutoff, $\varphi^X(\lambda)$ (equations [49] and [48]), and the dotted curves represent the domestic productivity cutoff at each level of quality, $\varphi(\lambda)$ (equations [47] and [48]), respectively. Figure 3 is graphed for a sector when its intermediate good intensity $\beta$ is varied from 0.2 (the blue curves) to 0.7 (the red curves).
It is shown that the export productivity cutoff by quality curve follows a V-shape, which confirms our predictions in Proposition 2. Particularly, for varieties in the high quality range, the export productivity cutoff coincides with the domestic productivity cutoff and follows the positive association with the quality. On the other hand, for the low quality varieties (below the export quality benchmark): the lower the quality is, the higher the productivity the exporter must have. The very steep export productivity cutoff branch for the low quality range suggests that firms need to have extremely high productivity to export the very low quality varieties, which explains why the low quality varieties are not popular in the export market.

The minimum export productivity cutoff $\varphi^X$ is at the bottom of this V-shaped export productivity cutoff. At each level of productivity, the horizontal distance between the two branches of the V-shaped export productivity cutoff curve constitutes the export quality range for a firm of that productivity. The least productive exporter (with $\varphi^X = 4.7$ in this calibration) only exports one variety $\lambda_{H} = 2.65$, whereas, a more productive firm exports a larger range of varieties (Proposition 1 and Proposition 3).

It can be seen that in the low to middle quality range ($\lambda$ is from 0 to approximately 16.5), all the three productivity cutoffs, i.e., the domestic productivity cutoff, the minimum export
productivity cutoff and the export productivity cutoff by quality, increase as $\beta$ decreases. This evidence is consistent with Proposition 5, which suggests that for the low quality varieties, the export productivity cutoff is higher in the labour intensive sector than in the capital-intensive sector, and correspondingly, a firm’s export quality range is smaller in the former sector compared to the latter sector. Hence, it is more difficult to export in the labour intensive sector than in the capital intensive sector.

However, the closer the quality level is to the world frontier of quality, $\lambda = 30$, as shown in Figure 3, the relative positions of the two export productivity cutoff curves corresponding to the two sectors start to switch. Particularly, the productivity cutoffs to produce and export the top quality varieties are higher in the capital-intensive sectors, which are completely consistent with Proposition 5.2.

7. Conclusion

This paper develops a new theoretical model that unites three rather separated branches of literature in international trade so far, i.e., the product vertical differentiation literature, the product horizontal differentiation literature and the literature on heterogeneous firms and the comparative advantage analysis. By incorporating an intermediate sector of heterogeneous suppliers to the standard heterogeneous firm model, we have shown that the new model has a greater capacity to analyse firms’ behaviour from more aspects simultaneously and provide good supports to a number of recent empirical findings in international trade.

With this more realistic model structure, the discussion may not be limited to the mentioned results. Particularly, some of the possible extensions of the model would be: (i) to extend the model to fit the two asymmetric countries case; (ii) to relax the assumption of no FDI activity to analyze the product quality choice of a multinational in facing two strategies - exporting versus FDI, and (iii) to allow for trade in the intermediate sector and analyze the trade liberalization effect on the intermediate goods which have become more dominant in the global trade flow recently.
References


APPENDIX

Appendix 1 (Proof of Proposition 1): to prove that \( \lambda_H = \left( \frac{\theta - 1}{\theta - 1} \right)^\frac{1}{r} \) is the benchmark quality level to export and \( \varphi^X = \varphi(\lambda_H) \), given by [24], is the minimum export productivity cutoff.

For convenience, the firm notation is switched from \( c \) to \( \varphi_c \), particularly, \( \pi_c^X(\lambda, \varphi_c) \) is equivalent to \( \pi^X(\lambda, \varphi_c) \). For low productive firms with \( \varphi_c < \varphi^X \), they can only produce varieties of low quality \( \lambda < \lambda_H \), while the export profit is increasing in \( \lambda \) and \( \varphi \), so it follows that

\[
\pi^X(\lambda, \varphi_c) < \pi^X(\lambda_H, \varphi_c) \quad \forall \lambda < \lambda_H,
\]

and

\[
\pi^X(\lambda_H, \varphi_c) < \pi^X(\lambda_H, \varphi^X) \quad \forall \varphi_c < \varphi^X.
\]

Since \( \pi^X(\lambda_H, \varphi^X) = 0 \), by transitivity:

\[
\pi^X(\lambda, \varphi_c) < 0 \quad \forall \lambda < \lambda_H, \varphi_c < \varphi^X.
\]

Hence, low productive firms can only produce varieties \( \lambda < \lambda_H \), which fail to be exported. ■

Appendix 2 (Proof of Proposition 2): to show that the productivity cutoff \( \varphi_f(\lambda) \), \( \varphi(\lambda) \) are increasing in \( \lambda \), \( \varphi^X(\lambda) \) is increasing in \( \lambda \) \( \forall \lambda \geq \lambda_H \), and decreasing in \( \lambda \) \( \forall \lambda < \lambda_H \).

(2.1) Since \( \frac{r}{(\theta - 1)(e - 1) - \beta(k + e + 1)} > 0 \) under condition [60], and \( Z(\tau) \) does not depend on \( \lambda \) so \( \varphi(\lambda) \) given by [48], and \( \varphi_f(\lambda) \), given by [50], are both increasing in \( \lambda \).

(2.2) \( \varphi^X(\lambda) = \varphi(\lambda) \forall \lambda \geq \lambda_H \)

So the export productivity cutoff for a high quality variety is increasing in \( \lambda \).

(2.3) The export productivity cutoff for a low quality variety, given by [49] can be written as follow:

\[
\varphi^X(\lambda) = \lambda^\frac{r}{\theta - 1}(e - 1) - \beta(k + e + 1) \frac{r}{\theta - 1} Z(\tau) \lambda_H^\frac{r}{\theta - 1} \left[ 1 + \tau^{-\theta} \left( \frac{\lambda}{\lambda_H} \right)^\frac{r(k - \theta + 1)}{\theta - 1} \right]^{-\beta(k + e + 1)}.
\]

\( r < \frac{(e - 1)(\theta - 1)}{\beta(k + e + 1)} \) under condition [60] so it follows that
\[
\frac{(r - \theta + 1)(\varepsilon - 1)}{(\theta - 1)[(\varepsilon - 1) - \beta(k - \varepsilon + 1)]} - \frac{r}{\theta - 1} = \frac{r \beta (k - \varepsilon + 1) - (\varepsilon - 1)(\theta - 1)}{(\theta - 1)[(\varepsilon - 1) - \beta(k - \varepsilon + 1)]} < 0.
\]

At the same time, since
\[
\frac{-\beta(k - \varepsilon + 1)}{k[(\varepsilon - 1) - \beta(k - \varepsilon + 1)]} < 0 \quad \text{and} \quad \frac{r(\theta - 1)}{\theta - 1} > 0,
\]

hence, both
\[
\frac{(r - \theta + 1)(\varepsilon - 1)}{(\theta - 1)[(\varepsilon - 1) - \beta(k - \varepsilon + 1)]} \frac{r}{\theta - 1}
\]

and
\[
\left[1 + \tau^{-\theta} \left(\frac{1}{\lambda_H}\right)^{\theta - 1}\right]^{r(\theta - 1) \frac{(1 - \beta)(k - \varepsilon + 1)}{k[(\varepsilon - 1) - (1 - \beta)(k - \varepsilon + 1)]}}
\]

are decreasing in \(\lambda\), so \(\phi^X(\lambda)\) is decreasing in \(\forall \lambda \geq \lambda_H\).}

**Appendix 3. Proof that \(\lambda_{cX}^I < \lambda_{cX}\)**

Let \(\pi_{c_X}^X(\lambda)\) and \(\pi_{c_H}^X(\lambda)\) be the functional forms of a firm’s export profit for the two quality ranges given by [53]. It can be easily checked that
\[
\pi_{c_X}^X(\lambda) < \pi_{c_H}^X(\lambda) \quad \forall 0 < \lambda < \lambda_H,
\]

so the following inequality also holds:
\[
\pi_{c_X}^X(\lambda_{c_X}) < \pi_{c_H}^X(\lambda_{c_X}).
\]

Since \(\pi_{c_X}^X(\lambda_{c_X}) = 0\), it follows that \(\pi_{c_H}^X(\lambda_{c_X}) > 0\).

Let \(\lambda_{cX}^I\) be the quality level that \(\pi_{c_H}^X(\lambda_{c_X}) = 0\), so
\[
\pi_{c_H}^X(\lambda_{c_X}) > \pi_{c_H}^X(\lambda_{c_X}^I).
\]

\(\pi_{c_H}^X(\lambda)\), whereas, is also a strictly increasing function of \(\lambda\), therefore, \(\lambda_{cX} > \lambda_{cX}^I\), and \(\lambda_{cX}^I\) can be used as a bounded level for \(\lambda_{cX}\).}

**Appendix 4 (Proof of Proposition 4.2):** to show that the quality adjusted price of a variety is decreasing in its quality.

From the price function in [56] and [57], the quality-adjusted price of a variety is derived as follow:
\[
p_c^X(\lambda) = \begin{cases} 
\left(\frac{\tau B A^B}{\phi_c} \frac{r \beta (k - \varepsilon + 1)(\theta - 1)(\varepsilon - 1)}{k[\theta - 1][\varepsilon - 1] - \beta(k - \varepsilon + 1)} \right)^\frac{\beta}{\phi_c^k} \left[1 + \tau^{-\theta} \left(\frac{1}{\lambda_H}\right)^{\theta - 1}\right] & \forall \lambda \geq \lambda_H \\
\left(\frac{\tau B A^B}{\phi_c} \frac{r \beta (k - \varepsilon + 1)(\theta - 1)(\varepsilon - 1)}{k[\theta - 1][\varepsilon - 1] - \beta(k - \varepsilon + 1)} \right)^\frac{\beta}{\phi_c^k} \left[1 + \tau^{-\theta} \left(\frac{1}{\lambda_H}\right)^{\theta - 1}\right] & \forall \lambda < \lambda_H.
\end{cases}
\]

Under condition [60],
\[
\frac{r(k - \varepsilon + 1)(1 - \beta)}{k[(\varepsilon - 1) - (1 - \beta)(k - \varepsilon + 1)]} < 0, \quad \frac{-\beta(k - \varepsilon + 1)}{k[(\varepsilon - 1) - (1 - \beta)(k - \varepsilon + 1)]} < 0\quad \text{and} \quad \frac{r(\theta - 1)}{\theta - 1} > 0,
\]

so the quality adjusted \(p_c^X(\lambda)\) is decreasing in \(\lambda\) \(\forall \lambda > 0\).
Appendix 5: to proof that $Z(\tau)(1 + \tau^{-\theta})^{-\beta(k-e+1)/\beta(k-e+1)}$ is decreasing in $\beta$.

From the expression of $M(\tau)$ in [43], we have:

$$
M(\tau)(1 + \tau^{-\theta})^{-\beta(k-e+1)/\beta(k-e+1)} = \int_0^{\lambda_H} \lambda^{r(k-\theta+1)/(\theta-1)((c-1)-\beta(k-e+1))} \left[1 + \tau^{-\theta}\left(\frac{\lambda}{\lambda_H}\right)^{r(k-\theta+1)/(\theta-1)((c-1)-\beta(k-e+1))} \frac{\beta(k-e+1)}{(c-1)-\beta(k-e+1)}\right] \tau^{1-\theta}\left(\frac{\lambda}{\lambda_H}\right)^{\theta-1} d\lambda.
$$

Note that $0 < \frac{\lambda}{\lambda_H} < 1$, and by condition [60], $\frac{r(k-\theta+1)}{\theta-1} > 0$, so that the following inequality holds:

$$
0 < 1 + \tau^{-\theta}\left(\frac{\lambda}{\lambda_H}\right)^{\theta-1} < 1.
$$

Since $\frac{\beta(k-e+1)}{(c-1)-\beta(k-e+1)}$ is increasing in $\beta$, it follows that $\left[1 + \tau^{-\theta}\left(\frac{\lambda}{\lambda_H}\right)^{\theta-1} \frac{\beta(k-e+1)}{(c-1)-\beta(k-e+1)}\right]$ is decreasing in $\beta$.

At the same time, $\lambda^{r(k-\theta+1)/(\theta-1)((c-1)-\beta(k-e+1))}$ is decreasing in $\beta \forall \lambda \geq 1$, so $M(\tau)(1 + \tau^{-\theta})^{-\beta(k-e+1)/\beta(k-e+1)}$ is decreasing in $\beta$.

And $Z(\tau)(1 + \tau^{-\theta})^{-\beta(k-e+1)/\beta(k-e+1)} = \left[1 + \tau^{-\theta}\left(\frac{\lambda}{\lambda_H}\right)^{\theta M(\tau)} \frac{k \varphi^k}{k-\theta+1} \left(\frac{\lambda}{\lambda_H}\right)^{r(k-\theta+1)/(\theta-1)((c-1)-\beta(k-e+1))}\right]$ is also decreasing in $\beta$.

∎