Trade liberalization, Agglomeration of Firms, and Unemployment

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Abstract

We construct a two-country model with search friction in the labor market to investigate the effect of trade liberalization on the unemployment rates in both countries. In this model, there are two sectors: homogenous good sector and differentiated products sector. We assume that in the homogenous good sector there is no labor market friction and trade costs whereas in the differentiated products sector, there is labor market friction and trade costs between countries. In both short-run, trade liberalization increases the number of firms and lowers the unemployment rates in the larger country whereas decreases the number of firms and increases the unemployment rates in the smaller countries. On the other hand, in the long-run, trade liberalization increases (decreases) the unemployment rates in the larger (smaller) country.

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Key words: Agglomeration of firms, International Trade, Search friction, Unemployment.

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1 Introduction

The aim of this paper is to investigate the effect of trade liberalization on unemployment. In the international trade theory, we often set up the model which ignore imperfection of labor market. Considering trade policy like reducing trade cost, however, each government often pays attention to the protection for employment of domestic industry. We cannot avoid the problem of employment in international trade with expanding globalization.

There are some literature which focus on the relationship between unemployment and trade. Main concern in this context is whether trade openness and unemployment is positively related or not. Matusz(1996) develops the model of monopolistic competition with a efficiency wage model and shows trade openness leads to decrease unemployment. Felbermayr et al.(2011) incorporates search friction into Melitz model to analyze the effect of trade liberalization on unemployment. This paper concludes that trade liberalization lowers employment. They analyze symmetric countries and do not discuss the effect of difference in market size on economic performance in each country. Since market size affects the number of firm which generates employment, we consider the effect of the difference in market size is important. On the other hand, Dutt et al.(2009) shows trade liberalization and unemployment are negatively related for labor-scarce country from theoretical and empirical aspects by developing Heckscher-Ohlin model with comparative advantage.

In this paper, we incorporate two-country NEG model into search friction to analyze the effect of trade liberalization on unemployment, number of firms, or other economic performance. In the short-run, trade liberalization induce firms to locate and lowers the unemployment rates in the larger country whereas increases the unemployment rates in the smaller countries. In the long-run, trade liberalization increases the number of firms and the unemployment rates in the larger country whereas decreases the unemployment rate in the smaller country.

The remainder of this paper is as follows. First, we describe the NEG model with search friction. Second, we characterize the equilibrium in the short-run and long-run and analyze the effect of the trade liberalization on the equilibrium. Finally, we conclude.

2 The model

There are two countries, 1 and 2. Variables that refer to country 1 have the subscript 1 and those that refer to country 2 have the subscript 2. Each country is endowed with a fixed amount of labor, $L_1$ and $L_2$, respectively, while country 1 is larger than country 2: $L_1 > L_2$. We assume that agents in both countries obtain utility from the consumption of homogeneous goods and differentiated products. In the homogenous goods sector, there is no search friction whereas in the differentiated products sector, there is search friction. While labor can be mobile between sectors in the same country, it cannot be mobile between
2.1 Household problem

The utility function of the agent in country $i$ ($i = 1, 2$) is given by

$$U_i = A_i + \mu \log C_i,$$

where

$$C_i = \left[ \int_0^n m_i(\omega)^{\frac{\sigma - 1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma - 1}}, \sigma > 1.$$

$A_i$ and $m_i(\omega)$ represent the consumption level of homogenous goods by the agent in country $i$ and of differentiated products of $\omega$ by the agent in country $i$, respectively. The budget constraint of the agent in country $i$ becomes

$$E_i = A_i + \int_0^n p_i(\omega)m_i(\omega)d\omega,$$

where $E_i$ denotes the expenditure by the agents in country $i$ and $p_i(\omega)$ denotes the price of differentiated products of $\omega$ in country $i$. We take homogeneous goods as the numeraire. From the utility maximization problem, the demand functions can be obtained as follows:

$$C_i = \frac{\mu}{P_i},$$

$$m_i(\omega) = \frac{\mu p_i(\omega)^{-\sigma}}{P_i^{1-\sigma}},$$

$$A_i = E_i - \mu,$$

$$P_i = \left[ \int_0^n p_i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

Then, substituting the demand functions into the utility function, we can obtain the indirect utility function as follows:

$$U_i = E_i - \mu(1 - \log \mu) - \mu \log P_i.$$

2.2 Production

The homogenous goods market is perfectly competitive. In the homogenous goods sector, there are no labor market friction. We assume that in both countries, one unit of homogenous goods is produced with one unit of labor and that the interregional trade of homogeneous goods incurs no transportation costs. Therefore, the equilibrium wages in the homogenous goods sector in both countries are both one: $w_1 = w_2 = 1$. 
In the differentiated product sector, firms operate under Dixit-Stiglitz (1977)-type monopolistic competition. Each differentiated product firm produces differentiated products, and each variety is produced by one firm. In the differentiated products sector, there are labor market friction. Then, when a differentiated product firm and a worker match, the differentiated product firm can produce the goods. If differentiated products firms and worker do not match, they can not produce the differentiated products. To produce one unit of differentiated products requires one unit of homogenous goods. The profits of differentiated products firms which succeed matching is given by

$$\pi_i(\omega) = p_i^i(\omega)x_i^i(\omega) + p_i^i(\omega)p_i^i(\omega) - (x_i^i(\omega) + \tau x_i^i(\omega)), \quad (7)$$

where $p_i^i(\omega)$ denotes the price of variety $\omega$ sold in country $i'$ and produced in country $i$ and $x_i^i(\omega)$ denotes the quantity of variety $\omega$ sold in country $i'$ and produced in country $i$. We assume that international shipping of a differentiated product is costly as it implies an "iceberg" cost: if a differentiated products firms sends one units of goods to a foreign country, it must dispatch $\tau > 1$ units of goods.

The equilibrium in each variety market of country $i$ implies that demand and supply of each variety $\omega$ should be equal in both countries:

$$x_i^i(\omega) = m_i(\omega)L_i. \quad (8)$$

Profit maximization together with (3) and (8) lead to the following constant markup prices:

$$p_i^i(\omega) = \frac{\sigma}{\sigma - 1}, \quad (9)$$

$$p_i^i(\omega) = \tau p_i^i(\omega) = \tau \frac{\sigma}{\sigma - 1}. \quad (10)$$

Substituting these prices in to the price index in both countries, the following equation can be obtained:

$$P_i = \frac{\sigma}{\sigma - 1} \left[ n_i + \phi n_i' \right]^{1/\sigma}, \quad (11)$$

where $\phi \equiv \tau^{-(\sigma - 1)}$ represents the so-called freeness of trade. $n_j$ denotes the number of differentiated products firms succeeding in matching in country $j$. $\phi = 0$ describes the case of autarky, whereas $\phi = 1$ implies free trade. Profits of differentiated products firms in country $i$ is given by

$$\pi_i(\omega) = \frac{\mu}{\sigma} \left[ \frac{L_i}{n_i + \phi n_i'} + \frac{\phi L_i'}{\phi n_i + n_i'} \right] \equiv \pi_i. \quad (12)$$

2.3 Matching

The setting of search and matching in this paper has a similar model of Pissarides (2000, Ch. 1). In the differentiated products sector, there are search-and-matching frictions. Let the matching function be $M_i = g(u_i, v_i)$ where $M_i$ denotes the number of job matches, $u_i$ denotes unemployed workers and $v_i$ denotes
job vacancies engage in matching. \( g(u_i, v_i) \) is increasing in \( u_i \) and \( v_i \), strictly concave, and homogeneous of degree 1. We assume that \( 0 \leq g(u_i, v_i) \leq \min \{u_i, v_i\} \) and \( g(0, v_i) = g(u_i, 0) = 0 \). The probability of a worker finding a job is \( p(\theta_i) = \frac{M_i}{u_i} = g(1, \theta_i) \) where \( \theta_i = v_i/u_i \) and \( \theta_i \) represents the tightness of labor market. An increase in \( \theta_i \) increases the probability of a worker finding a job. The probability of a differentiated products firm finding a worker is \( q(\theta_i) = \frac{M_i}{v_i} = g(1/\theta_i, 1) \) and \( \frac{\partial q(\theta_i)}{\partial \theta_i} < 0 \) holds. The relationship between \( p(\theta_i) \) and \( q(\theta_i) \) is given by

\[
p(\theta_i) = q(\theta_i) \theta_i. \tag{13}
\]

Next, we focus on the value of workers and differentiated products firms. Let \( W_i \) and \( U_i \) be the present-discounted value of the expected income of an unemployed and employed worker, respectively. In this model, we assume that capital market is perfect. The unemployed worker can receive unemployment benefit of \( b < 1 \). Then, \( U_i \) is

\[
rU_i = b + p(\theta_i) (W_i - U_i), \tag{14}
\]

where \( r \) is the discount rate and the second term represents the capital gain from succeeding in matching. The value of \( W_i \) is given by

\[
rW_i = w_{Mi} + \delta(U_i - W_i), \tag{15}
\]

where \( w_{Mi} \) denotes the wage rate in the differentiated products sector in country \( i \) and \( \delta \) denotes the rate of job destruction. The rate of job destruction is exogenous variable. The second term represents the capital loss from losing their job. Let \( J_i \) and \( V_i \) be the present-discounted value of the expected profit of an occupied job and a vacant job, respectively. The value of a vacant job is given by

\[
rV_i = -k + q(\theta_i)(J_i - V_i), \tag{16}
\]

where \( k \) denotes the search cost. The second term represents the capital gain from succeeding in matching. The value of an occupied job is given by

\[
rJ_i = \pi_i - w_{Mi} + \delta(V_i - J_i). \tag{17}
\]

\((\pi_i - w_{Mi})\) represents the profits of differentiated products firms and the second term is the capital loss from losing their jobs.

In this model, we assume that the differentiated products firms enter the market freely. Then, from free-entry condition, the value of a vacant job is

\[
V_i = 0. \tag{18}
\]

From (16) and (18), the value of an occupied job becomes

\[
J_i = \frac{k}{q(\theta_i)}. \tag{19}
\]

\(^{1}\)If \( b \) is larger than 1, all of workers prefer to work in the manufacturing goods sector.
Substituting (19) into (17), the following equation can be obtained:

$$\pi_i - w_{M_i} = \frac{(r + \delta)k}{q(\theta_i)} = 0.$$  

(20)

We assume that workers and the products firm engage in wage bargaining. The wage rate in the differentiated products sector is determined by Nash bargaining. The worker’s share of the total surplus is $\beta$ and the differentiated products firm’s share of the total surplus is $1 - \beta$. Then, the following equation must hold:

$$W_i - U_i = \beta(J_i + W_i - V_i - U_i).$$  

(21)

From (15), the value of employed worker is given by

$$W_i = \frac{w_{M_i} + \delta U_i}{r + \delta}.$$  

(22)

From (17) and (18), the value of an occupied job is given by

$$J_i = \frac{\pi_i - w_{M_i}}{r + \delta}.$$  

(23)

Substituting (18), (22), and (23) into (21), the wage rate in the differentiated products sector becomes

$$w_{M_i} = \beta\pi_i + r(1 - \beta)U_i.$$  

(24)

Substituting (18) into (21), $(W_i - U_i)$ is given by

$$W_i - U_i = \frac{\beta}{1 - \beta}J_i.$$  

(25)

Substituting (14) into (25), the value of an unemployed worker becomes as follows:

$$rU_i = b + \frac{\beta}{1 - \beta} \frac{p(\theta_i) k}{q(\theta_i)} = b + \frac{\beta}{1 - \beta} \theta_i k,$$  

(26)

where last equality comes from (13). Then, to substitute (26) into (24), we can obtain the wage rate in the differentiated products sector as follows:

$$w_{M_i} = (1 - \beta)b + \beta(\pi_i + \theta_i k).$$  

(27)

In the steady-state, because the number of unemployment is constant, the following equation must hold:

$$p(\theta_i)u_i = \delta n_i.$$  

(28)

$L_{M_i}$ denotes the supply of workers in the differentiated products sector in country $i$. Some workers succeed in matching and the others become an unemployed worker. Then, in the labor market equilibrium condition in the differentiated products sector is given by

$$L_{M_i} = u_i + n_i, i = 1, 2.$$  

(29)
In this paper, we assume that in the short-run, the number of worker in the differentiated products sector is exogenous variable, whereas in the long-run, the number of worker in the differentiated products sector is endogenous variable. Therefore, in the short-run, $L_{Mi}$ is constant variables. In the long-run, the workers choose the supply of worker in the differentiated products good sector. In the long-run, because the value of an unemployed worker is equal to the value of worker in the homogenous goods sector, the following equation can be obtained:

$$rU_i = 1, i = 1, 2.$$  

(30)

3 Equilibrium

In this paper, we focus on the equilibrium that the differentiated products firms locate in both countries. Then, $J_1 = J_2$. From (19), we can obtain the following equation:

$$\frac{k}{q(\theta_1)} = \frac{k}{q(\theta_2)}.$$  

Therefore, $\theta_1 = \theta_2 \equiv \theta$ holds. Substituting (27) into (20), the profits of differentiated products firm is given by

$$\pi_i = b + \frac{k}{1 - \beta} \left[ \beta \theta + \frac{r + \delta}{q(\theta)} \right] \equiv \pi.$$  

Then, the profits of differentiated products firm is same in both countries. To substitute $\theta_i = \theta$ and $\pi_i = \pi$ into (27), we can obtain $w_{M1} = w_{M2} \equiv w_M$. From $\pi_i = \pi$ and (12), we can obtain the following equation:

$$n_2 = \frac{L_2 - \phi L_1}{L_1 - \phi L_2} n_1.$$  

(31)

When $\phi < \bar{\phi} \equiv L_2/L_1$, in the equilibrium, the differentiated products firms locate in both countries. When $\bar{\phi} < \phi < 1$, all of differentiated products firms locate in the country 1. Substituting (31) into (12), we can rewrite the profits of differentiated products firm as follows:

$$\pi = \frac{\mu}{\sigma n_1} \frac{L_1 - \phi L_2}{1 - \phi}.$$  

(32)

Substituting (32) into (20) and (27), we can obtain the following equations:

$$\frac{\mu}{\sigma n_1} \frac{L_1 - \phi L_2}{1 - \phi} - w_M - \frac{(r + \delta)k}{q(\theta)} = 0,$$

(33)

$$w_M = (1 - \beta)b + \beta \left( \frac{\mu}{\sigma n_1} \frac{L_1 - \phi L_2}{1 - \phi} \right).$$  

(34)
3.1 In the short-run equilibrium

In the short-run, the supply of labor in the differentiated products sector is constant. In the short-run equilibrium, from (28), (29), (31), (33), and (34), we can obtain \((n_1^*, n_2^*, \theta^*, w_M^*, u_1^*, u_2^*)\). From (33) and (34), the number of the differentiated products firms in country 1 is given by

\[
\begin{align*}
n_1 &= \frac{\mu \frac{L_1 - \phi L_2}{1 - \phi}}{b + \frac{\beta}{1 - \beta} \theta k + \frac{(r + \delta)k}{(1 - \beta)\eta(\theta)}} \equiv \Phi(\theta). \tag{35}
\end{align*}
\]

\[
\partial \Phi(\theta)/\partial \theta < 0, \lim_{\theta \to 0} \Phi(\theta) = \frac{\mu L_1 - \phi L_2}{\sigma (1 - \phi)} > 0, \text{ and } \lim_{\theta \to \infty} \Phi(\theta) = 0. \quad \text{From (28) and (29), the number of the differentiated products firms in country 1 is also given by}
\]

\[
\begin{align*}
n_1 &= \frac{p(\theta)L_M}{p(\theta) + \delta} \equiv \Psi(\theta). \tag{36}
\end{align*}
\]

\[
\partial \Psi(\theta)/\partial \theta > 0, \lim_{\theta \to 0} \Phi(\theta) = 0, \text{ and } \lim_{\theta \to \infty} \Phi(\theta) = L_M > 0. \quad \text{From (35) and (36), we can obtain the short-run equilibrium } \theta^* \text{ uniquely. From (33) and (34), the wage rate in the differentiated products sector in the short-run equilibrium is given by}
\]

\[
\begin{align*}
w_M^* &= b + \frac{\beta}{1 - \beta} \theta^* k + \frac{(r + \delta)k}{(1 - \beta)\eta(\theta^*)}. \tag{37}
\end{align*}
\]

Summarizing above results, we can obtain the following proposition.

**Proposition 1** In the short-run equilibrium, from (28), (29), (31), (33), and (34), \(n_1^*, n_2^*, \theta^*, w_M^*, u_1^*, u_2^*\) are determined uniquely.

From (35) and (36), the equilibrium level of \(\theta^*\) is determined as follows:

\[
A(\theta^*) = 0, \quad \text{where}
\]

\[
A(\theta^*) \equiv p(\theta^*)L_M[b + \frac{\beta \theta^* k}{1 - \beta} + \frac{(r + \delta)\theta^* k}{(1 - \beta)\eta(\theta^*)}] - \frac{\mu (L_1 - \phi L_2)}{\sigma (1 - \phi)}(p(\theta^*) + \delta). \tag{39}
\]

\(A(\theta^*)\) is increasing in \(\theta^*\). We investigate the effects of the trade liberalization on the number of firms, the tightness of labor market, and the wage rate in the differentiated goods sector. Then, we can obtain the following proposition (See the Appendix for Proof).

**Proposition 2** In the short-run equilibrium, from (28), (29), (31), (33), and (34), trade liberalization increases (decreases) the number of firms in country 1 (2), the tightness of labor market, wage rate in the differentiated goods sector, the unemployment rates in country 1 (2), and price index in country 2 (1).
3.2 In the long-run equilibrium

In the long-run, the supply of labor in the differentiated products sector is endogenous variables. In the long-run equilibrium, from (28), (29), (30), (31), (33), and (34), we can obtain \((n_1^L, n_2^L, \theta^L, w_M^L, u_1^L, u_2^L, L_{LM1}, L_{LM2})\). From (26) and (30), \(\theta^L\) in the long-run equilibrium is given by

\[
\theta^L = \frac{(1-b)(1-\beta)}{\beta k}.
\]

(40)

In the long-run, trade liberalization does not affect the tightness of labor market because the value of an unemployment worker is constant. Substituting \((??)\) into (35), we can obtain the number of differentiated product firms in country 1 in the long-run equilibrium as follows:

\[
n_1^L = \frac{\mu L_1 - \phi L_2}{\sigma} \frac{(1-\beta)q(\theta^L)}{(1-\beta)q(\theta^L) + (r+\delta)k^2}.
\]

(41)

The number of differentiated product firms in country 2 in the long-run equilibrium is given by

\[
n_2^L = \frac{\mu L_2 - \phi L_1}{\sigma} \frac{(1-\beta)q(\theta^L)}{(1-\beta)q(\theta^L) + (r+\delta)k^2}.
\]

(42)

Using (29) and (36), we can obtain the number of worker engaged in differentiated product sector and unemployment rates in each country in the long-run equilibrium as follows:

\[
L_{LM1}^L = \frac{p(\theta^L) + \delta}{p(\theta^L)} n_1^L, \quad L_{LM2}^L = \frac{p(\theta^L) + \delta}{p(\theta^L)} n_2^L,
\]

(43)

\[
u_1^L = \frac{\delta}{p(\theta^L)} n_1^L, \quad u_2^L = \frac{\delta}{p(\theta^L)} n_2^L.
\]

(44)

The wage rates in the differentiated product sector is given by

\[
w_M^L = 1 + \frac{(r+\delta)\beta k}{(1-\beta)q(\theta^L)}.
\]

(45)

From the long-run equilibrium, we can obtain the following proposition (See Appendix for proof):

**Proposition 3** In the long-run, trade liberalization increases \(n_1^L, L_{LM1}^L, u_1^L\), and decreases \(n_2^L, L_{LM2}^L, u_2^L\). Trade liberalization does not affect the tightness of labor market and wage rates in the differentiated product sector in the long-run equilibrium.
4 Conclusion

We construct a two-country model with search friction in the labor market to investigate the effect of trade liberalization on the unemployment rates in both countries. In this model, there are two sectors: homogenous goods sector and differentiated goods sector. We assume that homogenous goods is freely traded and in this sector, there is no labor market friction. On the other hand, international shipping of a differentiated product is costly and in the differentiated good sector, there is labor market friction. In the short-run, trade liberalization increases the number of the differentiated product firms and decreases the unemployment rates in the larger country. On the other hand, trade liberalization decreases the number of firms and the unemployment rates in the smaller country. In the long-run, trade liberalization increases the number of the differentiated product firms and unemployment rate in the larger country whereas decreases the number of firms and unemployment rate in the smaller country.

References


A Derivation of $A'(\theta^*) > 0$.

Differentiating $A'(\theta^*)$ with respect to $\theta^*$, we can obtain the following equation:

$$A'(\theta^*) = p'(\theta^*) \frac{\beta \theta^* k}{1 - \beta} \left[ b + \frac{\beta \theta^* k}{1 - \beta} - \frac{\mu (L_1 - \phi L_2)}{\sigma L_{M1}(1 - \phi)} \right] + \frac{L_{M1} k}{1 - \beta} \left[ \beta p'(\theta^*) + r + \delta \right].$$

(46)
Substituting $A(\theta^*) = 0$ into the above equation, we can rewrite $A^*$ as follows:

$$A'(\theta^*) = \frac{\delta p'(\theta^*) \mu(L_1 - \phi L_2)}{p(\theta^*)} + \frac{kL_{M1}(r + \delta)}{1 - \beta} \left[1 - \frac{\theta^* p'(\theta^*)}{p(\theta^*)}\right] + \frac{\beta k L_{M1} p(\theta^*)}{1 - \beta} > 0,$$

(47)

because $0 < \theta^* p'(\theta^*)/p(\theta^*) < 1$ holds.

\[\text{B \ Proof of Proposition 2}\]

From (38), we can obtain the relationship between transportation costs and the tightness of labor market as follows:

$$\frac{\partial \theta^*}{\partial \phi} = -\frac{\frac{\partial A(\theta^*)}{\partial \phi}}{A'(\theta^*)},$$

(48)

$\partial A(\theta^*)/\partial \phi$ is given by

$$\frac{\partial A(\theta^*)}{\partial \phi} = -\frac{\mu}{(p(\theta^*) + \delta)^2} [p(\theta^*) + \delta] < 0.$$ 

(49)

Therefore, because $A' > 0$ and $\partial A(\theta^*)/\partial \theta^* < 0$, $\partial \theta^*/\partial \phi$ has a positive value. Then, trade liberalization increases the tightness of labor market in the differentiated goods sector.

Next, we investigate the effect of trade liberalization on the number of firms. Differentiating (36) with respect to $\phi$, we can obtain the following equation:

$$\frac{\partial n_1^*}{\partial \phi} = \frac{\delta p'(\theta^*) L_{M1} \partial \theta^*}{(p(\theta^*) + \delta)^2} \frac{\partial \theta^*}{\partial \phi} > 0,$$

(50)

because $p'(\theta^*) > 0$ and $\partial \theta^*/\partial \phi > 0$. Then, trade liberalization increases the number of differentiated firms in country 1. We differentiate (31) with respect to $\phi$ as follows:

$$\frac{\partial n_2}{\partial \phi} = \frac{(L_2 - L_1)(L_2 + L_1)}{(L_1 - \phi L_2)^2} n_1^* + \frac{L_2 - \phi L_1}{L_1 - \phi L_2} \frac{\partial n_1^*}{\partial \phi}.$$ 

(51)

Substituting $\partial n_1^*/\partial \phi$ into the above equation, we can rewrite $\partial n_2/\partial \phi$ as follows:

$$\frac{\partial n_2}{\partial \phi} = \frac{(L_1 - L_2) n_1^*}{(L_1 - \phi L_2) A'(\theta^*)} B(\theta^*, \tau),$$

(52)

where

$$B(\theta^*, \tau) = \frac{\delta p'(\theta^*) \mu(L_2 - \phi L_1)}{p(\theta^*)} - \frac{(L_1 + L_2) A'(\theta^*)}{L_1 - \phi L_2}.$$ 

(53)
When \( B(\theta^*, \tau) \) is positive (negative), \( \partial n^*_1 / \partial \phi \) has also positive (negative) value. Substituting \( A^* \) into \( B(\theta^*, \tau) \), we can rewrite \( B(\theta^*, \tau) \) as follows:

\[
B(\theta^*, \tau) = \frac{\delta p(\theta^*)}{p(\theta^*)} \frac{\mu}{\sigma(1-\phi)^2} [L_2 - \phi L_1 - (1 - \phi)(L_1 + L_2)] \\
- \frac{L_1 + L_2}{L_1 - \phi L_2} \left[ \frac{k L_1 (r + \delta)}{1 - \beta} (1 - \theta^* p'(\theta^*)) + \frac{\theta^* k L_1 (p(\theta^*))}{1 - \beta} \right] \\
= - \frac{\delta p'(\theta^*)}{p(\theta^*)} \frac{\mu(L_1 - \phi L_2)}{\sigma(1-\phi)^2} - \frac{L_1 + L_2}{L_1 - \phi L_2} \frac{k L_1 (1 - \theta^* p'(\theta^*)) + \theta^* p(\theta^*)}{1 - \beta} < 0.
\]

Then, \( B(\theta^*, \tau) \) has positive value. Therefore, trade liberalization decreases the number of differentiated firms in country 2.

We differentiated the wage rate in the differentiated goods sector with respect to \( \phi \) as follows:

\[
\frac{\partial w^*_{M}}{\partial \phi} = \frac{\beta k}{1 - \beta} \left[ 1 - (r + \delta) \frac{q'(\theta^*)}{q(\theta^*)} \right] \frac{\partial \theta^*}{\partial \phi} > 0,
\]

because \( q'(\theta^*) < 0 \). Therefore, trade liberalization increases the wage rate in the differentiated goods sector.

We investigate the relationship between trade liberalization and unemployment in both countries. Differentiating (29) with respect to \( \phi \), we can obtain the following equations:

\[
\frac{\partial u^*_1}{\partial \phi} = - \frac{\partial n^*_1}{\partial \phi} < 0,
\]

and

\[
\frac{\partial u^*_2}{\partial \phi} = - \frac{\partial n^*_2}{\partial \phi} > 0,
\]

because \( \partial n^*_1 / \partial \phi > 0 \) and \( \partial n^*_2 / \partial \phi < 0 \). Then, trade liberalization decreases the unemployment rates in country 1 whereas increases the unemployment in country 2.

\[ \text{C Proof of Proposition 3} \]

In this Appendix, we prove Proposition 3. Differentiating (??) and (??) with respect to \( \phi \), we can obtain the following equations:

\[
\frac{\partial n^*_1}{\partial \phi} = \frac{\mu}{\sigma (1-\phi)^2} \frac{L_1 - L_2}{(1 - \beta)q(\theta^L)} + (r + \delta) k > 0,
\]

\[
\frac{\partial n^*_2}{\partial \phi} = - \frac{\mu}{\sigma (1-\phi)^2} \frac{L_1 - L_2}{(1 - \beta)q(\theta^L)} + (r + \delta) k < 0,
\]
because $L_1 > L_2$. Then, trade liberalization increases the number of firms in the larger countries whereas decreases the number of firms in the smaller country.

We investigate the effect of trade liberalization on the number of worker engaged in the differentiated sector and unemployment rates in country 1.

\[
\frac{\partial L_{M1}}{\partial \phi} = \frac{p(\theta^L) + \delta \frac{\partial n_{L1}}{\partial \phi}}{p(\theta^L)} > 0, \\
\frac{\partial u_{L1}}{\partial \phi} = \frac{\delta \frac{\partial n_{L1}}{\partial \phi}}{p(\theta^L)} > 0,
\]

where $\frac{\partial n_{L1}}{\partial \phi} > 0$. Trade liberalization increases the number firms and unemployment in country 1. The effects of trade liberalization on the number of worker engaged in the differentiated sector and unemployment rates in country 2 are given by

\[
\frac{\partial L_{M2}}{\partial \phi} = \frac{p(\theta^L) + \delta \frac{\partial n_{L2}}{\partial \phi}}{p(\theta^L)} < 0, \\
\frac{\partial u_{L2}}{\partial \phi} = \frac{\delta \frac{\partial n_{L2}}{\partial \phi}}{p(\theta^L)} < 0,
\]

where $\frac{\partial n_{L2}}{\partial \phi} > 0$. Trade liberalization decreases the number firms and unemployment in country 2.