Do Larger Countries Have Higher Welfare?

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Abstract

Existing intra-industry models with two countries and inelastic labor supply show that a larger country has a higher wage/income and higher welfare. We reexamine these outcomes by use of a model with endogenous labor supply. We show that each outcome in existing models can be reversed as well as that the larger country may have less-than-proportionate share of firms. Especially, when consumers' love of variety is strong and trade costs are low, the larger country has lower welfare since workers in the country work too much.

Key words: Country size, Endogenous labor supply, Welfare, Home market effect.

JEL Classification: F12, F21, R12.

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1 Introduction

Existing two-country intra-industry models show that a larger country has a higher wage/income and higher welfare. However, these models assume inelastic labor supply and, thus, we reexamine their outcomes by use of a model with endogenous labor supply. We show that each outcome in existing models can be reversed. Especially, when consumers’ love of variety is strong and trade costs are low, the larger country has lower welfare.

Since Krugman (1980) and Helpman and Krugman (1985), the issue of country-size effects in intra-industry models has been examined under various settings. Many researchers especially have focused on the home market effect (HME) in terms of firm share. This is defined as a phenomenon in which a country with a relatively larger local demand attracts a more-than-proportionate share of industry with increasing returns and transportation costs (Krugman, 1980, Section III; Helpman and Krugman, 1985, Section 10.4). For example, Head and Ries (2001), Feenstra et al. (2001), and Head et al. (2002) explore the pervasiveness of the HME in terms of firm share and show that it can be reversed in the cases with varieties differentiated by the nation of production.

The HME is sometimes defined in another way, i.e., other things being equal, a larger country has a higher wage (Krugman, 1980, Section II; Krugman, 1991, p. 491). The HME based on this definition is called the HME in terms of wages. The literature shows that the HME in terms of wages is more pervasive than that in terms of firm share. For example, Davis (1998), Yu (2005), and Takatsuka and Zeng (2012b) introduce trade costs of homogeneous goods into Helpman and Krugman (1985) and show that a larger country has a higher wage although it may not have more-than-proportionate share of firms. Furthermore, this result is robust even when we introduce mobile capital (Takatsuka and Zeng, 2012a; Takahashi et al., 2013) and/or preferences of variable elasticity of substitution (VES) (e.g. Chen and Zeng, 2014; Bykadorov et al., 2015).\footnote{Wang and Gibson (2015, Proposition 2) use a model with additively separable (AS) preferences and claim that a larger country can get a higher, lower, or same wage rate. However, they do not show an example and/or conditions of the reverse HME of wages. Meanwhile, Bykadorov et al. (2015) use augmented hyperbolic absolute risk aversion (AHARA) utilities (a class of AS) and through massive simulations, they show that a wage in a bigger country is higher.}

Examining whether the HME is pervasive is important as a policy issue as well as a positive one since it suggests that smaller countries may be relatively worse off via lower share of firms and lower wages by their size. In fact, the literature also supports that a larger country has higher welfare. While Yu (2005) and Takatsuka and Zeng (2012b) prove it in one-factor economies, Takahashi et al. (2013) show it by use of a two-factor model. Numerical simulations by Chen and Zeng (2014) and Bykadorov et al. (2015) suggest that it also holds under VES preferences.

The above country-size effects can be significantly influenced by endogeneity of labor supply. For example, imagine a situation with two countries and one differentiated-good sector. In
general, industrial location is determined by the balance of market access and production costs. In the case of free trade, production costs are dominant since two countries keep the same level of market access. Thus, to equalize wages across countries, firms are located proportionally to country size. What happens if transport costs rise marginally? In that case, the large country gains an advantage of market access and thus the wage in the country is higher to offset the advantage. Furthermore, if labor supply is fixed, a higher wage means a higher income, attracting more firms to the country and resulting in the HME in terms of firm share and higher welfare (Takahashi et al., 2013). However, if labor supply is variable and workers in the large country supply less labor force, individual income can be lower there, resulting in the reverse HME in terms of firm share. On the contrary, if workers in the large country supply more labor force, welfare can be lower there although both income and firm share are higher. Due to high access to foreign varieties and enjoying leisure, the smaller country can achieve higher welfare despite its lower individual income.

We can show that endogeneity of labor supply may generate the reverse HME in terms of wages. Specifically, when consumers’ love of variety is strong and trade costs are high, a large country can get a lower wage than a smaller country. This is also a novel result which existing studies have never shown. Furthermore, we find that the HME in terms of firm share may be reversed for any trade costs if consumers’ love of variety is weak.

Our model is a slightly modified version of Ago et al. (2014). Ago et al. (2014) introduce endogenous labor supply into a two-country trade model and examine the impacts of technological progress. Since their model is based on Ottaviano et al. (2002), it also features variable markups, which are empirically supported. We introduce mobile capital and asymmetry in country size into their model to focus on the examination of the HME. Due to mobile capital, trade is not necessarily balanced. Thus, it is possible to examine which country is the net exporter of differentiated varieties.

The remainder of this paper is organized as follows. In Section 2, we present the model described above. In Section 3, we examine the HME in the model. Finally, Section 4 is the conclusion.

2 The Model

We modify the two-country model of Ago et al. (2014) to allow for asymmetric country size and mobile capital. Specifically, the economy consists of two countries, called countries 1 and 2, and one differentiated-good sector. The total number of workers in the world is normalized to one and the share in country 1 is denoted as \( \theta \in (1/2, 1) \), which implies that country 1 is the larger one. Labor is immobile between countries. Meanwhile, capital is mobile across countries and is evenly held by all workers. Thus, the measure of the total amount of capital is also one.

Each worker is assumed to hold the identical preference, which is described by a quasi-linear
utility with quadratic subutility:

\[ U = \alpha \int_0^n x(i)di - \frac{\beta}{2} \int_0^n x(i)^2 di - \gamma \left[ \int_0^n x(i)di \right]^2 - l, \]  

(1)

where \( n \) is the number of varieties, \( x(i) \) is the consumption of variety \( i \in [0, n] \), and \( l \) is her amount of labor supply. Parameters \( \alpha, \beta, \) and \( \gamma \) are all positive and a higher \( \beta \) means her stronger love of variety. The last term implies that working generates disutility. Her budget constraint is expressed as

\[ \int_0^n p(i)x(i)di = y = wl + r, \]  

(2)

where \( x(i) \) is the price of variety \( i \), \( y \) is the income, \( w \) is the wage rate, and \( r \) is the capital return.

From the first order condition (F.O.C.) of utility maximization:

\[ \alpha - \beta x(i) - \gamma \int_0^n x(j)dj = \frac{p(i)}{w}, \]  

(3)

we have the demand function of each variety and the labor supply function:

\[ x(i) = \frac{\alpha \beta w + \gamma P}{(\beta + n\gamma)} - \frac{p(i)}{\beta w}, \]  

(4)

\[ l = \frac{\alpha}{\beta + n\gamma} \frac{P}{w} - \frac{\beta(S + 1)/n + \gamma S P^2}{\beta (\beta + n\gamma)} \frac{P^2}{w^2} - \frac{r}{w}, \]  

(5)

where \( P \equiv \int_0^n p(i)di \) is the price index and

\[ S = \frac{1}{P^2} \left[ n \int_0^n p^2(i)di - P^2 \right]. \]

In the production, each firm needs marginal costs of \( m \) units of labor and a fixed cost of 1 unit of capital. We choose the labor in country 2 as the numéraire, so that the wage rate there is \( w_2 = 1 \). The wage \( w_1 \) in country 1 is written just as \( w \). As in most related papers, we assume Samuelson’s iceberg trade costs. Specifically, \( \tau (> 1) \) units of the variety must be shipped for one unit to trade between two countries. Therefore, the pure profit of each firm in countries 1 and 2 are expressed as

\[ \pi_1 = (p_{11} - mw)x_{11}\theta + (p_{12} - mw\tau)x_{12}(1 - \theta) - r, \]  

(6)

\[ \pi_2 = (p_{22} - m)x_{22}(1 - \theta) + (p_{21} - m\tau)x_{21}\theta - r, \]  

(7)

respectively, where \( x_{rs} \) and \( p_{rs} \) are the individual demand and the price in country \( s \) for firms located in country \( r \). From the F.O.C. of profit maximization and (4), we have the following
equilibrium prices:

\[ p_{11}^* = \frac{[2\beta(\alpha + m) + \gamma m(2n_1 + n_2)]w + \tau \gamma mn_2}{2(2\beta + n\gamma)}, \quad p_{12}^* = p_{22}^* + \frac{m}{2}(\tau w - 1), \quad (8) \]

\[ p_{22}^* = \frac{[2\beta(\alpha + m) + \gamma m(n_1 + 2n_2)] + \tau \gamma mn_1 w}{2(2\beta + n\gamma)}, \quad p_{21}^* = p_{11}^* + \frac{m}{2}(\tau - w), \quad (9) \]

where \( n_r \) is the number of firms in country \( r \). The F.O.C. of profit maximization also implies that

\[ x_{11} = \frac{p_{11}^* - mw}{\beta w}, \quad x_{12} = \frac{p_{12}^* - mw\tau}{\beta}, \quad x_{22} = \frac{p_{22}^* - m}{\beta}, \quad x_{21} = \frac{p_{21}^* - m\tau}{\beta w}. \quad (10) \]

Plugging (10) into (6) and (7), the zero profit conditions are

\[ r = \frac{(p_{11}^* - mw)^2}{\beta w} + \frac{(p_{12}^* - mw\tau)^2}{\beta}(1 - \theta), \quad (11) \]

\[ r = \frac{(p_{22}^* - m)^2}{\beta}(1 - \theta) + \frac{(p_{21}^* - m\tau)^2}{\beta w}\theta. \quad (12) \]

Substituting (8) and (9) into (10), we have

\[ x_{11} = \frac{2\beta w(\alpha - m) + \gamma mn_2(\tau - w)}{2\beta w [2\beta + (n_1 + n_2)\gamma]}, \quad x_{22} = \frac{2\beta(\alpha - m) + \gamma mn_1(\tau w - 1)}{2\beta [2\beta + (n_1 + n_2)\gamma]}. \]

To guarantee positive demand for any \( \gamma > 0 \), we assume that \( \alpha > m \).

In equilibrium, net import (resp. export) value of capital is equal to net export (resp. import) value of varieties:

\[ (k - \theta)r = kp_{12}^* x_{12}(1 - \theta) - (1 - k)p_{21}^* x_{21}\theta, \quad (13) \]

where \( k \) is the share (number) of firms in country 1 so that the share of firms in country 2 is \( 1 - k \). Three endogenous variables \( k, w, \) and \( r \) are determined by (11), (12), and (13).

3 Results

3.1 Autarky

When \( \tau \) is sufficiently large, both countries cease trading all varieties, i.e., \( x_{12} = x_{21} = 0 \). From (13), we immediately have \( k = \theta \), suggesting that the HME in terms of firm share disappears.

In the autarky case, from (6) and (7), the operating profit of each firm in countries 1 and 2
are expressed as

\[(p_{11} - mw) x_{11} \theta = \frac{p_{11} - mw}{p_{11}} \times p_{11} x_{11} n_{1} = r,\]

\[(p_{22} - m) x_{22} (1 - \theta) = \frac{p_{22} - m}{p_{22}} \times p_{22} x_{22} n_{2} = r,\]

respectively. These equations show that individual expenditure multiplied by the markup ratio is equalized across countries. Noting the price index \(P_r\) is \(n_r p_{rr}\) in the autarky case, variety prices (8) and (9) are

\[p^*_{11} = \frac{[\beta(\alpha + m) + \gamma mn_{1}] w}{2\beta + n_{1} \gamma} = \frac{[\beta(\alpha + m) + \gamma m\theta] w}{2\beta + \theta \gamma}, \quad (14)\]

\[p^*_{22} = \frac{\beta(\alpha + m) + \gamma mn_{2}}{2\beta + n_{2} \gamma} = \frac{\beta(\alpha + m) + \gamma m(1 - \theta)}{2\beta + (1 - \theta) \gamma}. \quad (15)\]

Thus, the markup ratios are

\[\frac{p_{11} - mw}{p_{11}} = \frac{\beta(\alpha - m)}{\beta(\alpha + m) + \gamma m\theta} \quad \text{and} \quad \frac{p_{22} - m}{p_{22}} = \frac{\beta(\alpha - m)}{\beta(\alpha + m) + \gamma m(1 - \theta)}.\]

Since \(\theta > 1/2\), the markup ratio in country 1 is lower than that in country 2. This is the pro-competitive effect. Therefore, the operating-profit equalization suggests that individual expenditure (= individual income) in country 1 is higher than that in country 2. Specifically,

\[\frac{\beta(\alpha - m)}{\beta(\alpha + m) + \gamma m\theta} (wl_{1} + r) = \frac{\beta(\alpha - m)}{\beta(\alpha + m) + \gamma m(1 - \theta)} (l_{2} + r) = r. \quad (16)\]

It is noteworthy that \(wl_{1} + r = y_{1}\) and \(l_{2} + r = y_{2}\) are individual incomes in countries 1 and 2, respectively.

If labor supply is a common constant across countries, we immediately have \(w > 1\), i.e., the HME in terms of wages. This is similar to the case of Chen and Zeng (2014). Furthermore, if the markup ratio is identical across countries as in the case of constant elasticity of substitution (CES), we have \(w = 1\) (Takahashi, et al., 2013). However, markup ratios are different across countries and labor supply is endogenously determined in our case, suggesting that it is not necessarily holds that \(w \geq 1\). From (5), (14), (15), and (16), we have

\[w = \frac{(1 - \theta)(2\beta + \theta \gamma)^{2}}{\theta [2\beta + (1 - \theta) \gamma]^{2}}, \quad r = \frac{\beta(\alpha - m)^{2}(1 - \theta)}{[2\beta + (1 - \theta) \gamma]^{2}}, \quad (17)\]

\[l_{1} = \frac{m(\alpha - m)\theta}{2\beta + \theta \gamma}, \quad l_{2} = \frac{m(\alpha - m)(1 - \theta)}{2\beta + (1 - \theta) \gamma}. \quad (18)\]
We can easily show that $\partial w/\partial \beta < 0$ and

$$ w \geq 1 \iff \beta \leq \frac{\gamma}{2} \sqrt{\theta(1-\theta)} = \beta_1. \quad (19) $$

Finally, we examine the welfare. Substituting the above equilibrium variables into (1), the welfare levels in the two countries, denoted as $V_1$ and $V_2$, are

\begin{align*}
V_1 &= \frac{\theta(\alpha - m)^2(3\beta + \theta \gamma)}{2(2\beta + \theta \gamma)^2}, \\
V_2 &= \frac{(1 - \theta)(\alpha - m)^2[3\beta + (1 - \theta) \gamma]}{2[2\beta + (1 - \theta) \gamma]^2},
\end{align*}

respectively, and we can easily show that $V_1 > V_2$. This result is similar to the one in standard models of intra-industry trade.

In summary, we have the following proposition:

**Proposition 1** In autarky, (i) $k = \theta$, (ii) $y_1 > y_2$, (iii) $w < 1$ (resp. $w > 1$) if and only if $\beta > \beta_1$ (resp. $\beta < \beta_1$), and (iv) $V_1 > V_2$.

Why is the HME in terms of wages reversed when $\beta$ is large? Intuitively, if $\beta$ is larger, each worker in country 1 supplies more labor relative to that in country 2, resulting in a lower wage in country 1 despite that individual income is higher there as mentioned above. In fact, Equation (18) suggests that $l_1 > l_2$ and $\partial(l_1/l_2)/\partial \beta > 0$. We should remember that a higher $\beta$ means workers’ stronger love of variety. Since country 1 supplies more varieties (i.e., $n_1 > n_2$), demand of varieties is relatively larger while demand of leisure is relatively smaller there, resulting in relatively larger supply of labor. Without love of variety, we know that $l_1 = l_2 = m(\alpha - m)/\gamma$ from above equations.

Then, is individual income in the larger country always higher than that in the smaller country? In other words, does the HME in terms of *incomes* hold instead of the HME in terms of wages? This is not necessarily true. To see this point, we focus on the free-trade case in the next subsection.

### 3.2 Free Trade

When $\tau = 1$, from (6), (7), and (13), we have

$$ w = 1, \quad k = \theta, \quad r = \frac{(\alpha - m)^2 \beta}{(2\beta + \gamma)^2}, $$

suggesting that both types of the HME disappears. In the case of free trade, production costs (wages) are only drivers of firm location since two countries are indifferent in market access. Thus, firms eventually relocate to eliminate any wage differentials. Using these results and (5),
we also obtain \( l_1 = l_2 = m(\alpha - m)/(2\beta + \gamma) \). Labor supply per worker is also identical across countries and, thus, not only wages but also incomes are equalized between two countries.

How are wage and income differentials when trade costs increase marginally? Differentiating (6), (7), and (13) with respect to \( \tau \) at \( \tau = 1 \), we have

\[
\begin{align*}
\frac{dk}{d\tau} \Big|_{\tau=1} &= \frac{\theta(1-\theta)(2\theta - 1)[3(\beta + \gamma)m^2 - \alpha \gamma m + \alpha^2 \beta]}{m(\alpha - m)(2\beta + \gamma)}, \\
\frac{dr}{d\tau} \Big|_{\tau=1} &= \frac{\beta \theta(\alpha - m)[(\alpha - m)(2\theta - 1) - 4m(1-\theta)]}{(2\beta + \gamma)^2}, \\
\frac{dw}{d\tau} \Big|_{\tau=1} &= 2\theta - 1 > 0.
\end{align*}
\]

Furthermore, differentiating the relative income, \( (wl_1 + r)/(l_2 + r) \), with respect to \( \tau \) at \( \tau = 1 \), we have

\[
\frac{d}{d\tau} \frac{wl_1 + r}{l_2 + r} \Big|_{\tau=1} = \frac{(2\theta - 1)[3(\beta + \gamma)m^2 - \alpha \gamma m + \alpha^2 \beta]}{(\alpha - m)[\alpha \beta + (\beta + \gamma)m]}.
\]

From (20) and (23), we immediately have

\[
\frac{dk}{d\tau} \Big|_{\tau=1} \geq 0 \iff \frac{d}{d\tau} \frac{wl_1 + r}{l_2 + r} \Big|_{\tau=1} \geq 0 \iff \beta \geq \frac{\gamma m(\alpha - 3m)}{\alpha^2 + 3m^2} \equiv \beta_2.
\]

When trade costs increase, market access begins to play an important role for firm location. Thus, workers are expected to be better off in the larger country due to the larger local market. This advantage of the larger country will be offset by higher production costs there. In fact, (22) shows that the wage in country 1 rises with \( \tau \) at \( \tau = 1 \). However, this does not necessarily suggest that individual income in country 1 relatively rises. Inequalities (24) imply that if \( (\alpha > 3m \text{ and} \beta < \beta_2) \), the relative income in country 1 decreases with \( \tau \) at \( \tau = 1 \). This is evidently because labor supply becomes relatively smaller in the larger country. When \( \beta \) is small, consumers’ love of variety is weaker. Since country 2 supplies relatively cheaper varieties, demand of varieties is relatively larger there, resulting in relatively larger supply of labor. Therefore, the relative income is lowered in country 1 in spite that the wage is raised there.

Inequalities (24) also show that the direction of marginal change in firm share is the same as that in incomes. Intuitively, firms move to the country with more income. Therefore, if \( (\alpha > 3m \text{ and} \beta < \beta_2) \), firms relocate to the smaller country, suggesting that the HME in terms of firm share is reversed.

Finally, we examine the welfare. At \( \tau = 1 \), we have

\[
V_1 = V_2 = \frac{(\alpha - m)^2 (3\beta + \gamma)}{2(2\beta + \gamma)^2}
\]
and
\[
\frac{dV_1}{d\tau}_{\tau=1} = -\left(1 - \theta\right)(\alpha - m) \left\{ m \left[ 3\beta + 2\gamma (1 - \theta) \right] + \alpha \beta (2\theta - 1) \right\} \frac{1}{(2\beta + \gamma)^2} < 0,
\]
(26)
\[
\frac{dV_2}{d\tau}_{\tau=1} = -\theta(\alpha - m) \left\{ m(3\beta + 2\gamma \theta) - \alpha \beta (2\theta - 1) \right\} \frac{1}{(2\beta + \gamma)^2},
\]
(27)
\[
\frac{dV_1}{dV_2}_{\tau=1} = \frac{2(2\theta - 1) [2\gamma m - \beta (\alpha - 3m)]}{(\alpha - m)(3\beta + \gamma)}.
\]
(28)

From the last equation, if \(\alpha \leq 3m\), it holds that \(d(V_1/V_2)/d\tau_{\tau=1} > 0\). Meanwhile, if \(\alpha > 3m\),
\[
\frac{dV_1}{dV_2}_{\tau=1} < 0 \iff \beta > \frac{2\gamma m}{\alpha - 3m} \equiv \beta_3,
\]
(29)
where \(\beta_3 > \beta_2\) since from (24), a positive \(\beta_2\) exists and
\[
\beta_3 - \beta_2 = \frac{\gamma m \left[ (\alpha + 3m)^2 - 12m^2 \right]}{(\alpha - 3m)(2\beta + \gamma)} > 0.
\]

Equations (25) and (29) suggest that if \(\alpha > 3m\) and \(\beta > \beta_3\), the larger country has lower welfare when \(\tau > 1\) is sufficiently small. In this case, the larger country has more-than-proportionate share of firms and a higher per capita income since \(\beta_3 > \beta_2\) and (24). Nevertheless, the country has lower welfare than the small country. The reason is that workers in the larger country work harder relatively. In the case of free trade, \(l_1 = l_2\) holds as mentioned above. Since country 1 supplies more varieties (i.e., \(n_1 > n_2\)), when \(\tau\) rises marginally, demand of varieties is relatively larger in the country, where it should be noted that consumers’ love of variety is strong (i.e., \(\beta\) is high). This results in relatively larger supply of labor there. In fact, we have
\[
\frac{d\ln l_1}{d\tau}_{\tau=1} = \frac{(2\theta - 1) \left\{ \alpha^2 \beta - m \left[ (2\alpha - 5m)\beta + 2(\alpha - 2m)\gamma \right] \right\}}{m(\alpha - m)(2\beta + \gamma)},
\]
suggesting that \(d(l_1/l_2)/d\tau_{\tau=1} > 0\) when \(\alpha > 3m\) and \(\beta > \beta_3\).

In summary, we have the following proposition:

**Proposition 2** In the case of free trade, \(k = \theta\), \(w = 1\), and \(V_1 = V_2\). In the case that \(\tau > 1\) is sufficiently small, if \(\alpha \leq 3m\), we have \(k > \theta\), \(w > 1\), and \(V_1 > V_2\); otherwise, (i) \(w > 1\), (ii) \(y_1 < y_2\) (resp. \(y_1 > y_2\)) and \(k < \theta\) (resp. \(k > \theta\)) if \(\beta < \beta_2\) (resp. \(\beta > \beta_2\)), (iii) \(V_1 < V_2\) (resp. \(V_1 > V_2\)) if \(\beta > \beta_3\) (resp. \(\beta < \beta_3\)), where \(\beta_3 > \beta_2\).

### 3.3 Intermediate trade costs

Let \(\tau_a\) be the minimum transport cost under which trade does not occur. If \(\tau\) becomes lower than \(\tau_a\), trade starts between the two countries. If the economy has only immobile factor
(labor), both countries begin to export and import for $\tau < \tau_a$ because of the trade balance. However, since the economy has mobile factor (capital) in our case, this does not necessarily hold. In other words, there is a threshold value of trade costs $\tau_{ow} \in (1, \tau_a)$ such that one-way trade occurs for $\tau \in [\tau_{ow}, \tau_a)$ while two-way trade occurs for $\tau \in [1, \tau_{ow})$.

Let

$$\tau_{a1} = \frac{\theta [2\alpha \beta + \gamma m (1 - \theta)] [2\beta + (1 - \theta) \gamma]}{(1 - \theta)m(2\beta + \theta\gamma)^2} > 0,$$

$$\tau_{a2} = \frac{(1 - \theta)(2\alpha \beta + \gamma m\theta)(2\beta + \theta\gamma)}{\theta m [2\beta + (1 - \theta) \gamma]^2} > 0,$$

and

$$g(\beta) = \frac{\tau_{a1}}{\tau_{a2}} = \left(\frac{\theta}{1 - \theta}\right)^2 \left[\frac{2\beta + (1 - \theta) \gamma}{2\beta + \theta\gamma}\right]^3 \frac{2\alpha \beta + \gamma m (1 - \theta)}{2\alpha \beta + \gamma m\theta}.$$ 

Since

$$\lim_{\beta \to 0} g(\beta) = \left(\frac{1 - \theta}{\theta}\right)^2 < 1, \quad \lim_{\beta \to \infty} g(\beta) = \left(\frac{\theta}{1 - \theta}\right)^2 > 1, \quad g'(\beta) > 0,$$

there exists a unique root of $g(\beta) = 1$. We denote the root as $\beta_4$. Then, the following proposition is shown:

**Proposition 3** (i) If $\beta < \beta_4$ (resp. $\beta > \beta_4$), we have $\tau_a = \tau_{a2}$ (resp. $\tau_a = \tau_{a1}$) and only country 2 (resp. country 1) exports and the HME in terms of firm share is reversed (resp. appears) for $\tau \in [\tau_{ow}, \tau_a)$.

(ii) It holds that $\beta_4 < \beta_1$.

Proof. (i) First, we assume that only country 1 exports for $\tau \in [\tau_{ow}, \tau_a)$. When $\tau = \tau_a$, country 1 as well as country 2 ceases to export, i.e., $x_{12} = 0$. Thus, we have $p_{12}^* - mw\tau_a = 0$ from (10). Using this equation, (15), and (17), we have $\tau_a = \tau_{a1}$. Meanwhile, for $\tau \in [\tau_{ow}, \tau_a)$, consumers in country 1 have no incentive to buy any foreign goods, so that the F.O.C. of utility maximization (3) does not hold. Instead, we have $\alpha - \gamma n_1 x_{11} < p_{21}/w$. Using (10), (14), (17), and the fact that $n_1 = \theta$ at autarky, we can show that this inequality is equivalent to $\tau_{a1} > \tau_{a2}$ at $\tau = \tau_a = \tau_{a1}$. In summary, if country 1 exports for $\tau \in [\tau_{ow}, \tau_a)$, we have $\tau_a = \tau_{a1} > \tau_{a2}$. In a similar way, we can show that if country 2 exports for $\tau \in [\tau_{ow}, \tau_a)$, we have $\tau_a = \tau_{a2} > \tau_{a1}$.

By contraposition of these two facts, country 2 (resp. country 1) exports for $\tau \in [\tau_{ow}, \tau_a)$ and $\tau_a = \tau_{a2}$ (resp. $\tau_a = \tau_{a1}$) if and only if $\tau_{a2} > \tau_{a1}$ (resp. $\tau_{a1} > \tau_{a2}$). Furthermore, from the property of $g(\beta)$, it holds that $\tau_{a2} > \tau_{a1}$ (resp. $\tau_{a1} > \tau_{a2}$) if and only if $\beta < \beta_4$ (resp. $\beta > \beta_4$). Finally, from the trade and capital balance, the fact that only country 2 (resp. country 1) exports suggests that the country imports capital so that the HME in terms of firm share is reversed (resp. appears).
(ii) From (17) and (19), we have
\[ w = \frac{(1 - \theta)(2\beta_1 + \theta \gamma)^2}{\theta [2\beta_1 + (1 - \theta) \gamma]^2} = 1, \]
which leads to
\[ g(\beta_1) = \left( \frac{\theta}{1 - \theta} \right)^2 \left[ \frac{2\beta_1 + (1 - \theta) \gamma}{2\beta_1 + \theta \gamma} \right]^3 \frac{2\alpha \beta_1 + \gamma m(1 - \theta)}{2\alpha \beta_1 + \gamma m \theta}. \]
Substituting (19) into this equation, we have
\[ g(\beta_1) = \frac{\theta}{1 - \theta} \sqrt{\frac{1 - \theta}{\theta} \frac{\alpha \gamma \sqrt{\theta(1 - \theta)} + \gamma m(1 - \theta)}{\alpha \gamma \sqrt{(1 - \theta)} + \gamma m \theta}} \]
\[ = \frac{1 - \theta}{\alpha \sqrt{\frac{1 - \theta}{\theta} + m}} = \frac{\alpha + m \sqrt{\frac{1 - \theta}{\theta} + m}}{\alpha \sqrt{\frac{1 - \theta}{\theta} + m}} > 1, \]
where the last inequality is from our assumption of \( \alpha > m \).

As implied by Proposition 1, the wage in country 1 is higher when \( \tau \) is large and \( \beta \) is small. Since a small \( \beta \) means that consumers’ love of variety is weak, demand of varieties in country 2 is relatively large due to their lower prices. Thus, country 1 ceases to export its products to country 2 for an intermediate \( \tau \). The fact that country 2 is the net exporter of varieties suggests that it is the net importer of capital, leading to the reverse HME in terms of firm share. This is an economic intuition of Proposition 3 (i).

With Proposition 2, if \( \beta \) is sufficiently small, the larger country has less-than-proportionate share of firms when \( \tau \) is sufficiently small or \( \tau \in [\tau_{ow}, \tau_a] \). Then, is the reverse HME in terms of firm share possible for any \( \tau \)? The answer is yes. Figure 1 shows some simulation examples with different \( \beta \) ( = 0.5, 0.2, 0.1, 0.01). Other parameters are \( \alpha = 10, \gamma = 1, m = 1, \) and \( \theta = 0.7 \). In our setting, we have \( \beta_3 = 0.29, \beta_1 = 0.23, \beta_4 = 0.14, \) and \( \beta_2 = 0.068, \) and thus our four \( \beta \)'s correspond to the following four cases: (a) \( \beta > \max\{\beta_1, \beta_3\} \), (b) \( \beta \in (\beta_4, \beta_1) \), (c) \( \beta \in (\beta_2, \beta_4) \), and (d) \( \beta < \beta_2 \).

Case (a) with a large \( \beta \) shows the reverse HME in terms of wages for a large \( \tau \) (Proposition 1) and the larger country has lower welfare for a small \( \tau > 1 \) (Proposition 2). In this case, workers in the larger country always supply more labor as shown in Panel (a3). Cases (b) and (c) are similar to the results of existing studies such as Takahashi et al. (2013) and Chen and Zeng (2014), respectively. Case (b) shows both types of the HME while Case (c) supports only the HME in terms of wages. As shown in Proposition 3, a small \( \beta \) generates the reverse HME in terms of firm share for a larger \( \tau \) (Panel (c1)). Finally, Case (d) is an example that the larger
country has less-than-proportionate share of firms for any \( \tau \) (Panel (d1)). In this case, we can confirm that workers in the smaller country supply more labor for a wide range of \( \tau \) (Panel (d3)), leading to the reverse HME in terms of firm share occurs not only for a large \( \tau \) but also for a small \( \tau \).

Our model with endogenous labor supply shows that both types of the HME could be reversed and the HME in terms of firm share could be reversed for any \( \tau \). Then, is it possible that both types of HMEs are reversed simultaneously? Propositions 1 and 3 (ii) immediately show that

**Corollary 1**

(i) If the HME in terms of wages is reversed in autarky, the HME in terms of firm share appears for \( \tau \in [\tau_{ow}, \tau_a) \).

(ii) If the HME in terms of firm share is reversed for \( \tau \in [\tau_{ow}, \tau_a) \), the HME in terms of wages appears in autarky.

This suggests that if \( \tau \) is large, simultaneous reversal of two types of HMEs is impossible. This impossibility is supported by Proposition 2 for a small \( \tau \). Furthermore, Figure 1 and our simulation with many other sets of parameters support the impossibility for intermediate trade costs. In other words, the larger country exhibits a higher wage or a more-than-proportionate share of firms although one of them may not be observed.

4 Conclusion

We reexamine the country-size effects in a model with variable markups and endogenous labor supply. We can show that, contrastive to the existing studies, a larger country can have a lower wage/income and lower welfare. Especially, when consumers’ love of variety is strong and trade costs are low, the larger country has lower welfare due to its long working time. This may explain unhappiness prevailed in high-income countries such as Japan.

References


Figure 1: Simulation examples
Figure 1: Simulation examples (cont.)