Instructor’s Details

- **Instructor**: Dr Damien S. Eldridge.
- **Office**: H. W. Arndt Building, Room TBA.
- **Email**: TBA.
- **Consultation Hours**:
  - M, W and F from 4:00 pm to 5:00 pm.
  - By appointment at other times.

Class Details

- **Dates**:
  - Final Exam: Wednesday 10 February 2016.
- **Lecture and Tutorial Times**:
  - M, W and F from 10:00 am to 12:00 noon.
  - M, W and F from 1:00 pm to 3:00 pm.
  - In general, we will endeavour to start each two-hour session at five-past the hour, finish each session at five-to the hour, and hold a ten-minute break somewhere near the middle of each session.
- **Class Room**: TBA.
- **Exception**: There will be no class from 2:00 pm to 3:00 pm on Monday 11 January 2016.

This is an unofficial intensive bridging unit that covers some of the mathematical techniques that you will need during your graduate studies in economics and your career as an economist. It will not appear on your academic transcript. However, your results might be used internally as a diagnostic tool to determine if you need further training in these techniques as part of your official program of study.

A semester length unit in economics at ANU would normally consist of three hours of lectures per week for thirteen weeks and a one-hour tutorial per week for twelve weeks. (There typically would not be a tutorial in week one.) This amounts to 39 hours of lectures and twelve hours of tutorials. In other words, a
semester length unit in economics at ANU would normally involve 51 hours of class time. While this subject is being taught in an intensive mode rather than the standard mode, we will still hold 51 hours of class time.

Note that I have deliberately spaced the classes so that there is at least a one-day break between each class day. I have done this to allow you more time to review material and attempt problems. I hope that this will help overcome a major problem with classes taught in intensive mode; namely, the lack of time for assimilating material. The downside to this is that the classes are spread over a longer period.

Assessment

- Problem Set 1: 0 % or 15 %.
  - Distributed on Friday 15 January 2016.
  - Due at 3:00 pm on Friday 22 January 2016.
- Problem Set 2: 0 % or 15 %.
  - Distributed on Friday 22 January 2016.
  - Due at 3:00 pm on Friday 29 January 2016.
- Problem Set 3: 0 % or 15 %.
  - Distributed on Friday 29 January 2016.
  - Due at 3:00 pm on Friday 5 February 2016.
- Final Exam: 55 % or 70 % or 85 % or 100 %.
  - Time: Wednesday 10 February 2016 from 10:00 am to 12:00 noon.
  - Location: TBA.

Each of the problem sets is potentially worth 15 % of your overall mark, but is redemptive. The final exam is worth at least 55 %, and at most 100 % of your overall mark for this subject. If your percentage mark on the final exam exceeds your percentage mark on any of the problem sets, then the weight of that problem set (15 % of your overall mark for the subject) will be transferred to the final exam. This is described more formally below.

Let w be your percentage mark on problem set 1, x be your percentage mark on problem set 2, y be your percentage mark on problem set 3 and z be your percentage mark on the final exam. Define:

- \( A = (0.15)(w+x+y)+(0.55)(z) \);
- \( B = (0.15)(w+x)+(0.70)(z) \);
- \( C = (0.15)(w+y)+(0.70)(z) \);
- \( D = (0.15)(x+y)+(0.70)(z) \);
- \( E = (0.15)(w)+(0.85)(z) \);
- \( F = (0.15)(x)+(0.85)(z) \);
- \( G = (0.15)(y)+(0.85)(z) \); and
- \( H = z \).

Your overall percentage mark for this subject will be given by the formula:

- Subject Percentage Mark = Max(A,B,C,D,E,F,G,H).
Useful References

There are multiple editions of some of the books in this reference list. In general, any of the editions of such books will be suitable. A rough indication of the level of each reference is indicated by the following symbols:

- Basic (B);
- Intermediate (I); and
- Advanced (A).

Assumed Knowledge

I will be assuming that you are familiar with the material that is typically covered in introductory (first-year level) subjects in mathematical techniques for economics. If you need to brush up on this material, you might like to consult the following books.


Primary Textbook

There is no requirement to purchase any books for this subject. However, if you want to purchase a book for reference purposes, you will probably find that Simon and Blume (1994) is a good choice. This will certainly be the case if your previous training in mathematics does not extend much beyond the assumed knowledge for this subject.


Supplementary References


• [A]: Corbae, D, MB Stinchcombe and J Zeman (2009), *An introduction to mathematical analysis for economic theory and econometrics*, Princeton University Press, USA.

• [I]: Dixit, AK (1990), *Optimization in economic theory (second edition)*, Oxford University Press, Great Britain.

• [I]: Intriligator, MD (1971), *Mathematical optimization and economic theory*, Prentice-Hall, USA.


• [I]: Leonard, D, and NV Long (1992), *Optimal control theory and static optimization in economics*, Cambridge University Press, USA.


Subject Outline and Reading Guide

This is a rough outline of the topics that might potentially be covered in this unit and the sequence in which they might be covered. Coverage of topics is contingent on there being sufficient time available.

Topic 1: Foundations

Topic Outline

- Logic and methods of proof.
- Sets and set operations.
- Some commonly used sets (including natural numbers, integers, rational numbers, real numbers and complex numbers).
- Cartesian products of sets.
- Mappings, functions and correspondences.
- Set size (finite, countably infinite or uncountably infinite).
- Binary relations and their potential properties.
- Equations and inequalities.
- Upper contour sets, level sets and lower contour sets.
- Applications:
  - Consumption sets.
  - Preference relations and utility functions.
  - Indifference curves and weak preference sets.
  - Budget lines and budget sets.
  - Production technologies and production functions.
  - Production possibilities frontiers and production possibilities sets.
  - Isoquants and input requirement sets.
  - Isocosts.
  - The price simplex.

Topic Reading Guide

- Chiang and Wainwright (2005), Chapters 1-2 (pp. 2-28).
- Corbae, Stinchcombe and Zeman (2009), Notation (pp. ix-xxii) and Chapters 1-3 (pp. 1-105).
- Intriligator (1971), Chapters 7 to 8 (pp. 142-219) and Appendix A (pp. 450-475).
- Kolmogorov and Fomin (1970), Chapter 1 (pp. 1-36).
- Ok (2007), Chapters A and B (pp. 3-113).
- Silberberg (1990), Chapter 1 (pp. 1-25) and Section 1 of Chapter 10 (pp. 299-308).

- **Simon and Blume (1994), Chapter 1 to 5 (pp. 3-103), Chapter 10 (pp. 199-236), Chapter 13 (pp. 273-299) and Appendices A1 to A3 (pp. 847-886).**

- Sundaram (1996), Chapter 1 (pp. 1-73) and Appendices A and B (pp. 315-329).

- Sydsaeter, Hammond, Seierstad and Strom (2005), Appendices A and B (pp. 519-553).

- Takayama (1993), Section 1 of Chapter 1 (pp. 3-13).

**Topic 2: The Structure of Euclidean Spaces**

**Topic Outline**

- **Euclidean n-Space as a Vector Space.**
  - A Review of Vector Arithmetic.
  - Real Vector Spaces.
  - Subspaces of Real Vector Spaces.
  - Linear Combinations, Linear Independence and Spanning.
  - A Basis for a Real Vector Space.
  - The Dimension of a Real Vector Space.

- **Euclidean n-Space as an Inner Product Space.**
  - Norms for real vector spaces.
  - Inner products.
  - Length, angle and distance.

- **Euclidean n-Space as a Metric Space**
  - The Euclidean distance and its relationship to the Euclidean norm.
  - Sequences, convergence and limits.
  - Properties of some sets.
    - Open sets, closed sets and clopen sets.
    - Compact sets. (Emphasise the fact that while “closed and bounded” implies compactness in Euclidean spaces, this is not the case for other spaces.)
    - Connected sets.
    - Bounded sets.
    - Convex sets.
  - Some properties of particular functions.
    - Continuity of functions.
    - Convexity and quasi-convexity of functions.
    - Concavity and quasi-concavity of functions.

- **Orthogonality and Projection.**
  - Orthogonality of a set of real vectors.
  - Orthogonal basis for a real vector space.
  - Orthonormality of a set of real vectors.
- Orthonormal basis for a real vector space.
- Orthogonal projection.

- Matrix Determinants.
  - The determinant function.
  - Properties of determinants.
  - Finding determinants by row reduction.
  - Finding determinants by cofactor expansion.

- Matrix Inversion.
  - The Existence (or Otherwise) of an Inverse Matrix.
  - Matrix Inversion by Row Reduction.
  - Matrix Inversion by Cofactor Expansion.
  - The Partitioned Inverse Formula.

- Real vector spaces associated with matrices.
  - The row-vectors and the row-space of a matrix.
  - The column-vectors and the column-space of a matrix.
  - The relationship between the row-echelon form of a matrix and a basis for the row-space of the matrix.
  - The relationship between the row-echelon form of the transpose of a matrix and a basis for the column-space of the matrix.
  - The dimension of the row-space of a matrix, the dimension of the column-space of a matrix and the rank of matrix.
  - The Characteristic Equation, Eigen-values and Eigen-vectors and the Eigen-space of a matrix.
  - Orthogonal matrices and projection matrices.

- Systems of Linear Equations.
  - The Existence (or Otherwise) and Uniqueness (or Otherwise) of Solutions.
  - Augmented Row Matrices and Gauss-Jordan Elimination.
  - Matrix Equations and Inverse Coefficient Matrices.
  - Cramer's Rule.

- Quadratic Forms.
  - Representation of a quadratic form using arbitrary square matrices.
  - Representation of a quadratic form using symmetric square matrices.
  - The sign of a quadratic form and matrix definiteness.

- Applications.
  - Single Market Linear Marshallian-Cross Model.
  - Multiple Markets Linear Marshallian-Cross Model.
  - Comparative Statics in the Marshallian-Cross Model.
  - The Identification Problem for the Single Market Linear Marshallian-Cross Model.
  - Comparative Statics for Demand Systems.
  - The Geometry of Least Squares Estimation.

**Topic Reading Guide**

- Anton (1987), Chapters 1 to 4 (pp. 1-244) Chapters 6 to 7 (pp. 301-370).
Topic 3: Multivariate Differential Calculus

**Topic Outline**

- **Partial Differentiation.**
  - First-Order Partial Derivatives.
  - The Gradient Vector.
  - The Del Operator.
  - Obtaining the Gradient Vector by Using the Del Operator.
  - Second-Order Partial Derivatives.
  - The Hessian Matrix.
  - Obtaining the Hessian Matrix by Using the Del Operator.
  - Young's Theorem
    - The theorem itself.
    - The role of the “twice continuously differentiable” condition.
    - Implications for symmetry of the Hessian matrix.
  - Higher-Order Partial Derivatives.
- **Total Differentiation.**
  - Total Differentials.
- Total Derivatives.
- The Implicit Function Theorem.
- The Inverse Function Theorem.

- Other Material.
  - Homegeneity, Homotheticity and Euler's Theorem.
  - Taylor Series Approximations for Multivariate Functions.

- Applications.
  - Consumer Theory Applications.
    - Comparative Statics for Demand Systems.
    - The Derivation of Slutsky's Equation.
    - Cournot Aggregation.
    - Engel Aggregation.
    - Euler Aggregation.
    - Deriving a Marshallian Demand System from a Translog Indirect Utility Function via Roy's Identity.
  - Producer Theory Applications.
    - Product Exhaustion under Constant Returns to Scale and Perfect Competition.
    - Deriving an Output Conditional Input Demand System from a Translog Cost Function via Shephard’s Lemma.

**Topic Reading Guide**

- Basilevsky (1983), Section 4 of Chapter 4 (pp. 139-147).

- Chiang and Wainwright (2005), Chapters 6-8 (pp. 124-218) and Chapter 10 (255-290).

- Silberberg (1990), Chapter 3 (pp. 68-106).

- **Simon and Blume (1994), Chapters 12-15 (pp. 253-371) and Chapters 29-30 (pp. 803-844).**

- Spiegel (1981a), Chapters 6 to 8 (pp. 101-179).

- Spiegel (1981b), Chapters 3 to 4 (pp. 35-81).

- Sydsaeter, Hammond, Seierstad and Strom (2005), Chapter 2 (pp. 43-103).

- Takayama (1993), Section 5 of Chapter 1 (pp. 40-71).

**Topic 4: Static Optimisation**

**Topic Outline**

- Fundamental concepts.
  - Maximisation problems.
- Upper bounds.
- Supremum (or least upper bound).
- Maximum.
- The potential difference between a supremum and a maximum.
- Solutions (max value) and the arguments that yield a solution (arg max).
  - Minimisation problems.
    - Lower bounds.
    - Infimum (or greatest lower bound).
    - Minimum.
    - The potential difference between an infimum and a minimum.
    - Solutions (min value) and the arguments that yield a solution (arg min).
  - Other.
    - Choice sets.
    - Converting a minimisation problem into a maximisation problem.
    - Converting a maximisation problem into a minimisation problem.
    - The separation of convex sets.
- Unconstrained optimisation.
  - First-order conditions.
  - Second-order conditions.
- Equality constrained optimisation.
  - The Lagrangean function.
  - First-order conditions.
  - Second-order conditions.
  - Non-degenerate constraint qualifications.
- Inequality constrained optimisation.
  - The Lagrangean function with a complete set of multipliers (including those for non-negativity and non-positivity constraints on individual variables).
  - First-order conditions and complimentary slackness.
  - Deriving the Kuhn-Tucker first-order conditions.
  - The standard Lagrangean function.
  - The Kuhn-Tucker first-order conditions and complimentary slackness.
  - Non-degenerate constraint qualifications.
- Mixed (equality and inequality) constrained problems.
  - The standard Lagrangean function.
  - The Kuhn-Tucker first-order conditions and complimentary slackness.
  - Non-degenerate constraint qualifications.
- Properties of solutions.
  - Weierstrass’ Theorem of the Maximum.
  - Properties of value functions and control functions.
  - Interpretation of the optimal value of a Lagrange multiplier.
• Envelope theorems.
• Some duality relationships.

• Applications.
  • Consumer Theory.
    ▪ Various utility maximisation problems.
    ▪ Various expenditure minimisation problems.
  • Producer Theory.
    ▪ Various profit maximisation problems.
    ▪ Various cost minimisation problems.
  • Welfare Economics.
    ▪ Various Pareto efficiency problems.
    ▪ Optimal commodity taxation (Ramsey taxation).
    ▪ Optimal public utility pricing (Ramsey-Boiteux pricing).
  • Econometrics.
    ▪ Ordinary least squares estimation of the classical linear regression model.
    ▪ Maximum likelihood estimation of the classical linear regression model (given the likelihood function).
    ▪ Estimation of the classical linear regression model subject to linear restrictions on the coefficient parameters.
    ▪ Motivation for Lagrange multiplier tests of the validity of linear restrictions on the coefficient parameters in the classical linear regression model.

Topic Reading Guide

• Chiang and Wainwright (2005), Chapter 9 (pp. 220-254) and Chapters 11 to 13 (pp. 291-442).

• Corbae, Stinchcombe and Zeman (2009), Chapters 5 (pp. 172-258) and Chapter 6 (pp. 259-354).

• Dixit (1990), Chapters 1 to 9 (pp. 1-144) and the Appendix (pp.181-185).

• Intriligator (1971), Chapters 2 to 5 (pp. 8-105) and Chapters 7 to 8 (pp. 142-219).

• Leonard and Long (1992), Chapter 1 (pp. 1-86).

• Silberberg (1990), Chapter 4 (pp. 107-134), Chapters 6 to 14 (pp. 156-490) and Chapter 17 (pp. 573-612).

• Simon and Blume (1991), Chapters 16 to 22 (pp. 375-576), Section 8 of Chapter 23 (pp. 626-629) and Chapter 30 (pp. 822-844).

• Sundaram (1996), Chapters 2 to 10 (pp. 74-267).

• Sydsaeter, Hammond, Seierstad and Strom (2005), Chapter 3 (pp. 105-150) and Chapter 13 (pp. 459-492).
• Takayama (1985), Chapters 1 and 2 (pp. 59-294).

• Takayama (1993), Chapters 2 to 5 (pp. 75-321) and Appendices B and C (pp. 605-647).

**Topic 5: Advanced Topics**

These topics will only be covered if, and to the extent that, time permits.

**Topic Outline**

• Correspondences.
  - Upper hemi-continuity of correspondences.
  - Lower hemi-continuity of correspondences.
  - Continuity of correspondences.
  - Berge’s Theorem of the Maximum.

• Fixed point theorems.
  - The contraction mapping theorem (Banach’s fixed point theorem).
  - Brouwer’s fixed point theorem.
  - Kakutani’s fixed point theorem.
  - Applications:
    - The existence of a competitive equilibrium in a pure exchange economy (and a counterexample involving Scarf-Shapley-Shubik preferences).
    - The existence of a Nash equilibrium in a normal-form non-cooperative game (and a counterexample such as matching pennies when mixing is not allowed.)
    - Uzawa’s equivalence theorem.

**Topic Reading Guide**

• Corbae, Stinchcombe and Zeman (2009), Section 4 of Chapter 4 (pp. 120-123), Section 11 of Chapter 4 (pp. 154-167), and Sections 11 and 12 of Chapters 5 (pp. 239-258).

• Ok (2007), Chapters C-E (pp. 117-353),

• Sydsaeter, Hammond, Seierstad and Strom (2005), Chapter 14 (pp. 493-518).

• Takayama (1985), Sections E and F of Chapter 2 (including their Appendices) (pp. 255-294).