Quality differentiation and firm choice between online and physical markets*

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Abstract

Growing with the prominence of e-commerce is the concern of researchers and policy-makers regarding product quality in online markets. We develop a theoretical framework to study firms’ choices between online and physical markets with respect to product quality and competition. We investigate two contrasting driving forces. On the one hand, since consumers cannot fully inspect an online product’s quality prior to purchase, conventional wisdom and some of the literature suggest that this attracts low-quality products to the online market (a pooling effect). On the other hand, the literature on vertical product differentiation indicates that a firm with a lower-quality product may prefer to reveal its product quality in the physical market because quality differentiation helps alleviate price competition (a differentiation effect). We show that the two contrasting effects may give rise to a wide range of product quality—from low-end to high-end—in both the online and offline markets.

Keywords: online vs. offline competition; market choices with respect to quality and competition

JEL Classification: L13

1 Introduction

The growing prominence of e-commerce has drawn increasing attention from researchers and policy makers. Among the recent studies, there is a stream of literature that focuses on the quality of online products[1] Unlike at brick-and-mortar stores, where consumers easily obtain first-hand experience with product quality, an online buyer cannot physically inspect a product before placing an order[2] This raises the concern that online markets tend to drive away firms with high-quality products and accommodate those that offer inferior quality. For example, Jin

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[1] For a review of recent literature on online versus offline competition, see Lieber and Syverson (2011).

[2] While some online sellers offer return-and-refund services for certain products, consumers may still incur disutility from disappointment and delay of consumption.
and Cato (2006, 2007) show that ungraded sport cards sold online are of lower quality than those in the offline market. The rationale is akin to a benchmark result in the literature on "voluntary disclosure" (Grossman and Hart, 1980; Jovanovic, 1982): since selling in a physical market is usually more costly than selling online, only sellers with higher-quality products will bear the cost and disclose their qualities via consumer inspection in the physical market. The sellers with lower-quality products pool themselves in the online market—a result that we call the "pooling effect."

However, there may also exist an opposite driving force: while the "pooling effect" explains well why the online market is more attractive to "bads"—that is, products that consumers will not buy if they know the quality—in reality, there are many goods with qualities ranging from low to high. Taking into account quality differentiation of goods, the stream of literature following Shaked and Sutton (1982) shows that differentiation may help alleviate price competition, and, thus, a firm may voluntarily choose to provide a low-quality good if the competitor produces a high-quality product. In the context of market choice, this means that a low-quality firm may want to choose the physical market (and, thus, reveal its quality) if the competitor's quality is high. In other words, one can argue that the physical market has a "differentiation effect" that may also attract low-quality firms.

Since the pooling effect and the differentiation effect act in opposite directions, it is not clear how they together affect firms' market choices and the quality distribution between the markets. In this paper, we develop a theoretical duopoly framework to study how firms choose between the online and offline markets with respect to product quality and competition. We conduct the study in two steps. First, we investigate a situation in which an entrant chooses between online and offline markets to compete with an offline incumbent. Next, we examine the setting in which two firms simultaneously choose between the markets.

In the entrant-incumbent model, we show that, due to the differentiation effect, in the offline market, the entrant's profit increases with the difference between the quality of its product and the incumbent's. This drives the entrant with highly differentiated product qualities—which consist of not only the highest qualities, but also the lowest ones—to choose the offline market. However, the differentiation effect diminishes as the entrant's product quality becomes closer to the incumbent's. We assume that the offline market is more costly than the online market. Consequently, there is an interval of qualities such that at the two boundary points, the entrant will be indifferent between the two markets. For medium-differentiated qualities, which are slightly above the lower bound or below the upper bound of the aforementioned interval, the entrant will choose the online market mainly to save the costs of the offline market. The pooling effect then drives the low-differentiated qualities—namely, those near the incumbent's quality—to pool with the medium-differentiated qualities in the online market.

Moreover, the differentiation effect implies that, all else equal, the offline market will be more attractive to the entrant with higher qualities. As a result, the average quality of the online good is lower than the quality of the incumbent's. However, the upper bound of the above-mentioned quality interval is above the incumbent’s quality. Hence the actual quality of the online good

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3In this paper, the words "offline" and "physical" are interchangeable.
may be higher than the incumbent’s.

The simultaneous-move model is different from the previous setting in that now both firms’ market choices are endogenous. We construct an equilibrium in which the market choices can be characterized by three areas of product qualities. In the first area, the firms’ qualities are low, and the pooling effect dominates the differentiation effect. As a result both firms choose the online market to avoid the higher costs of the physical market. Secondly, consistent with the entrant-incumbent model, when the firms’ qualities are sufficiently different, the differentiation effect outweighs the pooling effect. Consequently, both firms choose the physical market. Thirdly, when the firms’ qualities are sufficiently high (higher than when both are online) and sufficiently close (closer than when both are offline), we show that the firm with the higher quality will choose the physical market, while the lower-quality firm will sell online. A remarkable result here is that, when both firms have the highest quality, they will choose different markets. In this case, the online market is no longer a shelter for low-quality products; rather, it is essentially used by high-quality firms to avoid direct price competition in the physical market.

Overall, we show that the pooling effect and the differentiation effect give rise to a large set of possible product qualities in each of the online and offline markets, ranging from low-end to high-end.

More generally, the physical and online markets can be regarded as two markets that differ in the extent to which they can reveal product qualities. This suggests another interpretation and practical implication of our results: to tackle the potential adverse-selection problem, online markets have implemented various mechanisms, including consumer review, seller ranking, etc., to enhance the transparency of product quality. Our results suggest that firms prefer the more transparent market when their qualities sufficiently differ, and they will choose different markets when their qualities are close. Therefore, implementing a quality-disclosure mechanism in a market may result in more-differentiated qualities in the market, but not necessarily in better average product quality. To evaluate the effectiveness of a quality-disclosure mechanism, it is worth bearing in mind that firms may choose between the market under study and alternative markets, and that there may be strategic interdependency between firms’ market choices.

In addition to the above-mentioned two streams of literature on voluntary disclosure and vertical quality differentiation, this paper is related to the theoretical literature on the link between online and offline business, especially those on the impact of consumer demand, including Dinlersoz and Pereira (2007), Kocas and Bohlmann (2008), and Loginova (2009). These studies make firms’ marketplaces exogenous to product quality. By contrast we endogenize the firm’s market choice with respect to product quality. The theme of this paper, therefore, is in line with Chen

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4Dinlersoz and Pereira (2007) study physical retailers’ adoption of e-commerce in a technology-adoption-race framework where some customers have loyalty for particular firms while others buy from the lowest-price firm. Using another model with loyal consumers versus price-sensitive "switchers," Kocas and Bohlmann (2008) study price dispersion between firms with homogeneous products. Loginova (2009) studies the strategic interactions between online and offline markets in a Salop (circular city) model.

5In Dinler and Pereira (2007), the online good’s quality differs from the offline good’s by an exogenous constant. In Loginova (2009), firms’ choice of market type is also exogenous to the product quality, which is identical across firms. Consumers’ uncertainty about an online product’s quality is modeled via her uncertainty about her own type, which determines how well the product fits her and, before purchase, can be found out only by visiting a physical store.
et al. (2015). The latter, however, is abstracted from competition, and, thus, the differentiation effect is absent by assumption. Furthermore, Board (2009) studies competing firms’ decisions on quality disclosure. He focuses on the welfare analysis of mandatory disclosure law, and the equilibrium is such that the higher-quality firm always discloses, while the lower-quality firm discloses if its quality is neither too close to nor too far below that of the higher-quality firm. Thus, our paper differs from his in both its theme and results. In particular, a mandatory disclosure law, in the context of market choice, will require all firms to choose the physical market, which clearly is not feasible.

The rest of the paper is organized as follows. In Section 2, we study the entrant-incumbent model. In Section 3, we investigate the situation in which the firms simultaneously choose between the markets. We discuss practical implications and extensions of the model in Section 4. Section 5 concludes.

2 Market Choice by Entrant

2.1 Set-up

There are two profit-maximizing firms \( i = A, B \). Firm \( A \) is an entrant that chooses between an offline market and an online market, and firm \( B \) is an incumbent that sells in the offline market. Each firm’s product quality \( q_i \) is an independent draw from a uniform distribution on \([q, \bar{q}]\) with \( 0 < q < \bar{q} \). A firm’s product quality is observable to the public in the offline market. But if firm \( A \) sells online, then \( q_A \) will be unobservable to consumers prior to consumption and to the incumbent as well. After firm \( A \) chooses the market, the firms simultaneously post their prices \( p_i \), which are publicly observable.

We assume that the fixed cost of selling on the offline market, including rent and utility expenses, is larger than the fixed cost of selling online. For simplicity, we assume that the fixed cost for the offline market is \( F \), and the fixed cost for the online market is zero. The marginal cost of production is constant and normalized to 0.

The firms compete for a continuum of utility-maximizing consumers, indexed by \( j \). Each consumer has a unitary demand and is characterized by a type \( \lambda_j \), which measures the consumer’s marginal utility of quality and is an independent draw from a uniform distribution on \([\underline{\lambda}, \bar{\lambda}]\) with \( 0 < \underline{\lambda} < \bar{\lambda} \). Consumers are of mass \( M = \bar{\lambda} - \underline{\lambda} \). For ease of illustrating the differentiation effect, we adopt a standard specification of consumer utility as in Belleflamme and Peitz (2010): a consumer’s (indirect) utility from buying firm \( i \)’s product is \( u(q_i, p_i, \lambda_j) = r + \lambda_j q_i - p_i \), where \( r > 0 \) represents the basic willingness to pay for the product, and is assumed to be sufficiently large so that all consumers will buy from one firm in equilibrium.

Denote firm \( i \)’s profit by \( \pi_i \). If a mass of \( m \) consumers buys from firm \( i \), then \( \pi_i = mp_i - F \) if the firm sells in the offline market, and \( \pi_i = mp_i \) if it sells online.

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6 We model uncertainty in the online market as consumers’ uncertainty about a firm’s product quality. In contrast, Loginova (2009) assumes that consumers are uncertain about their own types if they buy online. Therefore quality signalling by the firm’s market choice is abstracted from her model.

7 Although, in this section and the next, we maintain the assumption that the physical market is more transparent and more costly than the online market, in Section 4.1, we show that the model can be adapted to study alternative situations, such as the online market being less transparent but more costly.
2.2 Analysis

A strategy of the entrant (firm A) is to choose a market and then a price, and a strategy of the incumbent (firm B) is to choose a price given the entrant’s market choice. A consumer’s strategy is to decide which firm to buy from given the prices and the observability of product quality. Because the entrant’s product quality will be unobservable in the online market prior to purchase, the game is essentially a dynamic game with incomplete information, and our solution concept is the perfect Bayesian equilibrium (PBE), which consists of a sequentially rational strategy profile and a consistent belief system.

A PBE is separating if the entrant always chooses the offline market, as this will lead to perfect revelation of its product quality. An equilibrium is pooling if the entrant always chooses the online market, since the market choice will not reveal extra information about its quality. An equilibrium is partially pooling (partially separating) if the entrant chooses the offline market for some qualities and the online market for others.

In equilibrium, the entrant will choose a market if the profit from that market is higher than that from the other. We first study the firms’ equilibrium behaviors in two situations: (i) the entrant has chosen the offline market; and (ii) the entrant has chosen the online market.

Lemma 1 below shows the profits of the two firms if they compete in the offline market.

Lemma 1 Suppose that both firms sell in the offline market. Then, given \((q_i, q_{-i})\) with \(q_i > q_{-i}\), in equilibrium the firms’ profits are \(\pi_i = \delta_H (q_i - q_{-i}) - F\) and \(\pi_{-i} = \delta_L(q_i - q_{-i}) - F\), where \(\delta_L = \frac{1}{5}(\lambda - 2\lambda)^2\) and \(\delta_H = \frac{1}{5}(2\lambda - \lambda)^2\).

Proof. In the appendix.

Lemma 1 is a standard result in the literature of vertical product differentiation (Belleflamme and Peitz, 2010): in the offline market, if the firms have the same quality, they will essentially be engaged in a cut-throat Bertrand competition. But when their qualities differ, their equilibrium profits both increase with the quality difference \((q_i - q_{-i})\). Hence, quality differentiation helps alleviate price competition and leads to higher profits for both firms, with the higher-quality firm making more profit than the other.

Next, if firm A chooses the online market, then consumers have to base their purchase decision on the expected quality of the firm’s product. Suppose that there is a set \(Q_A \subseteq [\underline{q}, \bar{q}]\) such that firm A will choose the online market if \(q_A \in Q_A\). Denote \(Q_A = \mathbb{E}[q_A | q_A \in Q_A]\), which is the expected quality of firm A’s product from consumers’ perspective. The next corollary characterizes the firms’ equilibrium profits if the entrant chooses the online market:

Corollary 1 If firm A chooses the online market, then in equilibrium \(\pi_A = \delta_H (Q_A - Q_B)\) and \(\pi_B = \delta_L(Q_A - Q_B) - F\) if \(Q_A > Q_B\); \(\pi_A = \delta_L(q_B - Q_A)\) and \(\pi_B = \delta_H(q_B - Q_A) - F\) if \(Q_A < q_B\).

Proof. The proof essentially replicates that of Lemma 1 by replacing \(q_i\) and \(q_{-i}\) with \(Q_A\) and \(Q_B\), and thus is skipped.

The proposition below characterizes a partial pooling equilibrium in which the entrant chooses
the offline market if its product quality is sufficiently high or sufficiently low, while for intermediate qualities it chooses the online market.

**Proposition 1** Let \( q_A = q_B - \frac{2\delta_H - \delta_L}{\sigma_H + \sigma_L} \cdot F \) and \( \bar{q}_A = q_B + \frac{2\delta_H - \delta_L}{\sigma_H + \sigma_L} \cdot F \). If \( \bar{q} < q_A \) and \( \bar{q}_A < \bar{q} \), then there exists an equilibrium in which firm A will choose the offline market if \( q_A \in [q_B, \bar{q}_A] \) and the online market if \( q_A \in [\bar{q}_A, \bar{q}] \).

**Proof.** In the appendix. ■

![Figure 1: Entrant’s market choice and average quality of online good](image)

The differentiation effect and the pooling effect, together, give rise to the entrant’s market choice in Proposition 1, the intuition for which has several layers. First, as Lemma 1 shows, the differentiation effect increases with the difference in the firms’ product qualities. Hence, the entrant with a quality close to the high-end \( \bar{q} \) or the low-end \( \bar{q} \) has a stronger incentive to choose the offline market than does the entrant with a quality close to the incumbent’s. For the highest qualities \( [\bar{q}_A, \bar{q}] \) as well as the lowest ones \( [q_B, q_A] \), the entrant has no incentive to pool with the qualities in the online market because, if it does, it will be regarded as having an intermediate quality, which, due to competition with the incumbent, will lead to a less profitable price than in the offline market. Consequently, as shown in Figure 1, the entrant will choose the offline market despite its higher fixed cost.

Secondly, the differentiation effect diminishes as the entrant’s quality approaches the incumbent’s. For the entrant’s quality that is slightly above \( q_A \) or below \( \bar{q}_A \), if the two markets are equally costly the entrant will still make a higher profit from the offline market than from going online. However, the higher fixed cost \( F \) of the offline market now drives the entrant to choose the online market. Those medium-differentiated qualities then become the target for qualities around \( q_B \) to pool with in the online market. It is worth noting that, in contrast to the conventional wisdom, in this case, the higher qualities (those close to \( q_B \)) want to pool with the lower qualities (those close to \( q_A \)), not the other way around. Moreover, \( \bar{q}_A > q_B \) implies that the actual quality of the online product may be higher than the incumbent’s.

Thirdly, Proposition 1 implies that in equilibrium the expected quality of the online product is \( Q_A = q_B - \frac{2\delta_H - \delta_L}{\sigma_L(\delta_H + \delta_L)} \cdot F < q_B \), as shown in Figure 1. That is, the online product is, on average, lower-quality than the offline product. To see the intuition of this result, note that
because \( \delta_B > \delta_L \), for \( q_A' \) and \( q_A'' \) such that \( q_B - q_A' = q_A'' - q_B \) (i.e., \( q_A' \) and \( q_A'' \) are equally distant from \( q_B \)), the higher quality \( q_A'' \)'s profit from choosing the offline market is higher than that of the lower quality \( q_A' \). So the offline market is more attractive to \( q_A'' \) than to \( q_A' \). Hence, there are more \( q_A < q_B \) than \( q_A > q_B \) to choose the online market. Consequently, the expected quality of the online product falls below the quality of the incumbent’s product in the offline market.

It is worth noting that in Proposition 1 \( q_A \) and \( q_B \) are independent of \( q \) and \( \bar{q} \). Hence, the condition that \( q < q_B \) and \( q_A < q_B \), under which the partial-pooling equilibrium exists, is fairly general. Below is a numerical example in which \( [q_A, q_B] \) is characterized given \( q_B, \lambda, \bar{\lambda}, \) and \( F \).

Numerical Example: Suppose that \( q_B = 10, \lambda = 1, \bar{\lambda} = 20, \) and \( F = 5 \). Then Proposition 1 implies that \( q_A = 9.77, q_B = 10.05 \), and, thus, \( Q_A = 9.91 \). When the entrant chooses the online market—i.e., \( q_A \in [q_A, q_B] \)—the profits of the entrant and the incumbent are \( \pi_A = 3.24 \) and \( \pi_B = 10.23 \).

3 Simultaneous Market Choices

Next, we investigate the case in which firm \( A \) and \( B \) simultaneously choose between online and offline markets. All the settings replicate the previous model, except for the timeline, which is described as follows. First, nature independently draws each firm’s product quality \( q_i, i = A, B \), from a uniform distribution on \( [q, \bar{q}] \), which is observable to both firms but not to the public. Then, the two firms simultaneously choose between online and offline market and simultaneously post their prices afterwards. A strategy of each firm is to choose a market and then a price for any \( (q_A, q_B) \).

The quality is not observable to the public prior to purchase if a firm chooses the online market, and, thus, the consumers have to base their purchase decision on the expected quality of the firm’s product. From the consumer’s perspective, an information set of the game consists of the market choices of both firms, together with the quality of any offline firm(s). Specifically, there are three cases. Firstly, if both firms choose the offline market, then the products’ qualities \( (q_A, q_B) \) are fully revealed. We denote the information set as \( h = (q_A, q_B) \) in this case. Second, if firm \( i \) with quality \( q_i \) chooses the online market while its competitor chooses the online market, we denote the information set as \( h = (q_i, O_{-i}) \), where "O" means "online." In addition, in this case, we suppose that there is a set \( Q_{-i}(q_i) \subset [q, \bar{q}] \) such that firm \( -i \) will choose the online market if \( q_{-i} \in Q_{-i}(q_i) \). We denote \( Q_{-i}(q_i) = E[q_{-i}|q_{-i} \in Q_{-i}(q_i)] \), which is the expected quality of firm \( -i \)'s product from the consumer’s perspective. Thirdly, we denote the information set where both firms choose the online market as \( h = (O_A, O_B) \). In this case, we suppose that there is a set \( Q \subset [q, \bar{q}] \times [q, \bar{q}] \) such that both firms will choose the online market if \( (q_A, q_B) \in Q \); and we let \( Q_A = E[q_A|(q_A, q_B) \in Q] \) and \( Q_B = E[q_B|(q_A, q_B) \in Q] \) be the expected quality of the online product from the consumer’s perspective.

As in the previous section, we use PBE as our solution concept. In equilibrium, an information set is either on or off the equilibrium path. If an information set is on the equilibrium path, then

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8As in the previous section, we consider only the signalling effect of the marketplace.
the belief $Q_{-i}(q_i)$ or $Q$ should be consistent with the firm’s strategy profile. Proposition 2 below characterizes an equilibrium in which firms make entry decisions simultaneously.

**Proposition 2** There exists an equilibrium consisting of the following strategy profile together with the consumer’s belief system:

**Strategy profile:**

1. Market choice:
   1. Both the firms will choose the offline market if $|q_B - q_A| \geq \frac{2E}{s_L}$.
   2. Firm $i$ will choose the offline market and firm $-i$ will choose the online market if $|q_i - q_{-i}| < \frac{2E}{s_L}$ and $q_i \geq q + \frac{2E}{s_H}$, $i = A, B$.
   3. Both firms choose the online market if $q_i < q + \frac{2E}{s_H}$, $i = A, B$.

2. Pricing:
   1. If both firms choose the online market, then $p_i = 0$ for $i = A, B$.
   2. If firm $i$ chooses the online market, while firm $-i$ chooses the offline market, then $p_i = \frac{1}{2}(2\lambda - \Lambda)(Q_i(q_{-i}) - q_{-i})$ and $p_{-i} = \frac{1}{2}(\lambda - 2\Lambda)(Q_i(q_{-i}) - q_i)$.
   3. Both firms sell in the offline market, then, given $(q_i, q_{-i})$ with $q_i > q_{-i}$, $p_i = \frac{1}{2}(2\lambda - \Lambda)(q_i - q_{-i})$ and $p_{-i} = \frac{1}{2}(\lambda - 2\Lambda)(q_i - q_{-i})$.

**Belief system:**

1. If $h = (O_A, O_B)$, consumers hold the belief that $Q = [q, q + \frac{2E}{s_H}]^2$ and $Q_A = Q_B = q + \frac{E}{s_H}$.
2. If $h = (q_i, O_{-i})$, then consumers hold the belief that

$$Q_{-i}(q_i) = \begin{cases} [q, q + \frac{2E}{s_H}] & \text{if } q_i \in [q, q + \frac{2E}{s_H}] \\ [q_i, q] & \text{if } q_i \in [q + \frac{2E}{s_H}, q + \frac{2E}{s_L}] \\ [q_i - \frac{2E}{s_L}, q_i] & \text{if } q_i \in [q + \frac{2E}{s_L}, q] \end{cases}$$

and

$$Q_{-i}(q_i) = \begin{cases} q + \frac{E}{s_H} & \text{if } q_i \in [q, q + \frac{2E}{s_H}] \\ \frac{1}{2}(q + q_i) & \text{if } q_i \in [q + \frac{2E}{s_H}, q + \frac{2E}{s_L}] \\ q_i - \frac{E}{s_L} & \text{if } q_i \in [q + \frac{2E}{s_L}, q] \end{cases}$$

3. At $h = (q_A, q_B)$, consumers just fully observe the true qualities.

**Proof.** In the appendix. □

Figure 2, below, illustrates the outcome of the equilibrium characterized by Proposition 2. When $(q_A, q_B)$ is in region I, both firms choose the online market. When $(q_A, q_B)$ is in region II, firm $B$ chooses the offline market while firm $A$ chooses the online market; and when $(q_A, q_B)$ is in region III, firm $A$ chooses the offline market, while firm $B$ chooses the online market. In regions IV and V, both firms choose the online market. \[8\]

\[\text{Since the five regions of quality pairs in Figure 2 are mutually exclusive, consumers’ information sets that are on and off the equilibrium path can be completely characterized by these regions. Specifically, as described in detail in the proof of Proposition 2, the following information sets are on the equilibrium path: (i) $h = (q_A, q_B)$ when $(q_A, q_B)$ are in regions IV and V; (ii) $h = (q_i, O_{-i})$, $i = A, B$, when $q_i \in \left[q + \frac{2E}{s_H}, q\right]$; and (iii) $h = (O_A, O_B)$. If a firm deviates from the prescribed equilibrium strategy, the game can go to an information set either on or off the}\]

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Proposition 2 shows that, similar to Proposition 1, a firm’s market choice depends not only on its own quality but also on the quality and the market choice of its competitor. However, since the firms move simultaneously, the specific results of their market choices differ from those in the incumbent-entrant model. In particular, Proposition 2 implies that each market can accommodate the full range of product qualities. For example, consider the three polar cases in the above-discussed regions—namely, $q_i = q_{-i} = q$ in region I (both online), $q_i = q_{-i} = \bar{q}$ in regions II/III (one online, one offline), and $(q_i = \bar{q}, q_{-i} = \bar{q})$ in regions IV/V (both offline). We see that both the offline and online markets can accommodate the highest and the lowest quality.

Specifically, the intuition of Proposition 2 is as follows. Firstly, in contrast to the previous model in which the incumbent is fixed at the offline market, in the current setting, both firms may choose the online market. According to Proposition 2, this occurs when both firms have low qualities (region I). When both firms’ qualities are unobservable, consumers hold the identical expectation about the firms’ quality. Consequently the firms are engaged in a cutthroat price competition, which leads to zero profit. Although the differentiation effect implies that a firm can make higher revenue by switching to the physical market, neither firm has the incentive to deviate: since the firms qualities are low in region I, the increase of revenue from the offline market will not be enough to cover the market’s higher cost. In other words, in region I, the equilibrium path. For example, consider a quality pair $(q_A, q_B)$ in region V (both firms choose the offline market in the equilibrium), and suppose that firm $B$ deviates to the online market such that an information set $\hat{h} = (q_A, O_B)$ is reached. If $q_A < q + \frac{2F}{\delta_H}$, then this information set is off the equilibrium path. However, if $q_A \geq q + \frac{2F}{\delta_H}$, the information set is still on the equilibrium path (region III), and, thus, the consistency between the belief system and the strategy profile in the PBE requires that the consumers still hold the on-the-equilibrium-path belief at $\hat{h}$ and the firms’ optimal prices after the deviation remain the same as the information set appears on the equilibrium path when there is no deviation.
pooling effect outweighs the differentiation effect.

Secondly, regions II and III represent the area where the firms’ qualities are sufficiently high (compared with those in region I) and close (closer than those in regions IV and V). The firms’ strategies are symmetric in this area, with the higher-quality firm choosing the offline market and the lower-quality one going online. In comparison with region I, in this area, the firms will no longer stay together in the online market because the differentiation effect and the higher qualities imply that the revenue from switching to the offline market exceeds the market’s fixed cost.

On the other hand, since the qualities are close in regions II/III, the differentiation effect is not large enough for the offline market to accommodate both firms. In this case, the online market essentially provides a haven that allows firms to avoid the price competition in one market and even more so when the online firm’s quality is closer to that of the offline firm. While the intuition is similar to that in the incumbent-entrant model, the current model depicts a more dramatic scenario: when both firms have the highest quality $q$, one will choose the offline market, whereas the other will go online. In this case, the online firm would prefer to be regarded as having a lower (expected) quality than to enter the profit-eroding price competition in the physical market.

Lastly, in regions IV and V, the result is consistent with the entrant-incumbent model: when the qualities are sufficiently apart, the differentiation effect drives both firms to choose the offline market.

In summary, in contrast to the conventional wisdom and previous studies that are based on competition-free settings, the pooling effect and the differentiation effect give rise to a large set of possible product qualities in each of the online and offline markets, ranging from the low-end quality to the high-end.

4 Discussion

4.1 Market transparency and entry cost

In the main model we assume that offline product quality is more transparent than online and that the offline market has a higher entry cost. These conditions correspond to scenario 1 in Table 1. The model, however, has the flexibility to investigate alternative situations. In particular, one may argue that with the implementation of quality disclosure mechanisms, consumers may learn more about a product from the online market than from visiting a physical shop. For example, for digital electronic products such as a GPS or camera, online forums allow consumers to share the sort of vivid post-purchase experience that is almost unattainable in a physical shop. Moreover, with a steady increase of online expenses, such as advertisement fees, the entry cost for the online market may also dwarf that of physical stores. These arguments correspond to scenario 4 in Table 1. But it is straightforward to see that scenarios 1 and 4 are symmetric, and, thus, we need only to switch the notations of "online" and "offline" in the main model, and the rest of the analysis carries through.

It is also easy to see that scenarios 2 and 3 are symmetric. If the offline market is more transparent and the online market more costly (scenario 2), then the pooling effect will vanish.
see this, consider the entrant-incumbent model. The two cutoff levels \( \frac{q}{A} \) and \( \frac{q}{A} \), as characterized in Proposition 1, will no longer exist: they were the points at which the differentiation effect decreased until that the entrant was indifferent between the costly physical market and the free online market. But now, with the online market being more costly, at each quality level the entrant will prefer the physical market. Consequently, the entrant will completely shun the online market. This is analogous to the fully-unravelling result under zero disclosure cost in the literature of voluntary disclosure. Next, in the simultaneous-move model, the prevailing differentiation effect implies that both firms will choose the physical market, with the equilibrium outcome characterized by Lemma 1.

<table>
<thead>
<tr>
<th>Offline market more transparent</th>
<th>Offline market more costly</th>
<th>Online market more costly</th>
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<tbody>
<tr>
<td>1</td>
<td>Online market more costly</td>
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<td>2</td>
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<td>4</td>
<td>Online market more costly</td>
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Table-1: Alternative assumptions on market transparency and entry cost

4.2 Empirical implications: quality distribution vs. quality levels

To tackle the potential adverse selection problem, online markets have implemented various mechanisms, including consumer review, seller ranking, etc., to enhance the transparency of product quality. More generally, the physical and online markets in the model can be regarded as two markets that differ in the magnitude to which product qualities can be revealed. Our results have empirical implications for evaluating the effects of such mechanisms: the above results suggest that firms prefer the more transparent market when their qualities are sufficiently differentiated, and that they will choose different markets when the qualities are close.

Previous empirical studies on online product quality focus mainly on a single market, and thus ignore the possibility that sellers may choose among alternative markets (Hui et al. forthcoming, Saeedi 2014, Cabral and Hortacsu 2010). While Jin and Cato (2007) consider sellers’ choices between online and offline markets, their study is abstracted from strategic interactions among sellers. In contrast, our results suggest that to evaluate the effectiveness of the quality disclosure mechanisms, it is worth noting that firms may choose between the market under study and alternative markets, and that their choices can be interdependent. We show that the improved transparency of a market may result in more differentiated qualities but not necessarily in an increase in the quality levels in the market. Therefore, in addition to the impact on the quality levels, researchers can investigate how the quality distribution in the market changes with the implementation of a quality disclosure mechanism. A quantile regression may be utilized to examine whether the tails of the quality distribution become heavier as the market becomes more transparent.

4.3 Fraudulent product and low quality

To illustrate the differentiation effect on firms’ market choices, we adopt the modelling approach from the literature on vertical product differentiation. A facilitating assumption in the models is
that a product is always of value ($\lambda q_i > 0$), and, thus, provided that the price is sufficiently low, a consumer is willing to pay for it. The solution concept of PBE requires consumers’ belief about product quality to be consistent with the firms’ strategies. In the equilibria that we characterize, consumers base their expectations about online quality on firms’ market choices. Conditional on the other firm’s market choice and observability of that firm’s product quality, once a firm goes online, consumers’ expectation about its quality remains constant, regardless of its actual quality. That is, even if the firm is able to make a (costless) announcement about its product quality, consumers’ expectation will not be affected by the claim.

One direction to extend our model is to consider more-general cases, where firms may produce fraudulent products that are of zero or negative value to consumers. If a firm produces a fraudulent product, then it can sell only in a market in which the product’s quality is not observable—i.e., the online market. As a result, consumers will lower their expectation about online quality, which then makes the physical market more attractive to firms with normal qualities. Hence, we can conjecture that, in equilibrium, regions I, II, and III in Figure 2 will become thinner, while regions IV and V will expand. In other words, with the possibility of fraudulent products, the normal qualities in the online market will become less differentiated than when the defective goods are absent. A more detailed study would be a direction for future research.

5 Conclusion

Since consumers cannot physically inspect products when they purchase online, it has been a major concern that the online markets appear more attractive to sellers with low-quality products. We show that, by taking into account the effect of vertical product differentiation on firms’ market choices, both the online and the physical markets can attract a wide range of product quality. This is in contrast to the previous studies based on competition-free settings. Our findings can be a starting point for several directions of future study. First, we have investigated a polar case in which the product quality can be perfectly disclosed in one market and fully concealed in the other. Therefore, the results can be used as a benchmark for investigating more-general cases in which the markets’ quality transparency is in between the extreme cases. Moreover, one may endogenize the firms’ quality choices prior to their market choices. Furthermore, it will also be interesting to investigate in more detail how other features of the online market, such as market size, transportation costs, and specific quality disclosure mechanisms, affect firms’ market choices.

6 Appendix

6.1 Proof of Lemma 1

Proof. The proof is similar to that in Belleflamme and Peitz (2010) for a standard model of vertical product differentiation in an offline market: When both firms choose to sell in the offline market, $(q_B, p_B)$ and $(q_A, p_A)$ are common knowledge. Without loss of generality, given $(q_B, p_B)$ and $(q_A, p_A)$ with $q_B < q_A$ and $p_B < p_A$, there will be a consumer with $\hat{\lambda}$ who will be indifferent
between the two firms:

\[
\hat{\lambda} q_B - p_B = \hat{\lambda} q_A - p_A, \\
\Rightarrow \hat{\lambda} = \frac{p_A - p_B}{q_A - q_B}.
\]

Hence, firm B’s profit is \( \pi_B = (\hat{\lambda} - \hat{\lambda})p_B - F = (\frac{p_A - p_B}{q_A - q_B} - \hat{\lambda})p_B - F \), while firm A’s profit is \( \pi_A = (\hat{\lambda} - \hat{\lambda})p_A - F = (\hat{\lambda} - \frac{p_A - p_B}{q_A - q_B})p_A - F \).

Given \( q_B \) and \( q_A \), each firm chooses the price to maximize its profit. The first-order condition for firm B implies \( \frac{p_A - p_B}{q_A - q_B} - \frac{3}{2}(\hat{\lambda} - \frac{1}{2}q_A q_B) = 0 \), which leads to 

\[
p_B = \frac{1}{2} \left[ p_A - \frac{3}{2}(q_A - q_B) \right].
\]

The first-order condition for firm A implies \( \hat{\lambda} - \frac{p_A - p_B}{q_A - q_B} = 0 \), which implies 

\[
p_A = \frac{1}{2} \left[ \hat{\lambda}(q_A - q_B) + p_B \right].
\]

Solving the two first-order conditions for \( p_B \) and \( p_A \), we have

\[
p_B = \frac{1}{3} (\hat{\lambda} - 2\hat{\lambda})(q_A - q_B), \\
p_A = \frac{1}{3} (2\hat{\lambda} - \hat{\lambda})(q_A - q_B).
\]

Thus given \( q_B < q_A \), in equilibrium

\[
\pi_B(q_B, q_A) = \frac{p_A - p_B}{q_A - q_B} - \frac{3}{2}(\hat{\lambda} - \frac{1}{2}q_A q_B) p_B - F \\
= \frac{1}{3} (2\hat{\lambda} - \hat{\lambda})(q_A - q_B) - \frac{3}{2}(\hat{\lambda} - \frac{1}{2}q_A q_B)(q_A - q_B) - F \\
= \frac{1}{9} (2\hat{\lambda} - \hat{\lambda})^2 (q_A - q_B) - F
\]

\[
\pi_A(q_B, q_A) = (\hat{\lambda} - \frac{p_A - p_B}{q_A - q_B}) p_A - F \\
= (\hat{\lambda} - \frac{1}{3} (2\hat{\lambda} - \hat{\lambda})(q_A - q_B) - \frac{1}{3} (2\hat{\lambda} - \hat{\lambda})(q_A - q_B))(q_A - q_B) - F \\
= \frac{1}{9} (2\hat{\lambda} - \hat{\lambda})^2 (q_A - q_B) - F
\]

\section*{6.2 Proof of Proposition 1}

**Proof.** First suppose that \( Q_A < q_B \). We consider two possible cases.

**Case 1:** \( q_A \in [q, q_B] \).

If firm A chooses the offline market, then by Lemma 1, \( \pi_A = \delta_L(q_B - q_A) - F \). If firm A
chooses the online market, then, by Corollary 1, \( \pi_A = \delta_L(q_B - Q_A) \). Firm A will choose the online market if \( \delta_L(q_B - Q_A) \geq \delta_L(q_B - q_A) - F \); that is,

\[
q_A \geq Q_A - \frac{F}{\delta_L}.
\] (1)

**Case 2:** \( q_A \in (q_B, \overline{q}) \).

If firm A chooses the offline market, then \( \pi_A = \delta_H(q_A - q_B) - F \). If firm A chooses the online market, then \( \pi_A = \delta_L(q_B - Q_A) \). Firm A will choose the online market when \( \delta_L(q_B - Q_A) \geq \delta_H(q_A - q_B) - F \); that is,

\[
q_A \leq q_B + \frac{1}{\delta_H}[F + \delta_L(q_B - Q_A)].
\] (2)

(1) and (2) imply that \( Q_A = \frac{1}{2} \left[ \left( q_A - \frac{F}{\delta_L} \right) + \left( \frac{1}{\delta_H}[F + \delta_L(q_B - Q_A)] \right) \right] \). Solving this equation for \( Q_A \), we have

\[
Q_A = q_B - \frac{\delta_H - \delta_L}{\delta_L(\delta_H + \delta_L)} \cdot F < q_B.
\] (3)

Note that \( Q_A < q_B \) confirms our assumption.

Since firm A will choose to sell online when \( \pi_A \in [Q_A - \frac{F}{\delta_L}, q_B + \frac{1}{\delta_H}[F + \delta_L(q_B - Q_A)] \] plugging (3) into the lower bound and the upper bound of the set, we get

\[
\frac{q_A}{2} = q_B - \frac{2}{\delta_H + \delta_L} \cdot \frac{\delta_H}{\delta_L} \cdot F, \quad \overline{q}_A = q_B + \frac{2}{\delta_H + \delta_L} \cdot F.
\]

To complete the description of the PBE, we need to construct the consumers’ belief system. On the equilibrium path, the belief system is straightforward: by assumption, if the entrant chooses the offline market, consumers can verify her quality. On the other hand, if the entrant chooses the online market, consumers believe that her quality is \( Q_A \). Then, the characterization of \( \underline{q}_A \) and \( \overline{q}_A \) implies that an offline entrant has no incentive to deviate to the online market, and vice versa. A subtle issue remains in the off-equilibrium-path belief: since the entrant chooses a marketplace and then sets the price, a quality in the online market may want to post a different price in order to separate itself from other online qualities. In particular, \( q_A \) or \( \overline{q}_A \) has the strongest incentive to deviate in this way, because the deviation will allow them to reveal their types while not paying the offline-market cost \( F \). To deter this kind of deviation, it suffices to let consumers hold the off-equilibrium-path belief that any online price other than the one in equilibrium comes from an entrant with quality \( Q_A \) because, then, the most profitable price-deviation in the online market will lead to the same profit as staying on the equilibrium path.

Next, we will show that it cannot be \( Q_A > q_B \) in equilibrium: Suppose, toward a contradiction, that \( Q_A > q_B \). There are two cases:

**Case 1:** \( q_A \in [q, q_B] \).

If firm A chooses the offline market, then \( \pi_A = \delta_L(q_B - Q_A) - F \). If firm A chooses the online market, then \( \pi_A = \delta_H(Q_A - q_B) \). Firm A will choose the online market if \( \delta_H(Q_A - q_B) \geq \delta_L(q_B - q_A) - F \); that is, \( q_A \geq q_B - \frac{1}{\delta_L}[F + \delta_H(Q_A - q_B)] \).
Case 2: $q_A \in (q_B, \bar{q}]$.

If firm A chooses the offline market, then $\pi_A = \delta_H(q_A - q_B) - F$. If firm A chooses the online market, then $\pi_A = \delta_H(Q_A - q_B)$. Firm A will choose the online market if $\delta_H(Q_A - q_B) \geq \delta_H(q_A - q_B) - F$; that is, $q_A \leq Q_A + \frac{F}{\delta_H}$.

Hence, $Q_A = \frac{1}{2} \left[ \left( Q_A + \frac{F}{\delta_H} \right) + \left( q_B - \frac{1}{\delta_L} [F + \delta_H(Q_A - q_B)] \right) \right]$. Solving the equation for $Q_A$, we have $Q_A = q_B - \frac{\delta_H - \delta_L}{\delta_L \delta_H} : F < q_B$, contradicting $Q_A > q_B$. Therefore it cannot be $Q_A > q_B$ in equilibrium. ■

6.3 Proof of Proposition 2

We first consider the pricing strategy. If both firms choose to sell on the online market, due to the symmetry, consumers will hold the expectation that both firms’ product quality is $Q = Q_A = Q_B$. Then the firms will be essentially engaged in a standard Bertrand competition with identical products, and, thus, undercut each other’s price until it reaches 0. Apparently, both firms get zero profit in this scenario. When both firms choose the offline market, or when one firm chooses the online market while the other firm chooses the offline market, the proof for the pricing strategy part is basically the same as that in the proof of Lemma 1 and Corollary 1, and thus is omitted here. Given the consumer’s belief, firms’ profit is characterized by Lemma 1 and Corollary 1 in these two scenarios, as well.

Next, we denote the belief system in Proposition 2 as (BS) and will show that given (BS), it is optimal for both firms to choose the marketplaces as described, and (BS) is consistent with the strategy profile. This part of the proof consists of several lemmas below, which verify the rationality of the strategy profile and the consistency of the belief system in different areas of the $[q, \bar{q}] \times [q, \bar{q}]$ space.

Lemma 2 Given (BS), when $q_i \in [q, q + \frac{2F}{\delta_H})$, $i = A, B$ (Area (I) in Figure 2), firm $i$ optimally chooses the online market given that firm $\neg i$ is online.

Proof. Given that firm $\neg i$ is online, if firm $i$ chooses the online market, it gets zero profit. If firm $i$ chooses the offline market, then its profit is no greater than

$$\pi_i = \delta_H |Q_{\neg i}(q_i) - q_i| - F.$$

When $q_i \in [q, q + \frac{2F}{\delta_H})$ and $Q_{\neg i}(q_i) = q + \frac{F}{\delta_H}$, $|Q_{\neg i}(q_i) - q_i| \leq \frac{F}{\delta_H}$, and thus,

$$\delta_H |Q_{\neg i}(q_i) - q_i| - F \leq 0.$$

Hence, firm $i$ will get no benefit by deviating to the offline market. ■

Lemma 3 Given (BS), for $i = A, B$, when $q_i \in [q + \frac{2F}{\delta_H}, q + \frac{2F}{\delta_L})$ and $q_{\neg i} \in [q, q_i]$ (the shaded parts of regions II and III in Figure 3), firm $\neg i$ will optimally choose the online market given that firm $i$ is offline, and firm $i$ will optimally choose the offline market when firm $\neg i$ is online.

Proof. We first show that given (BS), when $q_i \in [q + \frac{2F}{\delta_H}, q + \frac{2F}{\delta_L})$ and $q_{\neg i} \in [q, q_i]$, firm $\neg i$ will optimally choose the online market given that firm $i$ is offline. When firm $i$ is offline, if
firm \(-i\) chooses the offline market, then, by Lemma 1, \(\pi_{-i} = \delta_L(q_i - q_{-i}) - F\) since \(q_i \geq q_{-i}\). If firm \(-i\) chooses the online market, then, since \(Q_{-i}(q_i) = \frac{1}{2}(q + q_i) < q_i\), by Corollary 1, \(\pi_{-i} = \delta_L(q_i - Q_{-i}(q_i))\). Firm \(-i\) will choose the online market if \(\delta_L(q_i - Q_{-i}(q_i)) \geq \delta_L(q_i - q_{-i}) - F\); i.e.,
\[
q_{-i} \geq Q_{-i}(q_i) - \frac{F}{\delta_L} = q_i - \frac{F}{\delta_L}.
\]

When \(q_i < q + \frac{2F}{\delta_L}, \frac{1}{2}(q + q_i) - \frac{F}{\delta_L} < q_i\). This implies that any \(q_{-i} \in [q, q_i]\) will choose the online market when firm \(i\) is offline.

To see why it is also optimal for firm \(i\) to choose the offline market given that firm \(-i\) is online, first note that both firms get zero profit if they compete in the online market. Hence, when firm \(-i\) is online, firm \(i\) optimally chooses the online market when \(\pi_i = \delta_H(q_i - Q_{-i}(q_i)) - F \geq 0\), which is true when \(q_i \geq q + \frac{2F}{\delta_H}\).

![Figure 3:](image)

**Lemma 4** Given (BS), for \(i = A, B\), when \(q_i \in [q + \frac{2F}{\delta_H}, q]\) and \(q_{-i} \in [q_i - \frac{2F}{\delta_L}, q_i]\) (the un-shaded parts of regions II and III in Figure 3), firm \(i\) will optimally choose the online market given that firm \(-i\) is offline, and firm \(-i\) will optimally choose the offline market when firm \(-i\) is online.

**Proof.** We first show that given (BS), when \(q_i \in [q + \frac{2F}{\delta_H}, q]\) and \(q_{-i} \in [q_i - \frac{2F}{\delta_L}, q_i]\), firm \(-i\) will optimally choose the online market given that firm \(i\) is offline. When firm \(i\) is offline, if firm \(-i\) chooses the offline market, then, by Lemma 1, \(\pi_{-i} = \delta_L(q_i - q_{-i}) - F\) since \(q_i \geq q_{-i}\). If firm \(-i\) chooses the online market, then, since \(Q_{-i}(q_i) = q_i - \frac{F}{\delta_L} < q_i\), by Corollary 1, \(\pi_{-i} = \delta_L(q_i - Q_{-i}(q_i))\). Firm \(-i\) will choose the online market if \(\delta_L(q_i - Q_{-i}(q_i)) \geq \delta_L(q_i - q_{-i}) - F\); i.e.,
\[
q_{-i} \geq Q_{-i}(q_i) - \frac{F}{\delta_L} = q_i - \frac{2F}{\delta_L}.
\]
This implies that any \( q_{-i} \in [q_i - \frac{2F}{\delta_L}, q_i] \) will choose the online market when firm \( i \) is offline.

To see why it is also optimal for firm \( i \) to choose the offline market given that firm \( -i \) is online, first note that both firms get zero profit if they compete in the online market. Hence, when firm \( -i \) is online, firm \( i \) optimally chooses the offline market when \( \pi_i = \delta_H(q_i - Q_{-i}(q_i)) - F \geq 0 \), which is true when \( q_i \geq q + \frac{2F}{\delta_H} \).

**Lemma 5** Given \((BS)\), when \(|q_i - q_{-i}| \geq \frac{2F}{\delta_L} \) (Regions IV and V in Figure 2), firm \(-i\) will optimally choose the offline market given that firm \( i \) is offline, \( i = A, B \).

**Proof.** We consider the area in which \( q_B - q_A \geq \frac{2F}{\delta_L} \) (Area (V) in Figure 2). The discussion for the area in which \( q_A - q_B \geq \frac{2F}{\delta_L} \) (Area (IV) in Figure 2) follows similar arguments. First, when \( q_B - q_A \geq \frac{2F}{\delta_L} \), we have \( q_B \geq q + \frac{2F}{\delta_H} \), so firm \( A \) will optimally choose the offline market if \( q_A \leq q - \frac{2F}{\delta_L} \), according to the proof of the previous lemma. Hence, all we need to show is that firm \( B \) will optimally choose the offline market given that firm \( A \) is offline when \( q_B - q_A \geq \frac{2F}{\delta_L} \).

When firm \( A \) is offline, if firm \( B \) chooses the offline market, then its profit is \( \pi_B = \delta_H(q_B - q_A) - F \), which is no less than \( \frac{2\delta_H - \delta_L}{\delta_L} \cdot F \) since

\[
\delta_H(q_B - q_A) - F \geq \delta_H \cdot \frac{2F}{\delta_L} - F = \frac{2\delta_H - \delta_L}{\delta_L} \cdot F.
\]

If firm \( B \) chooses the online market, its profit can be one of the three cases below:

**Case 1:** \( q_A \in [q, q + \frac{2F}{\delta_H}] \)

In this case, if firm \( B \) chooses the online market, then \( Q_B(q_A) = q + \frac{F}{\delta_H} \). Thus, \( \pi_B = \delta_L(q_A - Q_B(q_A)) \) if \( q_A > Q_B(q_A) \), and \( \pi_B = \delta_H(Q_B(q_A) - q_A) \) if \( q_A \leq Q_B(q_A) \). But whether \( q_A > Q_B(q_A) \) or \( q_A \leq Q_B(q_A) \), \( \pi_B \leq \delta_H \cdot |Q_B(q_A) - q_A| \leq \delta_H \cdot \frac{F}{\delta_H} = F \). However, \( q_A > q = \frac{2F}{\delta_L} \) in this case.

**Case 2:** \( q_A \in [q + \frac{2F}{\delta_H}, q + \frac{2F}{\delta_H}] \)

This case can happen when \( q - q - \frac{2F}{\delta_H} > q + \frac{2F}{\delta_H} \) (see Figure 3). In this case, if firm \( i \) chooses the online market, then \( Q_B(q_A) = \frac{1}{2}(q + q_A) \) and \( \pi_B = \delta_L(q_A - Q_B(q_A)) = \delta_L \cdot \frac{1}{2}(q_A - q) \). Note that \( \pi_B < F \) since \( q_A - q < \frac{2F}{\delta_L} \) in this case.

**Case 3:** \( q_A \in [q + \frac{2F}{\delta_H}, \frac{q}{2} - \frac{2F}{\delta_H}] \)

This case can happen when \( q - q - \frac{2F}{\delta_H} > q + \frac{2F}{\delta_H} \) (see Figure 3). In this case, if firm \( i \) chooses the online market, then \( Q_B(q_A) = q_A - \frac{F}{\delta_L} \) and \( \pi_B = \delta_L(q_A - Q_B(q_A)) = F \).

In sum, since \( F < \frac{2\delta_H - \delta_L}{\delta_L} \cdot F \), in any of the three cases, the profit will be less than \( \frac{2\delta_H - \delta_L}{\delta_L} \cdot F \) and, thus, will be less the profit from choosing the offline market. Therefore, firm \( B \) will optimally choose the offline market when firm \( A \) is offline.

So far, we have shown that given the belief system, the strategy profile satisfies rationality. With this strategy profile, the information set \((q_i, O_{-i})\) is on the equilibrium path when \( q_i \geq q + \frac{2F}{\delta_H} \) and off the equilibrium path when \( q_i < q + \frac{2F}{\delta_H} \). When \((q_i, O_{-i})\) is on the equilibrium path, it is straightforward to check that the belief system is consistent with the strategy profile. Therefore, the equilibrium characterized by Proposition 2 turns out to be a PBE.
References


[6] Hui, Xiang; Saeedi, Maryam, Shen, Zeqian; and Sundaresan, Neel; "Reputation & Regulations: Evidence from eBay", forthcoming in Management Science


