Can We Rule Out Speculative Hyperinflations in Maximising Models? Yes, We Can.

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Abstract

A critique is advanced of the contention of Obstfeld and Rogoff (1983) that in a fiat money regime, ‘speculative hyperinflations can be excluded only through severe restrictions’ on preferences. It is maintained here, in contrast, that no more than the infinity of the marginal utility of real balances at zero real balances is sufficient to rule out speculative hyperinflations. What Obstfeld and Rogoff have successfully drawn attention to is the theoretical possibility of money having strictly zero purchasing power. But the phenomenon of zero purchasing power has no explanatory power for historically observed hyperinflations, or any historically observed modern economy.

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In the judgement of some theorists, models of monetary equilibrium possess the property of ‘speculative hyperinflations’. That is, with the nominal money supply constant, it appears possible that a sequence of ever rising price levels (and ever rising inflation rates) is consistent with maximisation and equilibrium. The logic of this apparent equilibrium goes as follows: a rise in prices reduces the supply of real balances, and the consequent required reduction in the demand for real balances is ‘ratified’ by an even greater rise in prices in the following period; which besides making rational the required reduction in money demand (by increasing the opportunity cost of holding money) also reduces the supply of real balances still further in the following period …. necessitating in that period a further reduction in the demand for real balances … that requires ratification by an even greater rise in prices in the period after that … and so on. There is an endless round of ever larger price rises; and any arbitrarily gigantic inflation rate that one cares to nominate would be ultimately be exceeded.

But are such ‘speculative hyperinflations’ admissible? Cannot ‘speculative hyperinflations’ be ruled out by invoking one of the standard elements in the canon of economists’ reasoning (maximisation, concavity, market equilibrium)? ‘No’, replied Obstfeld and Rogoff (1983). By means of a clear and demonstrative argument, based on a maximising model, they claimed that in order to preclude speculative hyperinflations it must be assumed that the total utility of real money balances approaches infinity as real balances approach zero. In other words, ‘if an agent is deprived of his real balances, no finite increase in his endowment of the consumption good can restore him to his previous utility level’. As this assumption seems certainly
to overstate the utility money, Obstfeld and Rogoff concluded that speculative hyperinflations cannot be ruled out.

The Obstfeld and Rogoff proposition has radical implications. The theoretical validity of ‘speculative hyperinflations’ – hyperinflation with a constant money supply - would be gravely damaging to the Quantity Theory of Money. How can the quantity of money be said to determine money’s value if a constant money supply was equally consistent with constant prices, or exploding prices? Further, the Obstfeld and Rogoff conclusion has in recent years gained greater purchase on our attention by its use to rationalise a ‘Fiscal Theory of the Price Level’ in place of the Quantity Theory (see, for example, Kocherlakota and Phelan 1999). Thus the Obstfeld and Rogoff proposition can be seen as one element of the recess in the Quantity Theory in recent decades. (On this recess, see Coleman 2007, which is broadly an exercise in rebutting that trend, and to that end contained some unripened remarks on speculative hyperinflations).

The contention of this paper is that speculative hyperinflations can be ruled out, and by no more than an assumption freely granted in Obstfeld and Rogoff’s analysis: the marginal utility of money approaching infinity as the value of money approaches zero. Even this assumption is only a sufficient to rule out speculative hyperinflations; it is not actually necessary to make that assumption. The necessary and sufficient condition is merely that there exists some level of real balances such that the marginal utility of money exceeds the marginal utility of consumption.
Such weakened conditions do not, however, rule out the phenomenon of money assuming a strictly zero value in equilibrium. That is a genuine possibility, and that is the possibility that Obstfeld and Rogoff successfully uncovered in their 1983 paper. But that possibility has nothing to contribute to the explanation of historically observed hyperinflations, or any historically observed modern fiat money economy.

The paper begins rehearsing a model that is Obstfeld and Rogoff’s, apart from some irrelevant simplifications to the structure of the real economy. It then presents the present paper’s argument that marginal utility the marginal utility of money approaching infinity as the value of money approaches zero is sufficient to rule out speculative hyperinflations.

It concludes by noting the zero value of money equilibrium identified by Obstfeld and Rogoff, and setting it aside as chimerical possibility.

1. A model of monetary equilibrium

The economy is assumed to be composed of infinitely-lived persons who share an identical utility function, in which utility is obtained from consumption, and holdings of real balances, h.

\[ U = \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(h_t)] \]
Obstfeld and Rogoff also assume the Inada condition,

$$\lim m \to 0, \varepsilon'(m) \to \infty$$

and we will also do so.

We will suppose that output is produced by a two factor, constant returns neoclassical production function with the usual properties. We also suppose that the capital stock is at its steady state value so that the stream of consumption is constant.

$$c_t = c \quad \text{all} \; c$$

The nominal money supply is fixed at 1,

$$M_t = 1 \quad \text{all} \; t$$

Money demand equals money supply,

$$h_t = m_t = \frac{M_t}{P_t} \quad \text{all} \; t$$

Agents maximise utility, and this yields a key equimarginal condition.

$$-u'(c_t) + \varepsilon'(h_t) + \frac{u'(c_{t+1})}{1 + \pi_t} = 0 \quad (1)$$
where \[ \pi_t \equiv \frac{P_{t+1}}{P_t} - 1 \]

Condition (1) can be understood as follows. The agent can always perturb their ‘plan’ by consuming one unit less in the current period, and thereby adding one unit to their real balances in the current period, and then spending the acquired balances on consumption in the following period. The utility impact of this perturbation is,

\[ -u'(c_t) + v'(h_t) + \frac{u'(c_{t+1})}{1 + \pi_t} \]

This magnitude must equal zero if they are optimising, and thus (1).

Further, given (i) the constancy of consumption, that implies \( u'(c_{t+1}) = \beta u'(c_t) = \beta u'(c) \), (ii) the constancy of the nominal money supply at 1, and (iii) monetary equilibrium, \( h_t = m_t \), we may write (1) as:

\[ m_t[-u'(c) + v'(m_t)] + \beta u'(c)m_{t+1} = 0 \] (2)

This is evidently a non-linear first order difference equation in \( m \); where \( m \) can be usefully interpreted as an index of the value of money.

There is clearly some magnitude of \( m \), \( m! \), such that \( m \) is unchanged over time. This \( m! \) satisfies,
m! is plainly a Quantity Theory solution: given a stable M, there is a stable value of money.

To illustrate this Quantity Theory solution with a simple example: let $c = 1$, $\beta = 0.5$ and,

$$U = \sum_{t=0}^{\infty} \beta^t [2c_i^{1/2} + 2h_i^{1/2}]$$

Then,

$$m![u'(c) + v'(m!)] + \beta u'(c)m! = 0$$

$$m! = 4 \quad \text{and} \quad P! = 1/4.$$

The equilibrium is recorded in Example 1.

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**Example 1: A Quantity Theory Equilibrium**

2. The invalidity of the speculative hyperinflations.
But will there not also be non-steady state solutions to (2)? The non-steady solutions of the difference equation can be analysed by defining,

\[ A_i \equiv m_i [u'(c) - v'(m_i)] \]

\[ B_i \equiv \beta u'(c) m_i \]

and rewriting the difference equation as

\[ A_i = B_{i+1} \]

B is easily plotted is a linear function of m.

Figure 1

A can also be plotted as a function of m. A is positive if \( v'(m_i) < u'(c) \); negative if \( v'(m_i) > u'(c) \); and zero if \( u'(c) = v'(m_i) \).
Notice that

\[ \frac{dA_m}{dm_i} = u'(c) - v'(m_i) - m_i v''(m_i) \]

Thus as long as \( A \) is positive (i.e. \( u'(c) > v'(m_i) \)) then \( \frac{dA_m}{dm_i} \) is positive. But as long as \( A \) is negative (i.e. \( u'(c) > v'(m_i) \)) then \( \frac{dA_m}{dm_i} \) could be positive, zero, or negative. Thus the \( A \) function could bottom out (\( \frac{dA_m}{dm_i} = 0 \)) with a sufficiently low value of \( m \), and begin to rise with a still lower value.
Indeed, $\frac{dA}{dm}$ could be so negative that as $m$ approaches zero, $A$ rises so that it approaches zero as $m$ approaches zero.\(^1\) This is a possibility which, as we shall see, Obstfeld and Rogoff attach great significance to.

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\(^1\) If $$U = \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^\alpha}{\alpha} + \frac{h_t^\beta}{\beta} \right]$$ then $$A_t = m_t [u'(c) - v'(m_t)] = m_t - m_t^{\beta^{-1}} = m_t - m_t^\beta$$

As long as $\beta$ is positive the limit of $A$ is zero. But if $\beta$ is negative then the limit of $A$ is negative infinity.$\beta$ may be negative, but $\beta < 0$ does imply the limit of $v(m_t)$ is negative infinity as $h$ approaches zero.
The dynamics can be analysed by plotting A and B together, as in Figure 5.
The previously noticed steady state, $m!$, is obviously at the intersection of these two schedules in Figure 5.

But we can now also analyse non-steady state paths. Consider some initial magnitude to of $m$, $m(0)$, less than $m!$. As $B_{r,j} = A_r$, $B$ in period 1 assumes the magnitude of $A$ in period 0. Thus the magnitude of $m$ implied by the magnitude of $B$ in period 1, $m(1)$ can read off the figure, in the way indicated in Figure 6.
Notice \( m(1) < m(0) \). In other words, a magnitude of the value of money, \( m(0) \), below the ‘Quantity Theory’ magnitude will be followed by a magnitude, \( m(1) \), still further below ‘Quantity Theory’ magnitude. This is the process of a ‘speculative hyperinflation’.

But will this process continue indefinitely without ever violating equilibrium, (as the thesis of ‘speculative hyperinflation’ contends)? Or will it ultimately produce disequilibrium? The key contention of this paper is that the ‘speculative hyperinflation solution’ ultimately will produce disequilibrium, and is, therefore, no solution at all. In the example above, it will produce disequilibrium in period 2.

Consider the solution the Figure 7 yields for \( m(2) \).
Why does $m(2)$ imply the violation of equilibrium? Because $m(2)$ implies $A$ is negative, and that implies

$$u'(c_t) - ν(m_t) < 0$$

which implies

$$-u'(c_t) + ν(h_t) > 0$$

And that violates equilibrium. For the perturbation of the consumer’ ‘plan’ instanced at the beginning

$$-u'(c_t) + ν(h_t) + \frac{u'(c_{t+1})}{1 + π_t}$$
is now always positive. The agent always wants to hold more money than they do. Thus the ‘speculative’ hyperinflation has lead to an excess demand for money.

It is not hard to see that all such speculative hyperinflation ‘solutions’ will end with an excess demand for money as long as

\[ \text{there exists some } h \text{ such that } v(h_x) > u'(c_x) \quad (3) \]

Can we expect (3) to hold? Yes. Clearly, the Inada condition – infinite marginal utility as the quantity of real balances approaches zero - is sufficient for (3). But, obviously, the Inada condition is not actually necessary.

### 3. The annihilation of the value of money.

The previous section has identified the Quantity Theory solution \((m_t = m! \text{ for all } t)\), and has exposed as delusive the alternative speculative hyperinflation ‘solution’

There is however another solution, apart from the Quantity Theory solution, and that solution Obstfeld and Rogoff successfully identified in their 1983 paper, that remains a genuine solution in spite of it being misidentified as a ‘hyperinflation solution’. What Obstfeld and Rogoff demonstrated is that it is possible that in the initial magnitude of money, \(m(0)\), is such that at some period in the future \(t\), \(m_t\) is such that,

\[ -u'(c) + v'(m_t) = 0 \quad (4) \]
The significant thing about (4) is that it does not violate optimisation over money demand, as it will satisfy the equimarginal condition,

\[-u'(c_t) + v'(h_t) + \frac{u'(c_{t+1})}{1 + \pi_t} = 0\]

as long as the rate of inflation is infinite; as long as the price level in the following period is infinite. To put the same point another way, if \(-u'(c) + v'(m_t) = 0\) the condition,

\[m_t[-u'(c) + v'(m_t)] + \beta u'(c)m_{t+1} = 0\]

can still be satisfied as long as,

\[m_{t+1} = 0\]

That is, by if money assuming a strictly zero value in the following and all subsequent periods.

The logic of the equilibrium characterised by (4) is that real balances are so low that the marginal utility of money is sufficiently high that it just matches the marginal utility consumption. And this level of real balances will be what we wish to hold as long as money is worthless the next period, because in this situation our sacrifice of unit of current consumption obtains us an extra unit of current real balances - but no
greater consumption in later periods - and, by assumption, the marginal unit of current consumption equals the marginal utility of current real balances.

The upshot is that one monetary equilibrium is a sequence of magnitudes of the value of money commencing at a value so low that \( u'(c) = v'(m) \), and becoming strictly zero in the following period. A numerical example of this equilibrium, using the assumptions of Example 1, is given below.

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Example 2 A Value Annihilating Monetary Equilibrium

There are, indeed, an infinity of sequences that will conclude in the pattern of Example 2. For there will always be some value of money in the period preceding the period in which \( u'(c) = v'(m) \) such that the inflation implied by that value is such that the supply of real money (at that value) equals the demand for real balances (at the inflation implied by that value). Given the assumptions used in example one, this value is \( \frac{1}{4 - 2\sqrt{3}} \). Thus another equilibrium sequence is,
Example 3. Another Value Annihilating Monetary Equilibrium

Such solutions we call Value Annihilating solutions.

Several points are worth registering:

- For a given a set of particular assumptions about the utility function etc, there is only such ‘value annihilating’ path. It is unique. By contrast, there are an infinity of ‘speculative hyperinflation paths’ given a set of particular assumptions about the utility function, etc. To put the point another way, almost no price levels will satisfy this ‘value annihilating’ equilibrium.

- The Value Annihilating solution exists under all utility functions we are considering.\(^2\)

\(^2\) Obstfeld and Rogoff restrict this solution to those utility functions that imply,

\[
\lim_{m \to 0} A_t \equiv m_t [u'(c) - v'(m_t)] \to 0
\]

It might be thought that the necessity of this restriction follows from the necessity of the optimising condition

\[
m_t [-u'(c) + v'(m_t)] + \beta u'(c)m_{t+1} = 0
\] (2)
The value annihilating solution does not describe a ‘hyperinflation’ as the word is typically used. A hyperinflation is an ‘extreme inflation’, an ‘out of control’ inflation. In standard usage, a hyperinflation does not denote a situation in which the value of money becomes strictly zero. Was there ever an inflationary episode in which the currency became strictly valueless?\(^3\)

But the real test of the Value Annihilating solution lies not in whether it may be called a ‘hyperinflation’ without violating existing usage; or whether it can help explain inflations that have been observed historically; but whether the solution has any claim to our attention as a possibility. This paper does not think so. Or, to be more precise, it has no more claim on our attention that the possibility that, upon awakening tomorrow, we will discover the value of the $US balances have zero real value. For if we think Value Annihilating solution – what Obstfeld and Rogoff miscall a Speculative Hyperinflation – is a significant possibility, then we are required to think money suddenly assuming a zero value tomorrow is a serious possibility.

But (2) need no longer hold when \(m = 0\). As explained in Section 4, the rationale for the optimisation condition (2) vanishes when \(m = 0\). (It might be argued by other paths that, under certain utility functions, optimisation is violated when \(m = 0\). See Footnote 4).

\(^3\) It is true that the hyperinflation have sometimes concluded with the introduction of a new currency (eg Germany in 1923). But this was not a situation where real money balances were zero. Germany of 1923 merely had a currency reform of the nature of the replacement in 1960 of the depreciated ‘old’ franc by the New Franc.
4. The zero value of money equilibrium.

The previous section has drawn attention to the fact that a certain value of money \( m = 1 \) in Example 2) can be an equilibrium as long as money has a zero value in the following period, \( m = 0 \). But can \( m = 0 \) itself be an equilibrium? If it cannot be an equilibrium, then even the Value Annihilating solution of section 3 is not an genuine solution. But a zero value of money \textit{can} be an equilibrium. Consider the economy we are analysing in this paper. Why cannot \( P = \infty \) be an equilibrium? If \( P = \infty \) then the maximisation of utility does not require,

\[
-u'(c_t) + v'(h_t) + \frac{u'(c_{t+1})}{1 + \pi_t} = 0
\]

It does not require this equimarginal condition, for the logic behind it has totally evaporated if \( P = \infty \). If \( P = \infty \), one does not ‘sacrifice of one unit of consumption to obtain one unit of real balances’: no amount of real balances can ever be obtained, by whatever means, since the value of money is zero.

Thus we believe that there is a third equilibrium in addition to the Quantity Theory solution, and the Value-Annihilating solution. It might be all the Value Annihilated solution. This simply \( m(0) = m(1) = m(2) = ... = 0 \). Money has zero value, now and forever. This is an equilibrium.

\[
\begin{array}{cccccc}
\text{t} & 0 & 1 & 2 & 3 & \ldots \\
\hline
\text{m} & 0 & 0 & 0 & 0 & \ldots \\
\end{array}
\]
Example 4: Value Annihilated Monetary Equilibrium

There is nothing to ‘stop’ this equilibrium. 4

Indeed, since bygones are bygones, there is nothing to stop this equilibrium suddenly occurring (as in Period 2 in Example 4).

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Example 5: From Quantity Theory to Value Annihilated Solutions

In every period, a zero magnitude of real balances is just as much an equilibrium as the Quantity Theory magnitude of real balances. 5 But is it an equilibrium that has any

4 Could the Value Annihilated solution can be ruled out by assuming that the total utility of money approaches negative infinity as m approaches zero (for example, in $U = \sum_{t=0}^{\infty} \beta^t [u(c_t) + \ln(h_t)]$)? This is, in effect, a claim of Obstfeld and Rogoff. Certainly, the notion of maximisation seems unintelligible in this circumstance; for under this circumstance utility is always infinitely negative no matter what decisions are made with respect to consumption. What can ‘maximisation’ be if utility is always the same (infinitely negative) no matter what choices one makes? So if we require an intelligible equilibria we can rule the Value Annihilated solution by assuming the total utility of money approaching negative infinity as m approaches zero. But, as Obstfeld and Rogoff observe, would we be willing to assume by the total utility of money approaches negative infinity as m approaches zero?
claim to our attention? Surely not. Historically, has money ever assumed a zero value? Certainly, new currencies have displaced old currencies; Euros replace Francs. But this is the change in the form of real balances, not the disappearance of real balances.

We conclude that the Value Annihilated solution, and the Value Annihilating solution, are a chimera. The Quantity Theory solution is the only (non-chimerical) solution to monetary equilibrium.

5 Of course, the sudden annihilation of the value of money would not an equilibrium if its occurrence (in Period 2 in the case of Example 4) was anticipated. Then the value of money in the preceding period (Period 1) would have to fall so low that marginal utility of real balances actually equalled he marginal utility of consumption (as there would be no possibility spending balances on consumption in Period 2). That is of course the Value- Annihilating solution. Thus The Value- Annihilating solution can be seen as the solution when the Value Annihilated solution is seen coming. The Value-Annihilating solution is just the long way of getting to the Value Annihilated solution.
References

