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Abstract We revisit the Heckscher-Ohlin-Samuelson model in the presence of labor market frictions à la Mortensen-Pissarides. Relaxing the assumption of the one-worker-one-firm matching rule, we show that the Stolper-Samuelson theorem and the Rybczynski theorem may not hold in specific circumstances. We also demonstrate that the Factor Price Equalization theorem is only valid for capital and unemployed labor across countries, but not for employed labor. In equilibrium, trade patterns are determined by countries’ factor endowments and relative factor intensities in sectors (independent of factor intensities in production). Finally, our results suggest an additional explanation for the “missing trade” phenomenon.

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1. Introduction

The aim of the paper is to examine how the main four trade theorems of the Heckscher-Ohlin-Samuelson model (HOS), namely the Stolper-Samuelson (1941) theorem, the Rybczynski (1955) theorem, the Factor Price Equalization theorem and the Heckscher-Ohlin-Samuelson theorem, fare in the context of a model where there exists frictional unemployment for labor. Our paper is not the first one to address this issue. Davidson et al. (1988) represent an early attempt to do so by incorporating labor market search frictions into a classical general equilibrium model. With the introduction of search unemployment, they show that the basic relationship between factor rewards and commodity prices may be different from that in a frictionless economy. Hosios (1990) specifies the necessary and sufficient conditions for obtaining constrained efficiency (i.e., the surplus sharing rules satisfy the well-known Hosios rule) and develops a simple general equilibrium model with search unemployment in parallel to Jones’ (1965) classical model. He shows that if search is constrained efficient then the Stolper-Samuelson and the Rybczynski effects are still present in the general equilibrium model with frictional unemployment. While Hosios (1990) shows that the Stolper-Samuelson theorem holds only for searching factors (i.e., unemployed workers and vacant firms), Davidson et al. (1999) and Davidson and Matusz (2004) further demonstrate that the real return to the matched factors (i.e., employed workers) is determined by a mix of the Stolper-Samuelson and Ricardo-Viner forces if bargaining between matched factors satisfies the Hosios rule. In addition, it is also shown that structural characteristics of the labor market such as job creation and job destruction rates can play a role in determining the pattern of trade.

The novelty of our paper is to introduce search unemployment into a general
equilibrium model while retaining the typical HOS feature of inter-sectoral
differences in factor intensities in production. We are able to do so because we
assume that the capital decision is independent of labor market frictions as in
Pissarides (2000): firms hire capital in a frictionless market only after successfully
hiring workers in a frictional market. Doing this allows for a model where factor
intensities in production are efficient and differ across sectors, which also makes
ranking differences possible between factor intensities in production and factor
intensities in sector such that the trade theorems may not hold.\textsuperscript{3} This is in contrast to
the literature whereby factor intensities in production are assumed to be the same
across sectors since search and matching between vacancy (capital) and worker forms
a one-to-one pair (Davidson et al., 1988, 1999; Davidson and Matusz, 2004; Hosios,
1990), from which it follows that if there are no search frictions, there will be
essentially neither trade nor its related Stolper-Samuelson and Rybczynski effects.
Although this assumption of equal factor intensities in production across sectors helps
highlight a new channel through which search frictions impact on trade, it effectively
shuts down the traditional channel through which the HOS model operates. In
particular, it is the cross-sector differences in factor intensities in production that
transmit the impact of changes in product prices (endowments) on factor rewards
(production and trade patterns) that lies at the heart of the HOS model.

We show that in a world with labor market frictions and search unemployment – both
of which may differ across sectors and countries\textsuperscript{4} – the four fundamental trade
theorems under the HOS framework are still valid in general, but have to be adjusted
quantitatively to a certain degree, and in some circumstances, they may not hold. We
demonstrate that employed labor’s return is tied to the return to capital, irrespective of
the sector to which it is attached, if labor market frictional costs are high. The return
to employed labor is a weighted sum of the return to unemployed labor and to capital
since workers cycle between employment and unemployment. With the existence of
labor market frictions and search unemployment, factor intensity in sector – which
accounts for both employed and unemployed labor – may be different from that in
production: namely, the sector producing capital- (labor-) intensive goods may
become relatively more labor- (capital-) intensive when this sector holds excessive
search unemployment. As a result, an increase in the endowment of labor (capital)
may lead to an increase in the output of either labor- or capital-intensive goods,
depending on the relative labor market frictional costs. Our model also shows that free
trade will equalize the factor price of capital and the expected lifetime income of
unemployed labor across countries but not the factor price of employed labor, and that
trade patterns are determined by countries’ factor endowments and relative factor
intensities in sector accounting for both employed and unemployed labors. In sum,
incorporating search unemployment into the HOS model sheds light on the
relationship between trade, employment and wage inequality which goes beyond what
the conventional trade theorems can explain with the assumption of frictionless labor
market. In particular, our model predicts richer trade patterns between countries with
similar endowments and provides an added explanation to the “missing trade”
phenomenon.

Our paper is related to the literature that discusses the impact of factor market
distortions on the pattern of trade, for example, Jones (1971), Magee (1973), Neary
(1978) and Fukushima (1985), among others. Our approach differs from this literature
in three ways. First, in our framework, each factor earns the value of its marginal
product such that the factor reward and its usage cost are the same. In contrast, the existing literature on factor market distortion assumes a differential between each factor reward and its usage cost or a differential of the reward to the same factor across sectors. Second, in our model there is no ranking difference between physical and value factor intensities; instead, a ranking difference between physical factor intensities in sectors and physical factor intensities in production arises from endogenous search unemployment. In contrast, the literature on market distortion relies on the notion of a ranking difference between physical and value factor intensities arising from exogenous distortions. Third, while the literature on market distortion also suggests that unemployment due to distortions in the economy may be a reason for traditional trade theory not to hold, our model differs substantially in that unemployment is endogenous and firmly based on the microeconomic foundations of search theory, which ensures that the equilibrium of our model is compatible with general equilibrium theory.

The structure of the paper is as follows. The next section develops a two-sector general equilibrium model incorporating search unemployment. Section III applies the model to examine how labor market frictions impact on the unit-cost function and revisits the four trade theorems. Section IV discusses the implications of our model in explaining the phenomenon of “missing trade”. The final section concludes.

2. A Simple Two-sector Search Model

2.1 The Economy

Consider an economy that produces two tradeable goods, \( Y_i = F(K_i, L_i), \ i = 1, 2, \) where \( Y_i \) is the output produced using capital \( K_i \) and labor \( L_i \). The production
function $F(.)$ is assumed to be increasing, concave, and homogenous of degree one in the inputs $(K_i, L_i)$. For simplicity, we assume there is no technological progress. Output per unit of employed labor, $y_i \equiv Y_i/L_i$, can then be written as, $y_i = f(k_i)$, where $k_i \equiv K_i/L_i$ is the capital-labor ratio in production for each sector, and the relative supply of the two goods is $Y_1/Y_2 = (f(k_1)L_1)/(f(k_2)L_2)$.

The economy features frictions associated with job search and matching and, therefore, there is search unemployment in the labor market. In such a frictional economy, it takes time and resources for a worker to find a job and for a firm to fill a vacancy in each sector which follows a constant-return-to-scale matching function, $m_i = m(v_i, N_i, u_i, N_i)$, where $m_i$ is the total number of matches, $v_i$ is the vacancy rate (defined as the number of vacancies divided by the total labor force) and $u_i$ is the search unemployment rate. $N_i$ is sectoral labor force, which includes both employed labor, $L_i$, and unemployed labor, $u_i N_i$, with $(1-u_i) N_i = L_i$. Let $\psi_i = v_i/u_i$ be the market tightness in sector $i$ (vacancy over unemployment). In a tight labor market with a high ratio of vacancies to unemployment, unemployed workers find it easy to locate new jobs. Thus, we have the rate at which vacant jobs become filled as $q(\psi_i) = m(1/\psi_i, 1)$ and the rate at which unemployed workers become employed as $\psi_i q(\psi_i) = m(1, \psi_i)$, where $q'(.) \leq 0$.

Consumers are assumed to derive utility $U$ from a composite good $Y$, $U(Y) = Y = ((Y_1^d)^a (Y_2^d)^{1-a})/(a^d(1 - a)^{1-a})$, where a superscript $d$ denotes demand for a good and $a \in (0,1)$. It is assumed that consumption is undertaken based on a
household with \( Z \) members and thus each worker can share his risk with other family members. This ensures the homogeneous preferences of individuals, and thus the relative demand for the two goods can be written as \( \left( Y_1^d / Y_2^d \right) = (\alpha \cdot P_2) / ((1 - \alpha) \cdot P_1) \), where \( P_i \) are the prices of good \( i \). It is easy to see that the relative demand for the two goods are a decreasing function of their relative prices, \( p = P_1 / P_2 \). The composite good \( Y \) is the numeraire good and the price index \( P = P_1^\alpha P_2^{1-\alpha} \) is the minimum expenditure such that \( Y = 1 \).

The aggregate endowment of labor in the economy is \( N = N_1 + N_2 \) and the total employment is \( L = L_1 + L_2 \). The total amount of capital is \( K \). Capital and unemployed labor are perfectly mobile between the two sectors while employed labor is not. Although unemployed workers can move freely to either sector, we assume that an unemployed worker has to decide which sector to move into and once the decision is made she will search in that sector only.\(^7\)

2.2 Production Decision

Following Pissarides (2000), a typical firm in each sector has jobs that are vacant and has to pay recruitment cost \( \gamma_i \) in terms of the numeraire good in order to fill a vacancy. During the process of hiring, workers arrive to vacant jobs at the rate \( q(\psi_i) \). When a firm and a worker meet and agree on an employment contract, a vacant job becomes occupied. The firm then rents capital for each hired worker, \( k_i \), and produces output, which is sold in competitive markets.
We consider the optimal decision of a typical firm in sector \( i \). Let \( V_i \) and \( J_i \) be the present-discounted value of expected profit for the firm from a vacant and occupied job respectively. \( V_i \) satisfies the Bellman equation, 
\[
    rV_i = -\gamma_i + q(\psi_i)(J_i - V_i).
\]
A job is an asset owned by the firm and is valued in a perfect capital market characterized by a risk-free interest rate, \( r \), which is equal to the exogenous discount factor. The asset value of a vacant job, \( rV_i \), is exactly equal to the rate of return on the asset: the cost of having a vacant position, \( \gamma_i \), plus the return of filling the job, \( J_i - V_i \), which will happen with probability \( q(\psi_i) \). In equilibrium, free entry and profit maximization ensure that the gains from job creation are always exhausted, so that jobs are created up to the point where \( V_i = 0 \), implying that 
\[
    J_i = \gamma_i / q(\psi_i).
\]

Since capital is costly and vacancies do not own capital, the firm will rent capital after the vacant job is filled (Pissarides 2000). As a vacant job is filled, the capital per hired worker rented by the firm becomes part of the value of the occupied job, which is now given by \( J_i + k_i \). Similar to the valuation of a vacant job, the asset value of an occupied job, \( r(J_i + k_i) \), satisfies the following Bellman equation,
\[
    r(J_i + k_i) = P_i f(k_i) - w_i - \eta_i J_i,
\]
where \( w_i \) and \( \eta_i \) are sector-specific wage rates and job destruction rates respectively. Since capital can be re-used by other firms, job destruction leads to the loss of \( J_i \) but not \( k_i \). Given the interest rate and wage rates, the firm’s first-order condition with respect to capital can be written as
\[
    P_i f'(k_i) = r, \tag{1}
\]
which has the standard interpretation whereby firms rent capital per worker \( k_i \) up to the point where the marginal product of capital is equal to the market rental rate, \( r \), as we assume that there is no friction in the capital market. Substituting (1) and the
equilibrium job creation condition \( J_i = \gamma_i/q(\psi_i) \) into the valuation function of an occupied job yields the equilibrium condition for the firm’s employment of labor:

\[
P_i f(k_i) - P_i k_i f'(k_i) = w_i + \frac{(r + \eta_i)\gamma_i}{q(\psi_i)}.
\] (2)

Equation (2) shows that the firm can post vacancies and once a vacancy is filled, the firm will hire the right amount of capital to the point where the benefit of hiring an additional worker, the marginal product of labor, is equal to the marginal cost – the recruitment-cost adjusted market wage. In cases where there is no recruitment cost (\( \gamma_i = 0 \)) or unemployed labor can find jobs instantaneously (\( q(\psi_i) = +\infty \)), the last term on the right hand side of (2) becomes zero and (2) reduces to the Euler equation for labor in a full-information, frictionless labor market.

2.3 Wage Determination and Employment

Workers (when unemployed) search for jobs in each sector and, once offered, have to make decisions to accept or reject the job offer. Again we illustrate a typical worker’s decision making in sector \( i \). Let \( U_i \) and \( E_i \) be the present-discounted value of the expected income stream of an unemployed and employed worker in each sector respectively. Given that workers meet job vacancies at the rate of \( \psi_i q(\psi_i) \), \( U_i \) satisfies the Bellman equation, \( rU_i = b + \psi_i q(\psi_i)(E_i - U_i) \), which states that the asset value of the unemployed worker’s human capital (or the permanent income that an unemployed worker expects to receive during search) is made up of the unemployment benefits, \( b \), measured in units of the numeraire good, and the expected capital gain from the change of employment state, \( \psi_i q(\psi_i)(E_i - U_i) \). Similarly, the asset value of an employed worker’s human capital satisfies the Bellman equation, \( rE_i = w_i + \eta_i(U_i - E_i) \), which says that the permanent income of an employed worker
is made up of the constant wage, \( w_i \), and the expected capital loss from the change of employment state, \( \eta_i(U_i - E_i) \). Combining the two Bellman equations, the permanent income of an unemployed and employed worker can thus be solved as

\[
ru_i = [(r + \gamma_i)b + \psi(q)w_i] / (r + \eta_i + \psi(q))
\]

and

\[
rE_i = [\eta_i b + (r + \psi(q))w_i] / (r + \eta_i + \psi(q)) , \text{ respectively.}
\]

As an occupied job yields returns that exceed the total expected returns of a searching firm and a searching worker, the pure economic rent needs to be distributed between the firm and the worker. A simple approach is to assume that the distribution process is defined by the Non-Cooperative Nash bargaining solution, and thus the representative worker’s wage in sector-\( i \) can be written as

\[
w_i = \text{arg max} \left\{ (E_i - U_i)\beta (J_i - V_i)^{1-\beta} \right\}, \text{ with threat points } U_i \text{ and } V_i \text{ for each job-worker pair and the worker’s bargaining power } \beta \in (0,1). \]

The solution to the first-order maximization problem satisfies \( E_i - U_i = \beta (J_i + E_i - V_i - U_i) \), which says that the worker receives his threat point \( U_i \), plus a share of the pure economic rent from the job match. Making use of the equilibrium condition \( V_i = 0 \) and \( J_i = \gamma_i / q(\psi_i) \), we have, after some manipulation

\[
w_i = (1 - \beta)b + \beta \gamma_i \psi_i + \beta [P(f(k_i) - rk_i)], \text{ or } w_i = ru_i + \frac{\beta(r + \eta_i)\gamma_i}{1 - \beta q(\psi_i)}
\]

(3)

where \( \gamma_i \psi_i \) is the average recruitment cost for each unemployed worker. The equilibrium unemployment in each sector will be constrained optimal if each worker’s bargaining power is equal to the matching elasticity with respect to unemployed workers given a constant-returns-to-scale matching function (Hosios, 1990).
Since job creation must be equal to job destruction in equilibrium, the unemployment rate in each sector can be written as

\[ u_i = \frac{\eta_i}{\eta_i + \psi_i q(\psi_i)}, \tag{4} \]

which shows that the unemployment rate in sector \( i \), \( u_i \), is increasing with respect to the job destruction rate \( \eta_i \) while decreasing with respect to market tightness \( \psi_i \).

### 2.4 Equilibrium

The factor market clearing conditions require that the total labor force in the economy exhausts employed and unemployed labor in both sectors:

\[ \frac{L_1}{(1-u_1)} + \frac{L_2}{(1-u_2)} = N, \tag{5} \]

and the economy’s total capital stock is given by

\[ k_1 \cdot L_1 + k_2 \cdot L_2 = K. \tag{6} \]

The no-arbitrage condition ensures that newly unemployed workers expect to receive the same lifetime income, \( \sigma \), in each sector. In equilibrium, we have \( rU_1 = rU_2 = \sigma \), which in turn links the market tightness in each sector with the relative recruitment cost \( \psi_1/\psi_2 = \gamma_2/\gamma_1 \). Intuitively, the higher the relative cost of recruitment in sector \( i \), the tighter the labor market in the same sector will be.

In equilibrium the goods markets clear. This requires that the relative demand and supply of the two goods are equal. From (5) and (6), we have the relative supply of the two goods

\[ \frac{Y_1}{Y_2} = \frac{[f(k_1)(1-u_1)(N(1-u_2)k_2-K)]/[f(k_2)(1-u_2)(K-N(1-u_1)k_1)]}{p}, \]

which is an increasing function of their relative prices \( p \). This, combined with the relative
demand for the two goods being a decreasing function of $p$, ensures that in equilibrium the goods markets clear so that the relative price $p^* = P_1^*/P_2^*$ is uniquely determined.

The general equilibrium of the two-sector model with unemployment is defined in a multi-dimension space ($p^*, k_i^*, \psi_i^*, \sigma^*, w_i^*, r^*, u_i^*, L_i^*$) that satisfies the optimization conditions for capital (1), the job creation condition (2), the wage equation (3), the flow equilibrium condition for labor (4), the factor market clearing conditions (5) and (6), the goods market clearing condition (defined above) as well as the no-arbitrage condition, $rU_1 = rU_2 = \sigma$. It can be solved as follows. Substituting (3) into (2), we obtain 

\[ (1 - \beta)[P_1f(k_i) - b] = P_1f(k_i)\gamma_i(f'(k_i) + \eta_i + \beta \psi_i q(\psi_i)) / q(\psi_i). \]

It follows that for a given $P_i^*$, $\psi_i$ is an increasing function of $k_i$ such that $\psi_i = \psi_i(k_i)$ (where $\psi_i'(k_i) > 0$) and thus $q'(\psi_i(k_i)) < 0$. Using (1) and (2) with the no arbitrage conditions for unemployed workers and capital, we have $P_1^* f'(k_i) = P_2^* f'(k_2) = r$ and

\[ P_1^* f(k_i) - P_1^* f'(k_i) - [P_1 f'(k_i) + \eta_i] \psi_i/q(\psi_i) = P_2^* f(k_2) - P_2^* f'(k_2) - [P_2 f'(k_2) + \eta_2] \psi_2/q(\psi_2) = \sigma. \]

Substituting $\psi_i = \psi_i(k_i)$ into the latter equation and making use of $P_1 f'(k_i) = P_2 f'(k_2) = r$, we can solve for $k_i^*$ with $\partial r / \partial k_i < 0$ and $\partial \sigma / \partial k_i > 0$ and thus $\psi_i^*$, $r^*$ and $\sigma^*$ can be derived respectively. We can then solve for $w_i^*$ from (3), $u_i^*$ from (4), and $L_i^*$ from (5) and (6).

We now introduce the following definitions.

**DEFINITION 1:** the “labor market frictional cost”, $\xi_i = [\beta \gamma_i] / [(1 - \beta) q(\psi_i)]$, is the cost associated with filling a vacant job with an unemployed worker. It is defined
as the product of a fixed recruitment cost, $\gamma_i$, and the average duration of a vacant job, $1/q(\psi_i)$, adjusted by the strength of the worker’s bargaining power.

Our definition of this labor market frictional cost deserves some explanation. It takes into consideration not only the conventional static cost – the fixed recruitment cost, $\gamma_i$, but also the benefits that a firm has to forgo if the vacant job is not to be filled during a certain period of time (the average duration of a vacant job). Obviously, the longer it takes to fill a vacant job, the higher the frictional cost to a firm would be, given a fixed recruitment cost. Given labor market frictional costs, unemployment is attached to each sector, which makes the factor intensity in each sector less than that in production – because the former includes unemployed labours in the intensity calculation.

**DEFINITION 2:** the “factor intensity in sector- $i$” is defined as

$$k_i' = K_i / N_i = k_i(1-u_i),$$

which differs from factor intensity in production $k_i$.

The existence of labor market frictions and unemployment create the potential for standard trade theorems not to hold. The following two assumptions are useful.

**ASSUMPTION 1:**

$$k_2 - k_1 \succ \xi_1 - \xi_2, \forall k_2 \succ k_1.$$

**ASSUMPTION 2:**

$$k_2' \succ k_1', \forall k_2 \succ k_1.$$

A sufficient condition for Assumption 1 is that the relative labor market frictional costs in the sector producing capital-intensive goods (sector 2) should be larger than that in the sector producing labor-intensive goods (sector 1). Since labor market frictions are negatively related to sector-attached unemployment, this implies that
unemployment in sector 2 relative to that in sector 1 should not be large enough to make the ranking difference between factor intensities in production inconsistent with factor intensities in sector (Assumption 2).

3. The Extended Fundamental Trade Theorems

3.1 Labor Market Friction and the Diversification Cone

As sectoral market tightness ($\psi_i$) is exogenous to a firm’s optimization for production due to the no-arbitrage condition for unemployed workers, the unit-cost function of each sector can be redefined, after some manipulation with (3) as:

$$
C_i(\sigma, r) = \min_{a_l, a_k \geq 0} \{ [(\sigma + \frac{\beta(r + \eta)}{(1-\beta)q_i})a_l + ra_{ik} | F_i(a_{il}, a_{ik}) \geq 1 \} \}
$$

where $C_i(\sigma, r)$ denotes the unit cost of production in sector-$i$, $a_{il}$ and $a_{ik}$ denote the employed labor and capital required for producing a unit of output, and $\sigma$ and $r$ denote the unemployed worker’s lifetime expected income and rental rate respectively, which are equal across sectors in equilibrium. Equation (7) shows that the introduction of search frictions modifies both the labor and capital usage terms as a change in the discount rate affects not only the cost of capital but also that of labor.

The effects of labor market frictions on factor intensity in production and sector are illustrated in Figure 1 where the horizontal axis denotes employed labor and the vertical axis denotes capital. The optimal factor intensity of production in sector-$i$ is determined by the tangency of its unit isoquant and iso-cost line. When there is no labor market friction, the factor price ratio is $w/r (= \sigma / r)$ so that tangent points A and
\(B\) determine the factor intensities in production, \(k_i\), which is the same as the factor intensities in sector \(k_i'\) with the cone of diversification being \(k_2ok_1\) (for \(k_1 < k_2\)).

However, if there are labor market frictions, both \(k_1\) and \(k_2\) will shift up in equilibrium to \(k_1'\) and \(k_2'\) since labor market frictions increase the marginal cost for using labor in production so that the wage-capital ratio, \(w_i/r\), may be different across sectors and is now higher than \(\sigma/r\).\(^{11}\) Firms then re-optimize their equilibrium factor intensities, shifting up factor intensities in production so that the new cone of diversification becomes \(k_2'ok_1'\) while the new equilibrium factor intensities in sector \((k_i'')\) may remain the same (illustrated) or different (not illustrated) from the initial \(k_i'\), depending on the sectoral unemployment. There is now a possibility that the ranking of factor intensities in production \((k_i)\) may be different from that in sector \((k_i')\), leading to richer results than what the standard trade theorems predict.

### 3.2 The Extended Stolper-Samuelson and Factor Price Equalization Theorems

For \(P_i\), the optimal condition requires unit output value equal to unit cost, which can be written as

\[
P_i = [\sigma + \frac{\beta(r + \eta_i)}{1 - \beta}q(\psi_i)]a_{il} + ra_{ik}. \quad (8)
\]

Totally differentiating (8) yields,

\[
\hat{P}_i = \theta_{il}\sigma + (1 + \kappa)\theta_{ik}\hat{r}, \quad (9)
\]

where \(\theta_{il} = a_{il}\sigma / P_i\), \(\theta_{ik} = a_{ik}r / P_i\), \(\theta_{il} + \theta_{ik} < 1\), \(\kappa = (a_{il} \cdot \beta \cdot \gamma_i) / (a_{ik} \cdot (1 - \beta) \cdot q(\psi_i)) = \xi_i / k_i\), and a circumflex above a variable represents a percentage change in that variable.
Equation (9) can be written in matrix form as

\[
\begin{pmatrix}
\dot{P}_1 \\
\dot{P}_2
\end{pmatrix} = \begin{pmatrix}
\theta_{1L} & (1 + \kappa_1)\theta_{1K} \\
\theta_{2L} & (1 + \kappa_2)\theta_{2K}
\end{pmatrix}
\begin{pmatrix}
\sigma \\
\dot{r}
\end{pmatrix}.
\]  

(9A)

It follows that

\[
\sigma = \frac{(1 + \kappa_2)\theta_{2K}\dot{P}_1 - (1 + \kappa_1)\theta_{1K}\dot{P}_2}{|\sigma|} 
\]

(10)

\[
\dot{r} = -\frac{\theta_{2L}\dot{P}_1 + \theta_{1L}\dot{P}_2}{|\sigma|} 
\]

(11)

where \( |\sigma| = (1 + \kappa_2)\theta_{1L}\theta_{2K} - (1 + \kappa_1)\theta_{1K}\theta_{2L} \).

**PROPOSITION 1 (the extended Stolper-Samuelson theorem):** In a world with labor market frictions and unemployment, an increase in the relative price of goods in the labor- (capital-) intensive sector increases the real return to unemployed labor (capital) and reduces the real return to capital (unemployed labor) if and only if Assumption 1 holds. The effect on the real return to employed labor is ambiguous as it is a weighted sum of the real return to unemployed labor and to capital.

**Proof:** Since \((1 + \kappa_2)\theta_{1L}\theta_{2K} = (k_2 + \hat{\xi}_2)a_{1L}a_{2L} \cdot \sigma \cdot r/(P_1P_2)\) and

\[(1 + \kappa_1)\theta_{1K}\theta_{2L} = (k_1 + \hat{\xi}_1)a_{1L}a_{2L} \sigma \cdot r/(P_1P_2)\]

for \(k_2 > k_1\), we have

\[(1 + \kappa_2)\theta_{1L}\theta_{2K} > (1 + \kappa_1)\theta_{1K}\theta_{2L}\] provided that Assumption 1 holds. It follows immediately that \(\sigma\) is increasing and \(\dot{r}\) is decreasing with respect to \(\dot{P}_1\) (holding \(P_2\) constant) \(\sigma = (1 + \kappa_2)\theta_{2K}\dot{P}_1/[(1 + \kappa_1)\theta_{1L}\theta_{2K} - (1 + \kappa_1)\theta_{1K}\theta_{2L}] > 0\) and

\[
\dot{r} = -\theta_{2L}\dot{P}_1/[(1 + \kappa_2)\theta_{1L}\theta_{2K} - (1 + \kappa_1)\theta_{1K}\theta_{2L}] < 0.\] The return to employed labor is

\[r_E = \sigma + \hat{\xi}_i r.\]  

QED
Proposition 1 suggests that in an economy with frictional labor markets, there are two forces affecting the Stolper-Samuelson relationship. Apart from the conventional Stolper-Samuelson channel whereby the relative factor intensities of sectoral production play a central role, the level of sectoral labor market frictional costs also has an impact. The Stolper-Samuelson effect is reinforced by the labor market frictional effect when labor market frictional costs in the sector producing capital-intensive goods (say sector 2) is higher than that in the sector producing labor-intensive goods (sector 1) ($\xi_2 > \xi_1$). This is a sufficient condition for Assumption 1 to hold, which implies tighter labor market in sector 1 (relatively more vacancies than unemployment, i.e., $\psi_1 > \psi_2$). In such a case, an increase in $P_1$ encourages expansion of sector 1 which opens more vacancies. To facilitate the expansion of sector 1, more unemployed workers will need to be released from sector 2. The newly unemployed factors (labor and capital) will be attracted to sector 1 where jobs are easy to find while it takes a longer time to fill a vacancy, leading to a higher expected income for unemployed workers (\(\bar{\sigma}\)) and a lower return to capital (\(r\)). Moreover, the return to capital would be lower due to the Stolper-Samuelson effect: the contraction of sector 2 releases more capital than is required by sector 1, driving down \(r\). Thus, the Stolper-Samuelson theorem is still valid for the reward to capital and unemployed labor.

However, if Assumption 1 does not hold, the labor market frictional effect may dominate the Stolper-Samuelson effect. A necessary condition for Assumption 1 not to hold is for $\xi_1 < \xi_2$, which implies tighter labor market in sector 2 ($\psi_1 < \psi_2$). In such a case, an increase in $P_1$ encourages the expansion of sector 1. On one hand,
since there are more workers relative to available jobs in sector 1, it is difficult for workers to find a match while it is easy for vacancy to be filled. This leads to a lower expected income for unemployed labor and a higher return to capital \((r)\). On the other hand, the Stolper-Samuelson effect implies that the contraction of the capital-intensive sector releases more capital than is required by sector 1, which tends to drive down \(r\). When the labor market friction effect dominates, the Stolper-Samuelson theorem may not hold.

The effect on the real return to employed labor, \(rE_i = \sigma + \xi r\), is ambiguous. The return to employed labor is a weighted sum of the real returns to unemployed labor and to capital. The weight given to capital is the sectoral labor market frictional cost. Intuitively, since workers cycle between employment and unemployment and when employed they become attached to capital. As such, employed labor’s real return has to reflect both situations and is jointly determined by the expected income of unemployed labor and the compensation for the possibility of their being employed. Both of these are related to sector-specific labor market frictions as well as the return to capital. For example, an increase in the interest rate not only drives up the discounted opportunity costs of the unemployed and their expected income but also allows newly employed labor to bid for higher wages since the expected value of vacancy increases at higher interest rates.

If the labor market frictional cost is low, the first term of the equation for \(rE_i\) will dominate. The real return to employed labor will be similar to that of unemployed labor. However, if the labor market frictional cost is high, the second term of the equation for \(rE_i\) will dominate. In both cases, the extent of the labor market frictional
costs determines the weighting of the workers’ income on the return to capital. Thus, our model predicts that employed workers’ permanent income is tied to the return to capital rather than the sector to which they are attached. This prediction of “sector irrelevance” is different from the prediction of Davidson et al. (1999) whereby the return to employed workers is dependent on the sector to which they are attached.\textsuperscript{14}

What does the empirical evidence tell us about sectoral preferences with regard to trade liberalization in industries with high labor market frictional costs? Recent work by Magee et al. (2005, P.167) suggests that “there is no significant difference between capital and labor groups (either in exporting and import-competing industries) in their support for representatives voting in favour of trade liberalization” which is consistent with our prediction of sector irrelevance.\textsuperscript{15}

Applying the above analysis to a 2x2 case, we can also reformulate the Factor Price Equalization (FPE) theorem in a world with labor market frictions and unemployment as follows.

**PROPOSITION 2 (the extended Factor Price Equalization theorem):** In a 2x2 world with labor market frictions and unemployment, free trade equalizes the factor price of capital and the expected lifetime income of unemployed labor across countries, but not the return to employed labor.

Proof: Assume that free trade equalizes commodity prices ($P^W_i$) across countries. Given that firms earn zero profits and technologies are the same across countries ($C_i(\sigma, r) = P^W_i$), the factor prices of capital and unemployed labor ($r^+$ and $\sigma^+$) in
equilibrium can then be derived from (8). Define the Jacobian matrix of \( C_i(\sigma, r) \) as 

\[ A'(\sigma, r) \] 

where, by Shephard’s lemma, a representative element of \( A'(\sigma, r) \) is the input-output coefficient adjusted by labor market frictional costs. Thus (7) becomes

\[
\begin{pmatrix}
\frac{dC_1(\sigma, r)}{dr} & \frac{dC_2(\sigma, r)}{dr}
\end{pmatrix}
\begin{pmatrix}
\sigma - \sigma^* \\
r - r^*
\end{pmatrix}
\begin{pmatrix}
A_{11} & A_{12} + A_{13} \xi_1 \\
A_{21} & A_{22} + A_{23} \xi_2
\end{pmatrix}.
\]

(12)

Thus, the FPE theorem will hold only for capital and unemployed labor when (12) can be solved uniquely or when the unit-cost functions \( C_i(\sigma, r) \) are globally invertible. Moreover, the Gale-Nikaido condition (Gale and Nikaido, 1965) states that \( C_i(\sigma, r) \) is globally invertible if all principal minors of \( A'(\sigma, r) \) are positive. In our 2x2 case, we have the Gale-Nikaido condition is satisfied if and only if \( k_1 - k_2 = \xi_2 - \xi_1 \). \( QED \)

Proposition 2 suggests that the labor market friction cost affects the FPE theorem through two channels. First, in the absence of production specialization, labor market frictional costs affects the factor-price-equalization relationship for the return to capital and unemployed labor through changing the relative factor intensities across sectors. The factor price of capital and the expected lifetime income of unemployed labor will be equalized across countries only when sectors are different in their factor intensities. However, the reward to employed labor will not be equalized across countries as it is a weighted sum of the return to capital and unemployed labor adjusted by labor market frictional costs. Thus, our model suggests that the FPE relationship generally does not hold for employed labor, which is consistent with the empirical literature. Note that if factor intensities in sectors are equalized throughout the economy due to labor market frictional costs (or \( k_1 - k_2 = \xi_2 - \xi_1 \)), the FPE theorem may not hold.
Second, if we consider the case of production specialization, the labor market frictional cost affects the FPE relationship by changing the probability of production specialization, since labor market frictional costs induce firms to use more capital to substitute for labor in production. As there exist differences between factor intensities in production and in sector due to different sector-specific labor market frictional costs, production specialization will be determined only by factor intensities in sector. In that case, the FPE theorem may not hold for capital and unemployed labor but may hold for employed labor.

3.3 The Extended Rybczynski and Heckscher-Ohlin-Samuelson Theorems

From (5) and (6), the market clearing conditions can be re-arranged as

\[ \frac{a_{1L}Y_1}{1-u_1} + \frac{a_{2L}Y_2}{1-u_2} = N, \]  

(13)

\[ a_{1K}Y_1 + a_{2K}Y_2 = K. \]  

(14)

Let \( \varepsilon = N_1/N \). Totally differentiating (13) and (14) yields,

\[(1 + \pi_1)\lambda_{1L}\hat{Y}_1 + (1 + \pi_2)\lambda_{2L}\hat{Y}_2 = \hat{N} \]

(15)

\[ \lambda_{1K}\hat{Y}_1 + \lambda_{2K}\hat{Y}_2 = \hat{K} \]  

(16)

where \( \lambda_{1L} = a_{1L}Y_1/L \), \( \lambda_{1K} = a_{1K}Y_1/K \) and \( \pi_i = (u_i - u_2)(1 - \varepsilon)/(1-u_i) \), \( \pi_2 = (u_2 - u_1)\varepsilon/(1-u_2) \). In matrix form, (15) and (16) can be written as

\[ \begin{pmatrix} \hat{N} \\ \hat{K} \end{pmatrix} = \begin{pmatrix} (1 + \pi_1)\lambda_{1L} & (1 + \pi_2)\lambda_{2L} \\ \lambda_{1K} & \lambda_{2K} \end{pmatrix} \begin{pmatrix} \hat{Y}_1 \\ \hat{Y}_2 \end{pmatrix}. \]  

(16A)

It follows that

\[ \hat{Y}_1 = \frac{\lambda_{2K}\hat{N} - (1 + \pi_2)\lambda_{1K}\hat{K}}{\lambda}, \]  

(17)
\[
\hat{Y}_2 = -\frac{\lambda_k N + (1 + \pi_1)\lambda_{1L} \hat{K}}{\lambda},
\]
where \( |\lambda| = (1 + \pi_1)\lambda_{1L} \lambda_{2K} - (1 + \pi_2)\lambda_{2L} \lambda_{1K} \).

PROPOSITION 3: (the extended Rybczynski theorem): In a world with labor market frictions and unemployment, an increase in the endowment of labor (capital), holding product prices constant, leads to an increase in the output of goods in the labor- (capital-) intensive sector and a decline in the output of the other good if and only if Assumption 2 holds.

Proof: Since \((1 + \pi_1)\lambda_{1L} \lambda_{2K} = (1 - u_2 + (u_2 - u_1)\epsilon) \cdot a_{1L} Y_1 \cdot a_{2K} Y_2 / [(1 - u_1) \cdot L \cdot K]\) and \((1 + \pi_2)\lambda_{2L} \lambda_{1K} = (1 - u_2 + (u_2 - u_1)\epsilon) \cdot a_{2L} Y_2 \cdot a_{1K} Y_1 / [(1 - u_2) \cdot L \cdot K]\), we have

\[(1 + \pi_1)\lambda_{1L} \lambda_{2K} - (1 + \pi_2)\lambda_{2L} \lambda_{1K} > 0\]

provided that Assumption 2 holds.\(^{17}\) It follows immediately that \(\hat{Y}_1\) is increasing with respect to \(\hat{N}\) while \(\hat{Y}_2\) is decreasing with respect to \(\hat{N}\) (holding \(K\) constant), i.e., \(\hat{Y}_1 = \lambda_{2K} \hat{N} / [(1 + \pi_1)\lambda_{1L} \lambda_{2K} - (1 + \pi_2)\lambda_{2L} \lambda_{1K}] > 0\) and \(\hat{Y}_2 = -\lambda_{1K} \hat{N} / [(1 + \pi_1)\lambda_{1L} \lambda_{2K} - (1 + \pi_2)\lambda_{2L} \lambda_{1K}] < 0\). \(QED\)

Similar to the extended Stolper-Samuelson theorem, we continue to assume that sector 1 produces labor-intensive goods while sector 2 produces capital-intensive goods. Thus, Proposition 3 suggests that the response of sectoral output to changes in relative supplies of labor and capital now depends on the labor market frictional costs, in additional to the conventional channel – the relative factor intensities in production. Our framework allows the interplay of inter-sectoral difference of factor intensities in production and inter-sectoral difference of sector-attached unemployment arising
from labor market frictions. As a result, the relative factor intensities in sectors, \( k'_i = k_i(1-u_i) \), accounting for both employed and unemployed labors will be different from those in production, \( k_i \).

If Assumption 2 holds, i.e., the capital-labor ratio in sector 2 is higher than that in sector 1, the ranking of factor intensities in sector is consistent with that in production so that the standard Rybczynski theorem still holds. Thus, an increase in the endowment of labor (capital) will lead to an increase in the output of labor- (capital-) intensive goods. However, when Assumption 2 does not hold, the capital-labor ratio in sector 2 is lower than that in sector 1. The sector producing capital-intensive goods (sector 2) may become relatively more labor-intensive while the sector producing labor-intensive goods (sector 1) may become relatively more capital-intensive. As such, an increase in the endowment of labor (capital) will be absorbed by sector 2 (sector 1), which in fact produces capital- (labor-) intensive goods. This leads to an increase in the output of capital- (labor-) intensive goods, generating anti-Rybczynski effects.

Finally, extending the above discussion to analyze the impact of capital (labor) inflow on unemployment, we may find that unemployment may increase with the inflow of additional capital as long as the expanded capital- (labor-) intensive sector have relatively more sector-attached unemployment than the other sector.

Applying the above analysis to a 2x2 case, we can also reformulate the HOS theorem in a world with labor market frictions and unemployment as follows:
PROPOSITION 4 (the extended Heckscher-Ohlin-Samuelson theorem): In a world with labor market frictions and unemployment, a country will export the good that uses intensively its abundant factor, adjusted by labor market frictional costs. Trade patterns may be reversed if differences in labor market frictional costs across sectors lead to differences between the ranking of factor intensities in production and in sector. Under such a circumstance, a capital-abundant country may export labor-intensive goods while a labor-abundant country may export capital-intensive goods.

Proof: Follows immediately from Proposition 3. QED

Proposition 4 suggests that labor market frictional costs and unemployment play an important role in determining the comparative advantage of a country through affecting its input-output matrix and factor intensities in production and in sector. In our two-sector search model, the difference in the relative price of goods across countries is not only determined by the relative factor abundance and therefore the difference in the relative factor intensities in production across sectors, but also affected by sectoral unemployment. In fact, it is this difference in sectoral unemployment (which may be regarded as a necessary input) across sectors that leads to the change in the real user costs of labor and capital and thus different relative price of goods in autarky.

Two important observations are worth noting. First, if labor market frictional costs are the same across sectors within a country, a country with relative high (low) labor market frictional costs tends to export capital- (labor-) intensive goods. In particular, trade may take place between countries with identical factor endowments if their labor
market frictional costs are different. This result is consistent with the literature on trade and unemployment (see for example, Davidson et al. 1988, 1999, 2004 and Hosios 1990). Second, if labor market frictional costs are different across sectors within a country, trade across countries may display complex patterns as it reflects differences of factor intensities in sectors, which in turn are determined by both factor endowments and labor market frictional costs. Consequently, it will be difficult to predict the pattern of trade between countries with similar factor endowments but different labor market frictional costs and sector-attached unemployment across sectors. As an example, if the relative labor market frictions across sectors reverse the ranking of relative factor intensities in production in comparison with that in sector, a capital-abundant country may export labor-intensive goods and vice versa. This result is novel as it has not been suggested in the existing literature. The result also has important implications for the empirical literature as an additional explanation for the phenomenon of “missing trade”.

4. Implications for “Missing Trade”

The poor performance of the Heckscher-Ohlin model in predicting measured trade has been coined “missing trade” (Trefler, 1995). Various attempts have been made to explain this mystery. Some have focused on goods trade and its relationship to factor abundance (Leamer, 1984) and the factor content of trade itself (Bowen et al., 1987), while others allow for differences in cross-country productivity (Trefler, 1993). Recently, modifications have been suggested by Trefler (1995) and Davis and Weinstein (2001) to allow for home bias in consumption, non-traded goods, and models without factor price equalization. Still, there remains a huge gap between theory and reality (Estevadeordal and Taylor, 2002). In particular “why endowment
similarity is associated with (more) poor prediction (of the pure theory)?” (Trefler, 1995; P.1044). Our discussion so far suggests a potential explanation: cross-country differences in sector-attached unemployment arising from labor market frictions. We now show how empirical models should be adjusted in the presence of labor market frictions.

Let \( n \) index country and \( A'_n(W_n) \) be the technology matrix adjusted by labor market frictions, which is a function of factor prices \( W_n \) (where \( W_n \) is a \( 2 \times 1 \) vector of factor prices for unemployed labor and capital).\(^19\) By the factor market clearing condition, we have \( A'_nT_n = A'_nQ_n - A'_nC_n \) where \( T_n, Q_n, C_n \) are vectors of net exports, output and consumption.\(^20\)

First, suppose that there is no international technology difference so that the extended Factor Price Equalization theorem holds, we have \( A'_n(W_n) = A' \). Let \( \Lambda_n \) and \( \Theta_n \) be a \( 2 \times 1 \) vector of the employment of factors in production and factor endowments respectively and

\[
Z_n = \begin{pmatrix}
(1-u^u_i)e + (1-u^s_i)(1-e) & 0 \\
0 & 1
\end{pmatrix}
\]

be a matrix of factor employment rates (where \( u^u_i \) is sector \( i \)'s attached unemployment rates \( (i = 1, 2) \) and \( e = N_i/N \)), we have the relationship between factor employment in production and factor endowment in country \( n \) as \( \Lambda_n = Z_n \Theta_n \).

Similarly, we have \( \Lambda_w = Z_w \Theta_w \) where \( \Lambda_w \) and \( \Theta_w = \sum_j \Theta_j \) are \( 2 \times 1 \) vectors of factor employment in production and factor endowments respectively in the world.

The adding-up condition (or accounting identity) implies factors (both employed and
unemployed factors) absorbed in sector $A_n', Q_n$ should be equal to a country’s endowment, $A_n', Q_n = \Theta_n = Z^{-1}_n \Lambda_n$ and similarly factors (both employed and unemployed factors) absorbed in consumption $A_n', C_n = s_n \Theta_w = s_n Z^{-1}_W \Lambda_w$ since $A_n', C_n = s_n A_n' Q_w$ where $s_n$ is country $n$’s share of world GDP if trade is balanced. Thus, a test of “missing trade”, after taking into account of labor market frictions, will be

$$F_n = Z^{-1}_n \Lambda_n - s_n \sum_j Z^{-1}_j \Lambda_j$$

(20)

where $F_n = A_n'T_n$ is a $2\times1$ vector of factor content in trade flows and $Z^{-1}_w \Lambda_w = \sum_j Z^{-1}_j \Lambda_j$ is the sum of factor endowments of all countries. In the presence of labor market frictions, if a country is abundant in a factor $(Z^{-1}_n \Lambda_n > s_n \sum_j Z^{-1}_j \Lambda_j)$ then it exports the services of that factor on net. However, the degree of factor abundance now has to take into account unemployment and the factor content of trade has to adjust for labor market frictional costs.

If there are international technological differences in production so that the extended FPE theorem does not hold, we have, following Trefler (1995), $\Theta_n^* = \vartheta_n \Theta_n$ where $\Theta_n^*$ is the factor endowment adjusted by productivity vector $\vartheta_n$. The factor price vector is now $W_n^* = W_n / \vartheta_n$. The empirical tests will then follow

$$F_n^* = \vartheta_n Z^{-1}_n \Lambda_n - s_n \sum_j \vartheta_j Z^{-1}_j \Lambda_j \text{ and } W_n^* = W_j^*$$

(21)

where $F_n^* = \vartheta_n A_n'T_n$ is a $2\times1$ vector of factor content in trade flows. Equation (21) is isomorphic to that in Trefler (1995) but encompasses the case of factor market frictional costs and unemployment.
Equations (20) and (21) suggest that in the presence of labor market frictions and unemployment, adjustment of cross-country and cross-sector differences in unemployment (and related factor rewards) may be necessary to further uncover the mystery of “missing trade”. The reason for the adjustment comes from the fact that when unemployment becomes a type of necessary input in production, the factor content in trade and that in production should contain not only employed labor and capital in each sector but also unemployed labor attached to that sector. Since the existence of labor market frictions and sectoral unemployment may reverse the relative ranking of factor intensities in sector compared with that in production across countries, it is possible that this ranking difference may provide an explanation as to why existing empirical studies on factor content of trade fail to perform well for the sign tests among the South-South countries or the North-North countries and for the rank test among the South-North countries. Moreover, when the FPE theorem does not hold due to technological differences in production or relative labor market frictions, adjustments should also be made to the reward to factors. However, differing from Trefler (1993, 1995) and Davis and Weinstein (2001), those adjustments should be made to take into account the return to (or the expected income of) unemployed labor rather than employed worker’s wage and labor market friction costs should be incorporated in the estimation.

5. Conclusion

This paper incorporates search-related labor market frictions into the classical HOS framework. We ask whether the classical trade theorems still hold in such an environment. We show that the existence of labor market frictions leads to the divergence of a country’s factor intensities in production from factor intensities in
sector. Although the classical trade theorems are still valid in general, the existence of unemployment may change the fundamental relationships between factor prices and product prices, between output and factor endowments, and between the pattern of trade and factor endowments. When such changes occur, the classical trade theorems have to be modified quantitatively to a certain degree for them to hold. However, the classical theorems may not hold if such changes lead to differences in ranking between factor intensities in production and in sector.

In contrast to the literature where the return to employed labor is dependent on the sector to which they are attached, we show that employed labor’s return is tied to the return to capital irrespective of the sector they are attached to, as long as labor market frictional cost is relatively high, and that an increase in the endowment of labor or capital may lead to an increase in the output of either labor or capital-intensive goods, depending on the relative labor market cost across sectors. With labor market frictions, free trade will equalize factor prices of capital and the expected lifetime income of unemployed labor across countries but not the factor price of employed labor. Incorporating search unemployment into the HOS model sheds light on the relationship between trade, employment and wage inequality which goes beyond what the conventional trade theorems can explain with the assumption of frictionless labor markets. In particular, our model predicts richer trade patterns between countries with similar endowments and provides an added explanation to the “missing trade” phenomenon. We leave the empirical tests for future research.
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Figure 1 Cone of Diversification with/without Labor Market Frictions
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1 The literature on labor market search dated back to the 1970s (McCall, 1970; Diamond, 1982; Mortensen, 1982 and Pissarides, 1985).

2 For more recent works in this field, Cuñat and Melitz (2007) construct a model linking volatility and labor market flexibility with trade pattern and Helpman and Itskhoki (2007) use a monopoly competition model with search unemployment in one sector to explore the relationship between firms’ productivity, trade and employment.

3 In our model, the Hosios rule is neither a sufficient nor a necessary condition for efficient production and in deriving the HOS results. It is only a necessary condition for the efficiency in sector-attached unemployment. In contrast, Hosios (1990) and Davidson et al. (1999) require the Hosios rule as a necessary condition to derive the Stolper-Samuelson results as their assumption of Leontief production prevents firms from choosing optimal factor mix in production and therefore efficient production has to come from search and matching.

4 Botero et al. (2004) show that there are substantial differences across countries in labor market frictions.

5 In our model, both factor markets and goods market are perfectly competitive and thus, there will be no economic rents for each factor.

6 For simplicity, we use unemployment to refer to search unemployment thereafter.
We offer two explanations for the assumption that an unemployed labor cannot search in both sectors simultaneously. One possible reason is that perhaps there exists some learning-by-searching: once one has her first interview, one learns something about this sector that might be helpful for the next job talk. If these learning economies were large enough, it would induce her to stick to one sector to search. We thank one referee for suggesting this to us. Another possible reason is that there is a fixed cost in searching such as training, collecting sector-specific information etc. In the literature, Hosios (1990) makes similar assumption: “… any unattached agent in sector i can locate a trading partner in i only by participating in i’s matching process.” (P.329). Davidson et al. (1999) assume that “when unemployed (idle), workers (capital) must choose a sector in which to seek a job (rental opportunity)” (P.274).

8 See Appendix A for detailed derivation. All Appendixes (A to D) are available at http://myweb.polyu.edu.hk/~afxxu/appendix-ss.pdf

9 See Appendix B for a detailed derivation.

10 In fact, sectoral market tightness is exogenous to firms’ optimization process for production, but endogenous to labor market frictions.

11 Firms need to pay not only for compensating workers’ expected income including their wage and a premium for possible unemployment but also for their recruitment costs. The factor price ratio is now

\[ w_i / r = (\sigma + \beta (r + \eta) \gamma_i / [(1 - \beta)q(\psi_i)]) / r, \]

which is higher than \( \sigma / r \).

12 See Appendix C for detailed derivations.

13 Given that \( \xi = \beta \gamma_i / [(1 - \beta)q(\psi_i)] \), we have \( \beta \gamma_i / [(1 - \beta)q(\psi_i)] < \beta \gamma_2 / [(1 - \beta)q(\psi_2)] \) when \( \xi_1 > \xi_2 \). Since \( \gamma_1 = \gamma_2 \), it is easy to show that this condition implies \( q(\psi_i) < q(\psi_2) \). When labor market frictions are higher in sector 1, the probability of moving into employment for the unemployed in the sector is relatively low. Since \( q(\psi_i) \) is an increasing function of \( \psi_i \), we have that \( \psi_2 < \psi_1 \). In other words, the market tightness (in equilibrium) in each sector is negatively correlated to labor market friction costs in that sector.

14 The differences may arise from different modelling assumptions. In Davidson et al. (1999), labor and capital form matches and therefore both experience sectoral attachment. In contrast, capital is not sector-specific in our model. Note that our assumption of low (high) labor market frictional costs is
corresponding to the high (low) turn-over industries as defined in Davidson et al. (1999). We thank one referee for pointing this out to us.

15 Although there is a marginally significant (at 10 percent level of significance) difference between exporting and importing industries using the GATT vote, which Magee et al. (2005) interpret as being supportive to their sector-specific dependence prediction rather than our prediction of sector irrelevance.

16 See, for example, Davis and Weinstein (2001).

17 See Appendix C for detailed derivation.

18 In Hosios (1990), factor intensities in production are assumed to be the same across sectors since search between vacancy (capital) and labor follows the one-to-one rule. In effect, the only difference is the sectoral labor-unemployment intensities. Consequently, Hosios (1990) predicts that an increase in the endowment of labor, for example, will be absorbed by the sector whose matching process uses labor relatively intensively, generating a Rybczynski-type effect.

19 It can be easily extended to the case of multiple factors and sectors.

20 See Appendix D for detailed derivations.

21 For empirical purposes, the sector-specific unemployment rate is hard to obtain but wages and rewards to unemployed labor can be used as a substitute.

22 Equation (20) is the typical equation in the estimation of “missing trade” (Bowen et al., 1987 and Trefler, 1993) except that we adjust for the presence of labor market frictional costs on the LHS and search unemployment on the RHS of the equation.